The 19th International Conference on
“Technical and Physical Problems of Engineering”
ICTPE-2023
31 October 2023
International Organization of IOTPE

PROPER OSCILLATION OF AN INHOMOGENEOUS CIRCULAR PLATE TAKING INTO ACCOUNT VISCOELASTIC RESISTANCE

V.L. Sharifov R.A. Allahverdiyev B.M. Aslanov

Faculty of Transport, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan

- **Abstract-**As you know, circular plates made of various materials are widely used in mechanical engineering and instrumentation, in the construction of various kinds of engineering structures, etc. Due to the fact that artificial materials have been widely used in various fields of engineering and construction in recent years, the interest of engineering researchers is focused on creating new calculation methods that could take into account the real properties of the construction material and the environment as accurately as possible. This includes, in general, inhomogeneous, anisotropic materials and inhomogeneous anisotropic non-elastic bases. The theoretical and experimental study of the stability and vibrations of thin-walled plates and shells of various configurations with different physical and mechanical properties in different directions has attracted primary attention due to the widespread use of new materials in various structures, such as plywood, which has a sharp anisotropy. Currently, research in this area is intensively developing, the use of new materials in structures requires new approaches and the creation of new methods for design, and it becomes relevant due to the rapid development and introduction of fiberglass, which are also anisotropic.
- **Keywords:** *Stability, Round plates, Heterogeneous base, Orthotropic bodies, Bubnov-Galerkin method*

- **1. INTRODUCTION**
- Depending on the above and a number of other reasons, the mechanical properties of V.A. Lomakin plates [1] significantly depend on the coordinates of points in the directions of the stressed state. In this paper, we will assume that the modulus of elasticity and the specific density depend on the function of the thickness coordinate and the current radius, and the Poisson's ratio is a constant:
- As is known, in the design, calculation and construction of engineering structures for various purposes, mainline railways, bridges, tunnels, offshore stationary platforms, etc., structural elements made of various elastic and inelastic materials are widely used.
- Currently, in connection with the construction of large-scale construction complexes and a number of other industries, designers and calculators are faced with an increased requirement for the most correct and efficient use of the material. In addition, there is a need for a more realistic description of the physical and mechanical properties of the material. One of the above materials is anisotropic materials. Note that anisotropic materials can be natural (for example, wood), obtained as a result of their manufacturing technology (for example, sheet metal, paper, concrete) and anisotropic-constructive (for example, reinforced plates).
- Summarizing all the above, it can be concluded that anisotropic materials are divided into three classes: "naturally anisotropic", "technologically anisotropic" and "structurally anisotropic". It should be noted that such a division into classes is conditional and often a new anisotropic material is created as a result of the development of a new technological process for the implementation of a new "design" of a material from known anisotropic and isotropic materials.

- In the monographs [2, 3], the theory of anisotropic materials was presented for the first time (within the limits of elasticity).Some issues of bending anisotropic plates are considered and a number of recommendations are given that may be useful for research engineers. This is emphasized in the name of one of the types of plastic "FAM", which means, glass is a “fibrous anisotropic material". Such materials are widely used not only in the construction of engineering structures. Currently, artificial materials are used in all spheres of the national economy.
- The monograph [3] is a more fundamental monograph devoted mainly to the development of theoretical aspects from the theory of complex media (within elasticity). Numerous problems of the stressed deformed state of anisotropic bodies are considered in [3].
- Various theoretical and experimental issues were dealt with by [8], [5], [6] and many others. In these and a number of other scientific studies, issues related to load-bearing capacity, strength and other sections of structural mechanics are widely covered.
- However, these works do not investigate the issues of bending and stability, taking into account the influence of various kinds of external resistance.

- In this review, we will consider those works that were devoted to the generalized plane stress state, which is closely related to the study of stability problems and
- vibrations of anisotropic plates (along the way, those works that are related to the stability of elastic plates, taking into account and without taking into account the resistance of the external environment, will also be discussed). As noted above, a new anisotropic material is created by developing a new technological process for the implementation of new "structures" of materials known as anisotropic and orthotropic. For example, in [7] an anisotropic material from unidirectional matrices is given.
- [9, 10] the problems of stability and vibrations of plates of different configurations under different resistance conditions were solved.
- [11, 12] works considered the question of vibrations of cylindrical shells when the inhomogeneity sharply depends on all three coordinates
- The works [13, 14] are mainly devoted to studies of vibrations of orthotropic inhomogeneous plates made of homogeneous materials without taking into account and taking into account the resistance of the Winkler base.

- **2. PROBLEM STATEMENT**

- As it was noted [1, 2, 3], in some cases, when designing and constructing various kinds of interesting structures, sea dams, etc., engineering tasks have to be solved taking into account the influence of an artificially created foundation.
- It is characteristic of such bases that they resist bending in various ways in the main directions.
- In this article, we will investigate the problem of proper oscillation of an anisotropic plate in the case when the specific density depends on the thickness coordinate. The coordinate system is chosen as follows: here, the right coordinate system   is selected for calculation, the origin of which is in the middle plane.
- The equation of motion, taking into account anisotropic resistance with inhomogeneity of specific density over the thickness of the plate, looks like this [1, 2]:

$$E = E_0 \chi_1(z); \qquad l = \mu_0 \xi_1(x) \xi_2(r);$$

$$\nu = const \cdot$$

Here E_0 and μ_0 correspond to the homogeneous case; χ_1, ξ_1, ξ_2 continuous functions.

Suppose that the plate performs a transverse axisymmetric oscillation and lies on a base that obeys the following law [8]:

$$K(w) = e_1 w + e_2 \frac{\partial^2 w}{\partial t^2}. \qquad (2)$$

Here is the deflection e_1, e_2 - determined by experiment, t - time.

The equation of motion taking into account (1) and (2) takes the

following form [2, 3]:

$$D_0 J(\Delta w)^2 + e_0 w + e_1 \frac{\partial^2 w}{\partial t^2} + \\ + \tilde{l}\, \xi_2(r) \frac{\partial^2 w}{\partial t^2} = 0. \quad (3)$$

The following designations are introduced here:

$$\tilde{l} = \frac{\mu_0}{n} \int_{-H}^{H} \xi_1(z)\,dz, \quad J = 12 \int_{-0.5}^{0.5} \chi(z) z^2 dz. \quad (4)$$

We will look for the solution (3) in the following form:

Here $v(r)$ - must meet the corresponding boundary conditions, ω - circular frequency, $H = h/2$ is half the thickness of the plate only plastics, L- plastic radius

We will assume that the plate is rigidly pinched along the entire contour, although this condition is not fundamental. Substituting (5) into (3) we get [9, 10]:

$$JD_0 (\Delta \mathbf{v}(r))^2 + e_1 \mathbf{v}(r) - e_2 \omega^2 \mathbf{v}(r) - \\ - \omega^2 \xi_2(r) \tilde{l} \mathbf{v}(r) = 0. \quad (5)$$

We introduce the following notation:

$$\overline{e}_1 = e_1 (JD_0)^{-1}, \quad \overline{e}_2 = a e_2 (JD_0)^{-1},$$

$$\tilde{l}_1 = \tilde{l} (JD_0)^{-1}, \quad D_0 = \frac{E_0 H^3}{12(1-\nu^2)}.$$

Then the equation of motion takes the following form:

$$(\Delta \mathbf{v}(r))^2 + \overline{e_1}\mathbf{v}(r) - \\ -\omega^2\left(e_2 + \xi(r)\tilde{l}_1\mathbf{v}(r)\right) = 0 \quad (6)$$

We will search for the solution of equation (6) using the Bubnov-Galerkin orthogonalization method, and $\mathrm{v}(r)$ we will search in the form of a series:

$$\mathrm{v}(r) = \sum_{i=1}^{n} c_i \varphi_i(r) \quad (7)$$

Here, $\varphi_i(r)$ each of them must satisfy the corresponding boundary conditions.
For simplicity of the analysis, we take the following notation:

$$\mathrm{P}_1(r) = (\Delta \mathbf{v}(r))^2 + \overline{e}_1\mathbf{v}(r), \quad (8)$$

$$\mathrm{P}_2(r) = \left(e_2 + \chi(r)\tilde{l}_1\right)\mathbf{v}(r). \quad (9)$$

Given (8) and (9) in (6) and using the orthogonalization method, we can write [4]:

$$\sum_{j=1}^{m} b_j \int_0^L \mathrm{P}_1(\beta_j)\,\beta_k(r)\,r dr - \\ -\omega^2 \sum_{j=1}^{n} b_j \int_0^L \mathrm{P}_2(\beta_j)\,\beta_k(r)\,r dr = 0 \qquad (k = \overline{1,..,n}). \tag{10}$$

Note that (10) is a linear inhomogeneous system of algebraic equations with respect to b_i . For the existence of a nontrivial solution, the main determinant composed of coefficients must be zero [15]:

$$\left\| \omega^2 \right\| = 0. \tag{11}$$

This equation is called the frequency equation.
For practical calculation, it is usually neglected to determine the pitch of the square of the frequency, which corresponds to the first approximation of the solution. In this case, to determine the frequency, we have the following equation:

$$\int_0^L \mathrm{P}_1(\beta_1)\,\beta_1(r)\,r dr - \omega^2 \int_0^L \mathrm{P}_2(\beta_1)\,\beta_1(r)\,r dr = 0. \tag{12}$$

From (12) we find:

$$\omega^2 = \frac{\int_0^L \mathrm{P}_2(\beta_1)\,\beta_1(r)\,r dr}{\int_0^R \mathrm{P}_1(\beta_1)\,\beta_1(r)\,r dr}. \qquad (13)$$

For the case of the Winkler base (13) takes the form:

$$\omega_b^2 = \frac{\int_0^L (\Delta\beta_1)^2\,\beta_1(r)\,r dr + \overline{b}_1 \int_0^L \beta_1^2(r)\,r dr}{\overline{l}\int_0^L \xi(r)\,\beta_1^2(r)\,r dr}. \quad 14)$$

$$\omega_v^2 = \frac{\int_0^L (\Delta\beta_1)^2\,\beta_1(r)\,r dr}{\overline{b}_2 \int_0^L \beta_1^2(r)\,r dr + \overline{l}\int_0^l \beta(r)\,\beta_1^2(r)\,r dr}. \qquad (15)$$

Without taking into account the resistance, we will have:

$$\omega_{\delta}^{2}=\frac{\int_{0}^{L}(\Delta\beta_{1})^{2}\,\beta_{1}(r)\,rdr}{\bar{l}\int_{0}^{R}\chi(r)\,\beta_{1}^{2}(r)\,rdr}. \qquad (16)$$

If the plate is inhomogeneous only in thickness, then we get:

$$\omega_{T}^{2}=\frac{\int_{0}^{L}(\Delta\beta_{1})^{2}\,\beta_{1}(r)\,rdr+\bar{b}_{1}\int_{0}^{L}\beta_{1}^{2}(r)\,rdr}{\bar{b}_{2}\int_{0}^{L}\beta_{1}^{2}(r)\,rdr+\bar{l}\int_{0}^{L}\chi(r)\,\beta_{1}^{2}(r)\,rdr}. \qquad (17)$$

If the plate is inhomogeneous only in radius, then formula (14) takes the following form ($J=1$):

$$\omega_{R}^{2}=\frac{\int_{0}^{L}(\Delta\beta_{1})^{2}\,\beta_{1}(r)\,rdr+b_{1}D_{0}^{-1}\int_{0}^{L}\beta_{1}^{2}(r)\,rdr}{b_{2}D_{0}^{-1}\int_{0}^{L}\beta_{1}^{2}(r)\,rdr+l_{0}D_{0}^{-1}\int_{0}^{L}\chi(r)\,\beta_{1}^{2}(r)\,rdr}. \qquad (18)$$

If the properties of the plate do not depend on the coordinates of the points, then the formula (18) takes the following form:

$$\omega_0^2 = \frac{\int_0^L (\Delta\beta_1)^2 \beta_1(r) r dr + b_1 D_0^{-1} \int_0^L \beta_1^2(r) r dr}{b_2 D_0^{-1} \int_0^L \beta_1^2(r) r dr + l_0 D_0^{-1} \int_0^L \beta_1^2(r) r dr}. \qquad (19)$$

In the case of Winkler, resistance (19) takes the following value:

$$\omega_0^2 = \frac{\int_0^L (\Delta\beta_1)^2 \beta_1(r) r dr + b_1 D_0^{-1} \int_0^L \beta_1^2(r) r dr}{l_0 D_0^{-1} \int_0^l \beta_1^2(r) r dr},$$

or

$$\omega_0^2 = \frac{\int_0^L (\Delta\beta_1)^2 \beta_1(r) r dr}{l_0 D_0^{-1} \int_0^l \beta_1^2(r) r dr} + \frac{b_1}{l_0}. \qquad (20)$$

From the relations (19) and (20) we obtain:

$$\left(\frac{\omega_R}{\omega_0}\right)^2 = \frac{b_2 D_0^{-1} \int\limits_0^L \beta_1^2(r)\, r dr + l_0 D_0^{-1} \int\limits_0^L \beta_1^2(r)\, r dr}{b_2 D_0^{-1} \int\limits_0^l \beta_1^2(r)\, r dr + l_0 D_0^{-1} \int\limits_0^L \chi(r)\, \beta_1^2(r)\, r dr},$$

$$\left(\frac{\omega_R}{\omega_0}\right)^2 = \frac{1 + l_0 b_2^{-1}}{1 + l_0 b_2^{-1} \int\limits_0^L \chi(r)\, \beta_1^2(r)\, r dr \left(\int\limits_0^L \beta_1^2(r)\, r dr\right)^{-1}}. \tag{21}$$

3. PROBLEM SOLUTION

To illustrate, consider the case of a hard pinching along the entire contour. The following conditions must be satisfied:

$$w = 0, \qquad \frac{dw}{dr} = 0 \qquad by \quad r = L \cdot \tag{22}$$

As an approximating function, we will take the following approximation:

$$\beta(r) = \chi_0 \left[1 - \left(\frac{r}{L} \right)^2 \right]^2 . \tag{23}$$

Note that we can also take the following approximation [12]:

$$\beta(r) = \chi_0 \left[1 - \left(\frac{r}{L} \right)^2 \right] \left(\frac{r}{L} \right)^{2(n-1)}, \; n = 2, 3, \ldots \cdot \tag{24}$$

However, the analysis will be conducted for the case (23).
Consider the case $\chi(r) = 1 + \delta \frac{r}{L}, \quad \delta \in [0, 1].$

To calculate, it is necessary to calculate the expression:

$$K = \int_0^L \chi(r)\,\beta_1^2(r)\,rdr \left(\int_0^L \beta_1^2(r)\,rdr \right)^{-1} . \quad (25)$$

For the linear case we get:

$$K = \int_0^L \left(1 + \delta \frac{r}{L}\right) \beta_1^2(r)\,rdr \left(\int_0^L \beta_1^2(r)\,rdr \right)^{-1} =$$

$$= \left[\int_0^L \beta_1^2(r)\,rdr + \delta \int_0^L \frac{r}{L} \beta_1^2(r)\,rdr \right] \left(\int_0^L \beta_1^2(r)\,rdr \right)^{-1} . (26)$$

From here we get

$$k = 1 + \delta \frac{\int_0^1 \overline{r}^2 \beta_1^2(\overline{r})\,d\overline{r}}{\int_0^1 \beta_1^2(\overline{r})\,\overline{r}d\overline{r}}, \qquad \overline{r} = \frac{r}{L} \cdot \qquad (27)$$

Taking into account (27), formula (21) can be given the following form:

$$\left(\frac{\omega_R}{\omega_0}\right)^2 = \frac{1+l_0 b_2^{-1}}{1+l_0 b_2^{-1} K}. \qquad (28)$$

We reveal the integrals that are included in the formula (respectively, the numerator and denominator) (27) and get:

$$\int_0^1 \overline{r}^2 \beta_1^2(\overline{r})\, d\overline{r} \quad \text{and} \quad \int_0^1 \beta_1^2(\overline{r})\overline{r}d\overline{r}:$$

$$F_1(\overline{r}^2, \overline{r}^4, \overline{r}^6) = \frac{31}{105}, \quad F_2(\overline{r}, \overline{r}^3, \overline{r}^4) = \frac{1}{6}. \qquad (29)$$

Substituting (29) into (28), we finally obtain an expression for determining the parameter proper oscillation of an inhomogeneous circular plate, taking into account viscoelastic resistance:

$$\left(\frac{\omega_R}{\omega_0}\right)^2 = \frac{1+l_0 b_2^{-1}}{1+l_0 b_2^{-1} K} =$$

$$= \frac{1+l_0 b_2^{-1}}{1+l_0 b_2^{-1}(1+1,953\delta)}. \qquad (30)$$

As can be seen from the graphs presented, taking into account the inhomogeneity of the resistance of the medium significantly affects the values of the critical parameters.

When $\delta = 0$ we get: $\left(\frac{\omega_R}{\omega_0}\right)^2 = 1$. The graph of the dependence of $\left(\frac{\omega_R}{\omega_0}\right) \sim \delta$, is shown in figure 1.

For specific values of the parameters E_0, l_0, ν and the selected forms of the function χ_1, ξ_1, ξ_2 numerical calculations are carried out and the results are presented in figure 1.

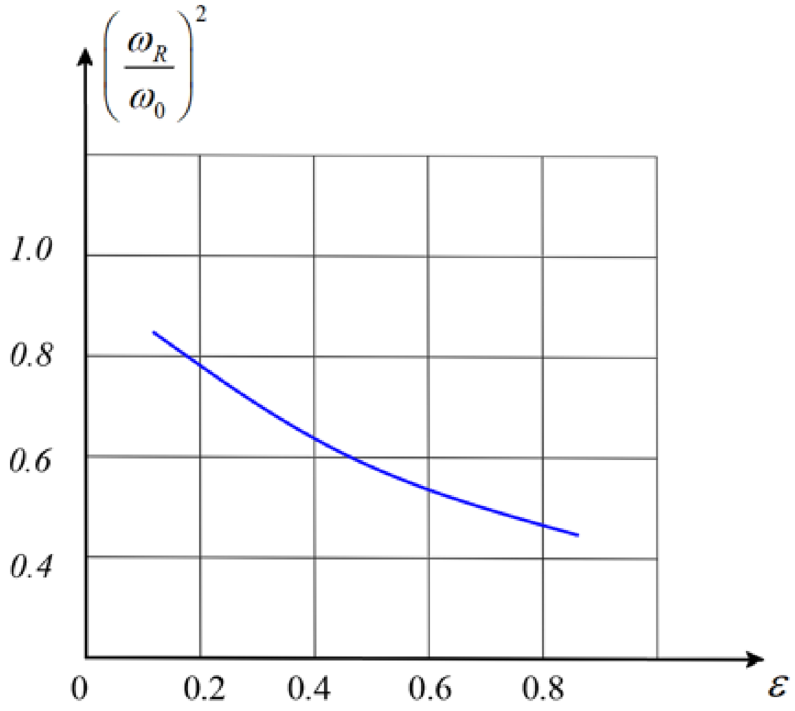

Figure 1. Graph of frequency dependence on the inhomogeneity parameter in a linear arrangement.

4. CONCLUSIONS

- Some problems of stability and natural oscillations of continuously in homogeneously anisotropic rectangular and round plates are formulated and solved, taking into account various kinds of external forces, the influence of environmental resistance, and a method for their solutions is constructed.
- Taking into account the two-constant, inhomogeneous and inhomogeneous viscoelastic modulus, specific formulas for determining the frequency value depending on the parameters of the base and the mechanical properties of inhomogeneous elastic orthotropic plates are found by approximate analytical methods.
- The formulation of the problems of natural oscillations of in homogeneously anisotropic rectangular and round plates with different homogeneous boundary conditions is considered for the first time and a method for their solutions is constructed.
- The problem of natural oscillations is solved for the first time, taking into account the influence of inhomogeneous viscoelastic resistance, when the properties of the material are anisotropic, their elastic characteristics and density are continuous functions of three spatial coordinates. With the use of approximate analytical methods, specific formulas for engineering calculation are obtained