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MOVEMENT OF WAVES OF INCOMPRESSIBLE TRANSVERSE FLUID IN A DEFORMABLE PIPE

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Abstract - In the work, the wave-like movement of the liquid in the deformed tube is considered. The mathematical model of the system used is described by the equation of motion of an incompressible viscous elastic fluid with the equation of discontinuity and the equation of dynamics for an isotropic linear viscous elastic tube with a variable cross-section. The solution of the problem is brought to the question of singular for the Sturm-Liouville equation.

The results of such studies form the basis of qualitative judgments of those or other facts characterizing the process of waves in the coating fluid system.

Harmonic analysis is used to generate complex impulses characteristic of wave processes, that is, impulses of complex shape are decomposed into sinusoidal components forming a Fourier series. Due to the linearization and homogeneity of the system, the passage of each harmonic is processed and the components corresponding to the given coordinate are summed to determine the shape of the pulse at any point.

It should be noted that the characteristics of hydroelastic problems are that it is necessary to solve two interrelated problems: the movement of the liquid and the coating under the influence of the hydrodynamic force is investigated. On the other hand, instead of classical hydrodynamic contact conditions, the condition of discontinuity of the velocity components should be satisfied at the liquid and coating boundary. As a result, the "coat-fluid" system is described by a system of equations that cannot be solved in general. For this reason, it is necessary to try to solve the problem based on a certain scheme by including the hypothesis of fluid and mesh.

Keywords: *viscous fluid, elastic tube, wave, discontinuity equation, equation of motion.*

1. INTRODUCTION

Currently, the problems of mathematical physics related to the description of wave motions of liquids of various physical nature attract much attention. This interest is due not only to the great applied significance of these problems, but also to their new theoretical and mathematical content, which often has no analogues in classical mathematical physics. Here, among the actual problems of hydromechanics, it is very important to study the flow of liquids in deformable tubes.

This statement is confirmed by the widespread use of fluid transportation systems in engineering and living organisms (pipeline transport, hemodynamics). When solving such problems, it is necessary to involve the equations of motion of the tube into consideration, taking into account the influence of the fluid moving in its cavity on the dynamics of the tube. The specifics of such studies, the roots of which are laid in the works of L.Euler, I.Gromek, E.Zhukovsky, are reflected in detail in the works [1,2,3,4]. However, taking into account a number of very important factors, such as the visco-elastic properties of the liquid and the tube material in combination with its narrowing, have not been sufficiently studied. In [5], on the basis of one-dimensional linear equations, an analytical solution of the problem of pulsating flow of a viscoelastic fluid in an elastic tube is constructed, taking into account the effect of narrowing. The proposed article considers the wave flow of a liquid enclosed in a deformable tube. The mathematical model of the system used is described by the equation of motion of an incompressible viscoelastic fluid together with the continuity equation and the dynamics equation for an isotropic linearly viscoelastic tube of variable circular cross-section. This problem leads to the solution of the singular boundary value problem of Sturm–Liouville.

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[3] Pedli T. “Hydrodynamics of large blood vessels” M.: Mir, 1983, 400 p.
[4] Lightvut E. “Transfer phenomena in living systems” M.: Mir, 1977, 520 p.
[5] Amenzade R.Yu. “Analytical solution of the problem of wave flow of a viscoelastic fluid in an elastic tube, taking into account the narrowing effect” DAN Russia, 2008, vol. 418, no. 3, pp. 327-330.

1. INTRODUCTION

In many practically important issues, it is necessary to study the properties of the deforming system in interaction with the surrounding medium - the liquid. In this case, the interaction force between the deforming system and the fluid usually depends significantly on the deformation of the system. Therefore, when studying the impact force of the liquid on the system, it is necessary to study the deformation of the system as well.

The study of multiphase system dynamics covers a wide range of issues related to science, technology, living organisms, and other fundamental problems. Despite the fact that the basic ideas and principles of liquid flow in deformable pipes are known, the multiphase nature of the liquid, as well as the influence of various factors on the characteristics of the liquid movement, are not well studied areas. Therefore, it is theoretically interesting and practically relevant to study more general regularities of wave propagation processes in liquids moving in deformable pipes. The results of such studies form the basis of qualitative considerations of these or other facts characterizing the wave-like process in the coating-liquid system.

2. NONLINEAR MOTOR DYNAMICS

Suppose that, $R = r(x)$ is a pipe with a semi - infinite cross-section variable, and h - is its thickness, where $R(x)$ is a monotonically decreasing function, and x is the coordinate with the longitudinal one. System of one-dimensional hydroelasticity equations from discontinuity equations [1,3]

$$\frac{\partial}{\partial x}(Su) + L\frac{\partial w}{\partial t} = 0 \qquad (1)$$

momentum equation

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(-p+\sigma) \qquad (2)$$

and the line consists of the equation of motion of the tube, which is for viscous elasticity

$$p = \frac{n}{R^2(x)}E^v w = \rho_* h\frac{\partial^2 w}{\partial t^2} \qquad (3)$$

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(3) when writing the equation, the tube was thin-walled and firmly fixed to the environment. As a result, the tube cannot move along the axis. Classical descriptions of the hydrodynamics of an Ideal and viscous Newtonian fluid are unacceptable when describing a whole flow of fluids with long macromolecular compounds. This fact is of paramount importance for many technological processes in the production of colloidal solutions, suspensions, emulsions, etc. this includes. For this, in order for the above equations to be correlated, we write the rheological relations of the fluid and take it as a linear viscous elastic.

$$\prod_{j=1}^{r}\left(1+\lambda_j\frac{\partial}{\partial t}\right)\cdot\sigma = 2\eta\prod_{j=1}^{s}\left(1+\theta_j\frac{\partial}{\partial t}\right)\cdot e \qquad (4)$$

(1)-(4) in the equations $u(x, t)$ is the flow rate of the liquid, $w(x, t)$ is the radial displacement of the walls of the pipe, $p(x,t)$ is the hydrodynamic pressure, $\sigma(x, t)$ voltage, ρ and ρ_* the density of the liquid and the material of the pipe, $e(x, t)$ is the speed of deformation, $S = \pi R^2$ - cross - sectional area, $L = 2\pi R(x)$ - length of pipe circumference, η - is the dynamic viscosity coefficient of the liquid. λ_j and θ_j characterize relaxation and retardation. In (3) E^v is a hereditary type operator [4].

[4] Lightvut E. “Transfer phenomena in living systems” M.: Mir, 1977, 520 p.

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$$E^{v} = E\left(1-\Gamma^{*}\right),\ \Gamma^{*}w(x,t) = \int_{-\infty}^{t}\Gamma(t-\tau)w(x,\tau)d\tau$$

where E is the modulus of elasticity, Γ^{*} is the relaxation operator, $\Gamma(t-\tau)$ is the difference core of relaxation. (3) in the open form it is written like this

$$p = \frac{h}{R^{2}(x)}E\left\{w(x,t) - \int_{-\infty}^{t}\Gamma(t-\tau)w(x,\tau)d\tau\right\} \qquad (5)$$

Given its equality $e = \partial u/\partial x$ in (4), it will be written as

$$\prod_{j=1}^{r}\left(1+\lambda_{j}\frac{\partial}{\partial t}\right)\cdot\sigma = 2\eta\prod_{j=1}^{s}\left(1+\theta_{j}\frac{\partial}{\partial t}\right)\cdot\frac{\partial u}{\partial x} \qquad (6)$$

The function $R(x)$ is written as $R(x) = R_{\infty}g(x)$, the function is a second-order differentiator. At infinity, the pipe has constant cross – section R_{∞}.

From here we find that,

$$\lim_{x\to\pm\infty} g(x) = 1 \qquad (7)$$

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At the same time

$$\lim_{x\to\infty} g'(x)=0,\ \lim_{x\to\infty} g''(x)=0 \qquad (8)$$

strokes denote differentiation with respect to the x coordinate. For example, this function can be shown as follows

$$g(x)=1+e^{-\beta x},\ (\beta>0) \qquad (9)$$

which indicates that the pipe is narrowed in the form of a cone due to its length. Then, taking into account (5) and (6), we get the next closed system of equations:

$$\frac{\partial u}{\partial t}+2\frac{g'(x)}{g(x)}u+\frac{2}{R_\infty g(x)}\frac{\partial w}{\partial t}=0 \qquad (10)$$

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial \sigma}{\partial x} \qquad (11)$$

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$$p=\frac{h}{R_\infty^2 g'(x)}E\left(w(x,t)-\int_{-\infty}^{t}\Gamma(t-\tau)w(x,\tau)d\tau\right) \quad (12)$$

$$\prod_{j=1}^{r}\left(\sigma+\lambda_j\frac{\partial\sigma}{\partial t}\right)=2\eta\prod_{j=1}^{s}\left(\frac{\partial u}{\partial x}+\theta_j\frac{\partial^2 u}{\partial x\partial t}\right) \quad (13)$$

Note that the linearization of hydroelastic equations in wave processes is valid $\left|u\cdot c^{-1}\right|<<1$ as long as there is inequality.

$$\left|\frac{u}{c}\right|<<1$$

c-is the complex propagation velocity of the wave (for all time). From the condition of kinematic impermeability, a linearization of the equations of the theory of viscous elasticity is obtained.
Solution of the issue. The Sturm-Luivil equation is brought to the question of the singular boundary. For this, let's use the Luivil substitution.

2.

$$y(x)=u_1\exp\frac{1}{2}\int\frac{G_2(x)}{G_1(x)}dx\equiv u_1(x)\chi(x) \quad (14)$$

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Therefore,

$$G_1(x)u_1'' + G_2(x)u_1' + G_3(x)u_1 = 0 \qquad (15)$$

it will be written like this (reduction form of the wave equation) [5]

$$y'' + I(x)y = 0 \qquad (16)$$

From (16) we get the differential equation:

$$y'' + \delta^2 y = \delta^2 q(x) y \qquad (17)$$

The integral condition is applied to the complex *q(x)* potential function [4]

$$\int_0^\infty |q(x)| dx < +\infty \qquad (18)$$

the obtained function *q(x)* together with (9) satisfies the condition (18).

[5] Amenzade R.Yu. “Analytical solution of the problem of wave flow of a viscoelastic fluid in an elastic tube, taking into account the narrowing effect” DAN Russia, 2008, vol. 418, no. 3, pp. 327-330.
[6] Rabotnov Yu.N. “Elements of hereditary mechanics of solids” M.: Nauka, 1977, 382 p.

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Then let's add the following boundary conditions for constructing the solution [6,7,8]

$$y(0) = y_0, \ \lim_{x\to\infty} y(x) = 0 \qquad (19)$$

the calculation of y_0 is depends on the operation of the system (different boundary conditions in the cross-section of the pipe). (19) indicates the limitation of the condition sought. Thus, the problem of hydroelasticity (17) and (19) was solved by bringing to the solution of the problem of Sturm-Luivil singular boundary.

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3. CONCLUSIONS

For the first class of the model, the attenuation coefficient is two orders of magnitude less than for the second class;
the amplitude of the "viscous voltage" in the second mode increases depending on ξ;
the amplitude of the "viscous voltage" in the first mode decreases depending on λ;
the non-Newtonian properties of the liquid are most significantly manifested when using the second class of the model.

In the case of non-symmetric propagation of waves to the axis during the movement of the liquid inside the coating, the difference between the speed of the waves propagating in the coating and the vortex waves is very small at different values of the number of waves formed. at this time, the speed of both waves almost coincides, as the amount of bubbles in the volume increases.

During axisymmetric wave motion of the liquid inside the cover, the change in the density of the liquid does not affect the frequency of the waves propagating in the cover, and the speed decreases with the increase in density. The change in density does not affect the frequency of eddy waves, their speed decreases. The speed of waves propagating in a liquid decreases by an average of 6% with increasing density.

The problem of axially symmetric and asymmetric propagation of waves, the effect of various factors on the characteristics of fluid motion in coatings with elastic, torque, torqueless, and at the same time taking into account sliding deformation, which allows not to limit the length of the wave, was investigated.