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WAVE PHENOMENA IN SHALLOW CHANNELS USING THEORY OF SHALLOW WATER

A.I. Khudiyeva

Azerbaijan University of Architecture and Construction, Baku, Azerbaijan

WAVE PHENOMENA IN SHALLOW CHANNELS, THE THEORY OF "SHALLOW WATER".

In this paper, the hydrological characteristics are based on the solution of the equations of the shallow water theory [Wol-tsinger et al., with parameterisation of the influence of the friction force on the bottom and taking into account the horizontal turbulent exchange In all these papers, the mathematical model of the processes of wave excitation and propagation is based on the shallow water theory, which, however, allows different in complexity and accuracy specific versions of systems of partial differential equations. The most accurate, and therefore the most complex and interesting of them is the two-dimensional quasi-linear system. Unfortunately, its use both in theoretical studies and in numerical calculations is extremely rare and is associated with great difficulties due to insufficient development of the qualitative theory of solutions of multivariate quasilinear hyperbolic systems. Without going into the details of this problem, we note that in such monographs as relatively fully studied only the cases of one quasilinear equation and a system of two quasilinear equations and only in one-dimensional version. The equations are obtained[1] by depth integration of the Navier-Stokes equations under the condition that the horizontal scale is much larger than the vertical scale. Under this condition, it follows from the law of continuity that the vertical velocities in the fluid are small, the vertical pressure gradients are close to zero, and the horizontal gradients are caused by the roughness of the fluid surface and the horizontal velocities are the same throughout the depth. When integrating vertically, vertical velocities are removed from the equations.

Although vertical velocities are not present in the shallow water equations, they are not zero. When horizontal velocities are obtained, vertical velocities are derived from the continuity equation.

Situations where the depth of the water area is much less than the horizontal dimensions are quite common, so the shallow water equations are widely used. They are used with Coriolis forces in atmospheric and ocean modelling as a simplification of the system of primitive equations describing atmospheric flows.

The shallow water equations consider only one vertical level, so they cannot describe factors that vary with depth. However, when the dynamics of flows in the vertical direction are relatively simple, vertical changes can be separated from horizontal changes, and the state of such a system can be described by several systems of shallow water equations.

The shallow water equations can be applied to model Rossby and Kelvin waves in the atmosphere, rivers, lakes, oceans, and smaller bodies of water such as swimming pools. In order for the application of the shallow water equations to be correct, the horizontal dimensions of the water area must be much greater than the depth. The shallow water equations are also suitable for modelling tides. Tidal movements with horizontal scales of hundreds of kilometres can be considered as shallow water phenomena, even if they occur over multi-kilometre ocean depths.

When modelling the motion of an incompressible fluid, a system of quasilinear equations of hyperbolic type is usually used, the solutions of which, generally speaking, are discontinuous (which corresponds to the "water jump" in the theory of shallow water). This circumstance complicates the application of difference schemes and imposes the requirement of conservativity. At the same time, in the problems of gas dynamics, precisely for the description of discontinuous solutions, the particle method turned out to be quite effective. It should be noted that the system of equations of the shallow water theory coincides with the system of equations of gas dynamics (isoentropic flow of an ideal gas with the adiabatic exponent) Therefore, it seems natural to apply the particle method to the study of models of the shallow water theory in order to describe the discontinuous solutions appearing there. To this type of problems we can refer the problems of dam failure, boron propagation in resting water, and the problem of hydraulic jump.

We will consider such gravitational waves in which the velocity of moving particles is so small that in the Euler equation we can neglect the term $(u\nabla)u$ compared to $\partial u / \partial t$.

During an interval of time of the order of the oscillation period τ of the fluid particles in the wave, these particles travel a distance of the order of the a wave amplitude. Therefore, their velocity is of the order of $u \sim a/\tau$. The velocity u changes noticeably during time intervals τ of order and during distances of order λ along the direction of wave propagation (λ - wavelength).

Therefore, the time derivative of the velocity is of - order u/τ , and the coordinate derivative is of - order u/λ . Thus, the condition $(u\nabla)u << \partial u / \partial t$ is equivalent to the requirement

$$\frac{1}{\lambda}\left(\frac{a}{\lambda}\right)^2 << \frac{a}{\tau}\frac{1}{\tau} \qquad \text{or} \qquad a << \lambda$$

i.e. the amplitude of oscillations in the wave must be small compared to the wavelength.

Consider the propagation of waves in a channel (directed along the axis Ox) when the fluid flows along the channel. The cross-section of the channel may have an arbitrary shape and vary along its length as the liquid level changes, the cross-sectional area of the liquid in the channel is denoted by $h = h(x,t)$. The depth of the channel and the pool are assumed to be small compared to the wavelength.

Let's write Euler's equations in the form

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

where ρ - density, p - pressure, g - free fall acceleration. The velocity quadratic terms are omitted, since the wave amplitude is still considered small.

From the second equation we have that on the free surface $z = h(x,t)$, where $p = p_0$ the condition must be fulfilled:

$$p = p_0 + \rho g(h - z)$$

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}$$

To determine u and h, let us use the continuity equation as applied to the case under consideration. Consider the volume of liquid enclosed between two cross-sectional planes of the channel, located at a distance dx from each other. For a unit time through one cross-section x will enter the volume of liquid equal to $(hu)_x$. At the same time through the cross-section $x+dx$ will exit $(hu)_{x+dx}$. Therefore, the volume of liquid between the planes will change by

$$(hu)_{x+dx} - (hu)_x = \frac{\partial(hu)}{\partial x}dx$$

Due to incompressibility of liquid this change can occur only due to change of its level. Change of volume of liquid between considered planes in unit of time is equal to

$$\frac{\partial h}{\partial t}dx$$

Hence, it can be written:

$$\frac{\partial(hu)}{\partial x}dx = -\frac{\partial h}{\partial t}dx \quad \text{or} \quad \frac{\partial(hu)}{\partial x} + \frac{\partial h}{\partial t} = 0 \quad \text{or} \quad t > 0,\ -\infty < x < \infty$$

Since $h = h_0 + \xi$, where through h_0 - denotes the ordinate of the free surface of the fluid being in a state of relative equilibrium and changing exclusively under the action of gravity, then

$$\frac{\partial \xi}{\partial t} + h_0 \frac{\partial u}{\partial x} = 0$$

Thus, we obtain the following system of equations describing the fluid flow in the channel:

$$\frac{\partial \xi}{\partial t} + h_0 \frac{\partial u}{\partial x} = 0 , \qquad \frac{\partial u}{\partial t} + g \frac{\partial \xi}{\partial x} = 0 , \qquad t > 0 , \ -\infty < x < \infty$$