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MULTIPHASE FLUID MOTION IN A PIPE OF VARIABLE CROSS-SECTION WITH HYDRAULIC RESISTANCE

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Abstract - Mechanics of fluid-deformable tube system in conjunction with modern methods finds application in mathematical modeling of dynamic processes of blood circulation in vessels and organisms. The study of regularities of interaction of fluids with the body, the surface of which they wet in the process of flow, is carried out for various fluid models (viscous fluid, non-viscous fluid, vapor-liquid mixture, fluid with bubbles, fluid with solid particles etc.). On the other hand, when calculating losses in pipelines in which a real fluid with viscosity flows, it is necessary to take into account hydraulic losses, i.e. irreversibly lost part of energy. Calculation of hydraulic losses in pipelines is one of the main tasks of hydrodynamics. Hydraulic losses in the movement of real fluid are caused, firstly, by the manifestation of viscous forces in the fluid, i.e. friction losses, and secondly, by the presence of various regulating and measuring valves on the pipeline - the so-called local resistances - sections of the hydraulic network, where there is a change in the flow velocity in magnitude and/or direction.

In this paper on the basis of the Rakhmatulin model, shock wave propagation in multiphase and deformable pipes is considered. In turbulent flow of subsonic flows the equation of momentum, the law of conservation of masses and the equation of state of the medium are constructed. The law of mass flow rate variation depending on time and on the pipe cross-section is obtained. An efficient analytical method has been developed to solve the problem.

Keywords: *multiphase media, mass flow rate, "live" section, a drag coefficient, hydraulic radius of the section.*

1. INTRODUCTION

In most cases, when conducting studies of the interaction of bodies with fluids, various simplifications and assumptions are adopted to obtain a "convenient" model for analyzing the mathematical model of the "body-liquid" system. Thus, it is accepted that if the body is elastic, the deformation of the supporting body is practically negligible. This makes it possible to study the resulting dynamic processes on the basis of linear differential equations of motion. In this case, the hypothesis of small wave-like motion of the fluid is accepted. As a result, the motion of the fluid phase leads to the application of linearized differential equations [1]. In [2] the first step in solving the inverse problem of controlling the flow of viscous incompressible fluid through a system of pipelines was made. According to a given flow rate at one of the outlets, the required pressure at the inlet is calculated and then the "pump" pressure is gradually brought to this level. In [3] a parametric model for the study of vortex steady-state flow in such fittings as bends and tees was proposed. Water was considered as a pumped medium. It was established that the choice of the branch radius was determined by the requirement of minimum pressure drop at its outlet. The coupled problem of hydroelasticity for a circular tube of annular cross-section with an absolutely rigid inner cylinder and an elastic, geometrically irregular outer shell, freely supported at the ends of the tube, was obtained in [4].

[1] Y.M. Sevdimaliyev, G.M. Salmanova, R.S. Akbarli, N.B.Nagieva, “Propagation of waves in a hydroelastic shell-viscous liquid system, in the presence of gas bubbles”, Mathematical Methods in the Applied Sciences.
[2] N.V. Sorokina, “A problem of control of the viscous incompressible liquid motion through a pipeline system” Systems analysis in design and management.
[3] K.A.Stepanovich, K.A.Ivanovna, M.A.Pavlovich, “Simulation of turbulent flow of viscous incompressible liquid in pipe fittings” International Journal of Applied Science and Technology.
[4] D.V.Kondratov, Y.N.Kondratova, V.S.Popov, I.V.Plaksina, “The Problem of a Hydroelasticity for a Tube Ring-type a Profile with Elastic, Geometrically Irregular Outer Shell at Pressure Influence”, News of the Saratov University, Series Mathematics, Mechanics, Informatics.

1. INTRODUCTION

Studies show that in thin-walled elastic structures, the force resulting from the interaction of a multiphase medium depends on the deformation of the structure [5, 7]. The construction of a mathematical model and solution of a specific problem in hydroelastic systems consisting of a multiphase fluid and a deformable solid body is relevant in theory and application. Mathematical modeling of multiphase and multicomponent medium motion consists of the equations of mass balance, balance of momentum, law of energy conservation and second laws of thermodynamics for each phase and components. It is necessary to supplement the equations of the mathematical model with specific relations. These relations include mechanical, thermodynamic, sometimes electromagnetic, chemical and other properties of the environment. The multiphase nature of the medium complicates the study of hydrodynamic processes and motions occurring in this medium. This is more sharply manifested in the propagation of waves arising from vibrations and shocks. The study of the regularities of these processes is of exceptional importance in modern technologies for the creation of energy devices, as well as in the military, etc. in the development of new methods and the creation of scientific bases for their analysis. As a rule, the results of many sections of physics, including shock wave physics, gas dynamics, explosion physics, hydraulics, thermophysics, and filtration theory, are jointly discussed in studies conducted from the position of continuum mechanics in the hydrodynamics of heterogeneous media.

[5] I.A.Charny, “Unsteady motion of real fluid in pipes”, State publishing house of technical and theoretical literature.
[7] R.S. Akbarly, G.M. Salmanova, “Pulsational motion of a mixture in a vibrating medium with consider flow rate”, International Journal on Technical and Physical Problems of Engineering (IJTPE).

2. FORMULATION OF THE PROBLEM

In the presented work, the pulsating flow of multiphase fluid in a thin-walled elastic tube of circular cross-section is investigated. On the basis of the single-velocity theory of multiphase media, the hydraulic shock during the movement of a mixture in a circular pipe is considered. Assume also that the mass average temperature of the mixture is constant. The cross-sectional area of the pipe changes, if the x-axis is directed along the pipe axis, then $R = R\ (x)$ The pressures of all phases of the multiphase media are considered to be coincident and equal to the pressure of the medium. In addition to this f_j - porosity, represents the volume fraction of the phases of the mixture will be [6]:

$$f_j = \frac{\rho_j}{\rho_j^0}$$

Wherein ρ_j -reduced density of the j- th phase;
Then the density of the medium is defined by: $\rho = \rho_1 + \rho_2 + \cdots + \rho_n = \rho_1^0\, f_1 + \rho_2^0\, f_2 + \ldots + \rho_n^0\, f_n$

here $\sum_{j=1}^{n} f_j = 1;\ \rho_j^0 = \varphi_j(p).$

[6] A.I.Umarov, U.D.Nurmatov, K.Z.Abidov, Sh.H.Akhmedov, “Hydrodynamics of Multiphase Mixtures in Tubes”, Tashkent State Technical University.

2. FORMULATION OF THE PROBLEM

Let us introduce the speed of sound in the medium, which, due to the arbitrariness of the initial state, can be rewritten in the form:

$$c^{-2} = \rho_0 \sum_{j=1}^{n} \frac{f_j}{\rho_j^0 c_j^2} .$$

However, if the pore pressure differs from the skeletal pressure, the formula is not applicable.

It should be noted that, when compiling the equations of motion, the characteristics of fluid resistances established for stationary motions also take place for unsteady flow currents. In this case, the law of conservation of mass and equation of momentum will be written as [5]:

$$-S(x)\frac{\partial p}{\partial x} = \frac{\partial M}{\partial t} + M\frac{\lambda u}{8\delta} + \gamma S(x)\sin\alpha + \frac{\partial}{\partial x}[(1+\beta)M u] \qquad \textbf{(1)}$$

$$-S(x)\frac{\partial p}{\partial t} = c^2\frac{\partial M}{\partial x}$$

[5] I.A.Charny, "Unsteady motion of real fluid in pipes", State publishing house of technical and theoretical literature.

2. FORMULATION OF THE PROBLEM

And so we obtain a system of two first-order partial differential equations of hyperbolic type. When writing this system of equations, it should be taken into account that for the motion of a real fluid in the case of small subsonic velocities, convective terms can be neglected in the equations of motion.

Here α - elevation angle of the element axis dx above the horizon, for our problem $\alpha = 0$ and therefore $\gamma\, S(x) \sin \alpha = 0$.

β the Coriolis correction for non-uniform velocity distribution in the expression of the momentum of the flow in terms of the average velocity and the average density in the section; in unsteady motion β will be a variable value depending on the nature of velocity distribution in the pipe section;

M- mass flow rate, which depends on time and on x, $M = M\,(x, t)$.

δ -hydraulic radius of the section;

$S = S(x)$ - area of the "live" section;

λ- a drag coefficient.

In this case, the drag coefficient and Coriolis correction in each specific problem are subject to determination. We will assume that the drag coefficient for unsteady motion is the same function of the Reynolds number as for steady motion.

We will study motion in long pipelines with subsonic velocity. For the flow of subsonic velocities in turbulent flow, let us average the term $\frac{\lambda\, u}{8\, \delta}$ as $[\frac{\lambda\, u}{8\, \delta}]_{av} = 4\frac{R}{\delta_0}$, here δ_0 -pipe wall thickness.

2. FORMULATION OF THE PROBLEM

Solving the system (1) for subsonic velocities, when we can neglect the change of velocity heads $\beta = 0$, we obtain:

$$\frac{\partial^2 M}{\partial t^2} + 4\frac{R(x)}{\delta_0}\frac{\partial M}{\partial t} + 16\,\frac{R^2(x)}{\delta_0 \lambda}\frac{\partial M}{\partial x} + 32\,\frac{M}{\lambda}R(x)\frac{\partial R(x)}{\partial x} - c^2\frac{\partial^2 M}{\partial x^2} + 2c^2\frac{1}{R(x)}\frac{\partial M}{\partial x}\,\frac{\partial R(x)}{\partial x} = 0 \qquad (2)$$

3. RESULTS AND DISCUSSIONS

In the special case when the “live” section of the pipe is unchangeable along the x-axis, R=const then integral equation (2) allows us to determine the mass flow rate in a pipe of unchanged cross-section. In this case, equation (2) takes the following form:

$$\frac{\partial^2 M}{\partial t^2} + 4\frac{R}{\delta_0}\frac{\partial M}{\partial t} + 16\,\frac{R^2}{\delta_0 \lambda}\frac{\partial M}{\partial x} - c^2\frac{\partial^2 M}{\partial x^2} = 0 \qquad (3)$$

For this case the boundary and initial conditions are respectively given as follows:

$$M(0,t) = 0, \qquad M(1,t) = M_0; \qquad M(x,0) = 0$$

3. RESULTS AND DISCUSSIONS

The flow of a three-phase fluid consisting of air, light oil [8] and water in a cylindrical pipe of constant cross-section is investigated for numerical report in a special case. The volume fractions of the phases are respectively taken as 0.1; 0.3; 0.6. The radius of the pipe is taken as 0.05m and the wall thickness 0.003m. In case of turbulent movement of the working flow, the hydraulic resistance factor for a circular cross-section main with smooth surfaces is taken as 0.0256. We consider that small excitations caused by a monochromatic source occur in a hydroelastic system consisting of a thin infinite cylindrical tube of circular cross section containing a three-phase medium. We have taken the origin where the source is located and direct the x-axis along the pipe axis and study the change of flow rate per unit length of the pipe.

The results obtained are presented in 3D graph format.

[8] Z.Wang, A.Nur, M.L.Batzie, “Acoustic velocities in petroleum oils” J. Pet. Technol.

3. RESULTS AND DISCUSSIONS

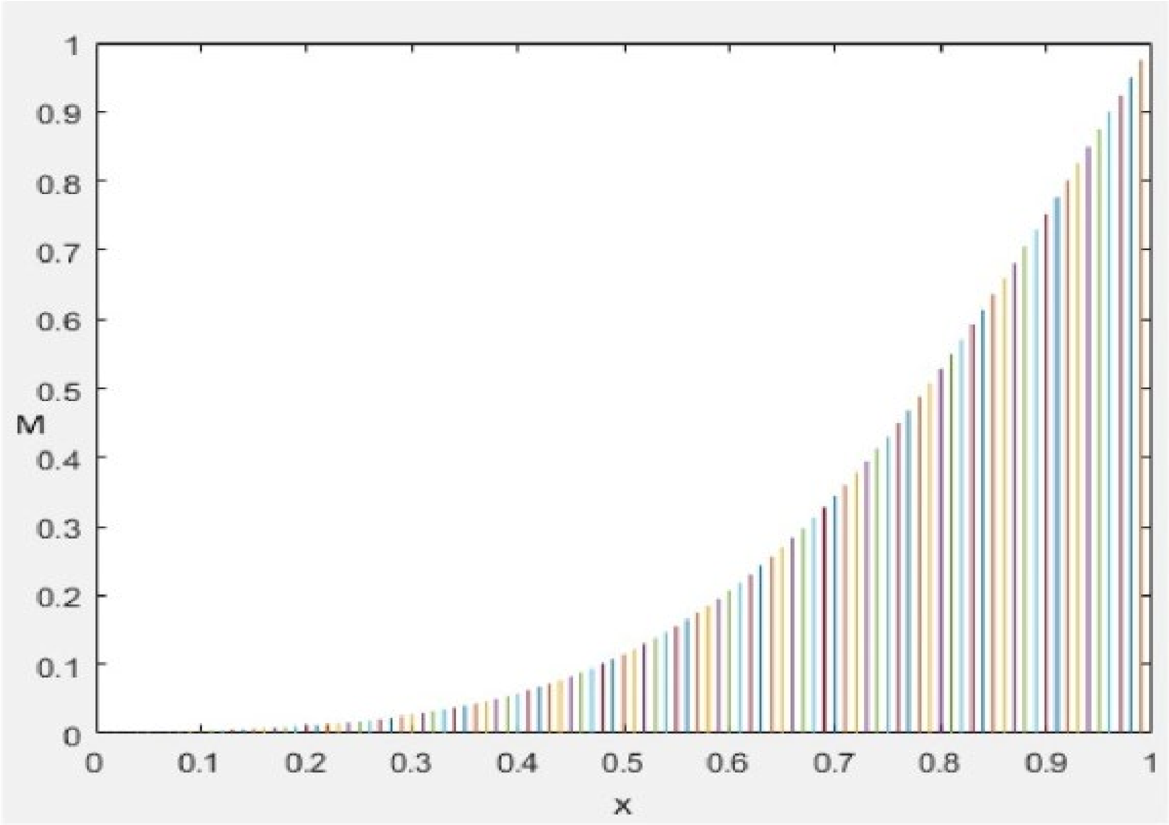

Figure 1. Variation of fluid flow rate along the axis of the tube

3. RESULTS AND DISCUSSIONS

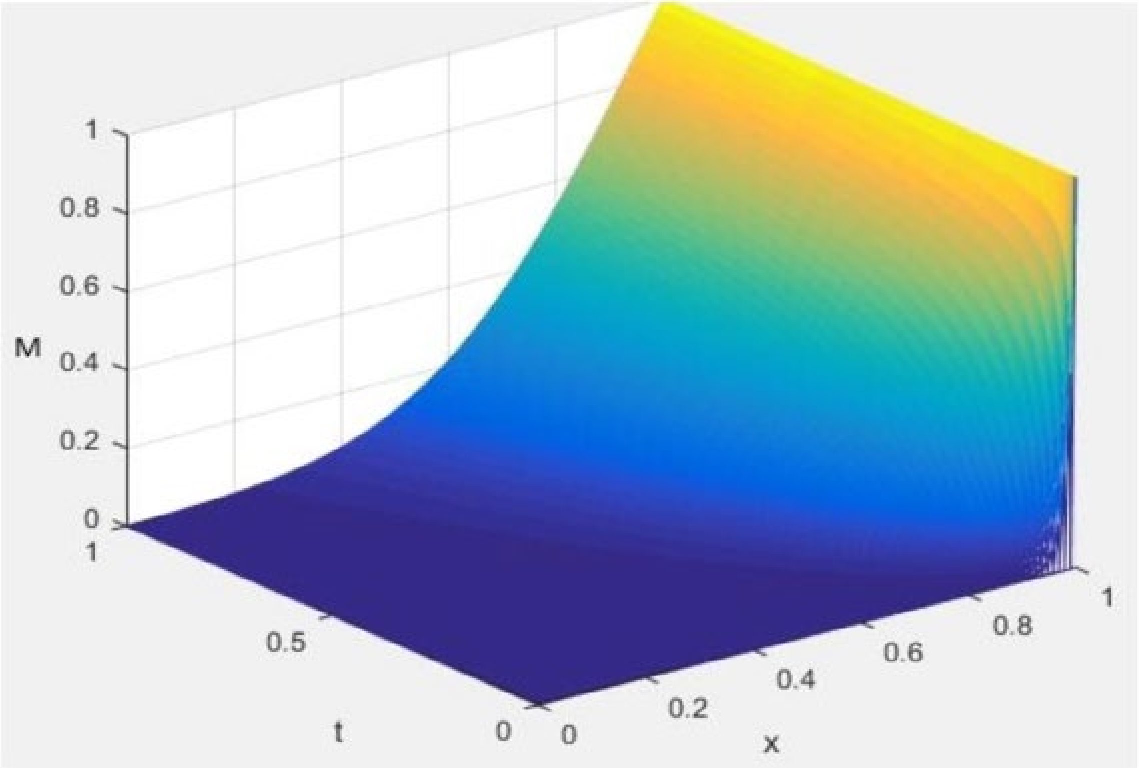

Figure 2. Dependence of flow rate on the x-coordinate directed along pipe axis and time

3. RESULTS AND DISCUSSIONS

For comparison, in article [7] considered the issue of flow discharge in the motion of two-phase viscous fluid in deformable elastic shells without taking into account hydraulic resistance. An interesting point in this work is how the amplitudes of the characteristics found using the flow rate vary as a function of the number of bubbles per unit volume and the density of the fluid. For this purpose, the behavior of four liquid samples containing air bubbles (water, glycerol, ethanol and oil) was considered for numerical studies.

[7] R.S. Akbarly, G.M. Salmanova, “Pulsational motion of a mixture in a vibrating medium with consider flow rate”, International Journal on Technical and Physical Problems of Engineering (IJTPE).

4. CONCLUSIONS

The paper considers a specific problem of motion of three incompressible and compressible media in pipes, on the basis of which the regularities of flow of mixtures are revealed. An effective numerical method for the calculation of real physical natural processes and techniques is created.

The developed mathematical models of motion of multiphase media and their solutions can find applications in the creation of biomechanical theory, in solving the modern problem of gas and oil mechanics and in the transportation of mixtures.

Knowing the law of variation of the “live” section of the pipe, we can specify by using the integral equation (2) the changes of mass flow rate as a function of time and along the pipe axis. The equation is solved by combining the boundary and initial conditions.

Numerical results are performed for a pipe of constant cross-section in which real multiphase fluid flows. The flow rate variation depending on the pipe axis and in the unsteady case on time is shown in the graphs. Reports are made for turbulent flows considering hydraulic resistance in a pipe with a smooth inner surface.

It is shown that the multiphase property is fundamentally different from the effect of wave propagation in a tube filled with a single-phase fluid. Although this feature complicates the mathematical solution of this problem, it plays an important role in the study of dynamic processes.