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THE PROBLEM SOLUTION OF NON-STATIONARY GAS FILTRATION IN THE CIRCULAR CLOSED VARIABLE THICKNESS BED AT NONLINEAR LAW OF RESISTANCE

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Abstract- flow of gases in a bed occurs at nonlinear law of resistance. This paper describes a solution of problems for variable thickness at the linear law of filtration of gases in a bed. The constant weight discharge of gas wells flow is analyzed in two stages. In the first stage of flow related equations are solved. The investigating gas flow in duration of the first stage is found out in the second stage. The results show that inconstancy of thickness essentially influences contour pressure decline and well production rate of contour pressure. Therefore, the thickness alteration of gas bed based on the solved equations influences well production rate and also bed pressure changing.

Keywords: Nonlinear Law of Resistance, Gas Filtration, Thickness Bed, Non-Stationery, Isothermal, Discharge Flow, Bed Distribution Pressure.

I. INTRODUCTION

In a nature oil and gas deposits are usually of variable thickness. It is known, that flow of gases in a bed occurs at nonlinear law of resistance. Therefore, the solution of problem about filtration of gases in view of change of bed thickness and nonlinearity of the law of resistance is great practical and theoretical interest. The solution of problems for variable thickness at the linear law of filtration is given in reference [1].

In this article a problem about gas filtration in the closed variable thickness bed at the nonlinear law of resistance is solved by a method of integral ratios, when constant weight discharge G and constant pressure p_w are set in a well.

Assume that circular closed gas-bearing bed of variable thickness of radius R exists. Before drilling-in the pressure in a bed is everywhere identical and equal to p_0. In the centre of a bed the single well of radius r_w is running [2].

II. THEORY SCHEMES

The differential equation of non-stationary, isothermal gas filtration in a variable thickness bed can be represented as

$$\frac{1}{F(r)}\cdot\frac{\partial}{\partial r}\left[F(r)\left(-a+\sqrt{a^2+4b\frac{\rho_{ta}}{p_{ta}}p\frac{\partial p}{\partial r}}\right]=2mb\frac{\rho_{ta}}{p_{ta}}\frac{\partial p}{\partial t} \quad (1)$$

where

$\frac{\partial p}{\partial r}=\frac{\mu}{k}v+b\rho v^2$ - the gradient of pressure;

p - pressure;

r - the current radius;

v - the filtration rate;

μ - the coefficient of gas viscosity;

k - the permeability index;

b - the turbulence factor;

$a=\frac{\mu}{k}, m$ - the factor of porosity of a bed;

$F=2\pi rh(r)$ - the current area of cross section of a bed;

$h(r)$ - the bed thickness.

Let's consider a case when it is given the constant weight discharge of a well. In the first stage of flow the solution of equation will be

$$p^2(r,t)=p_1(t)+p_2(t)\ln\frac{r}{l(t)}+p_3(t)\frac{l(t)}{r} \quad (2)$$

where $l(t)$ - the boundary of domain of influence moving in time and p_1, p_2, p_3 - some unknown factors which are defined from the following boundary conditions:

$$p[l(t),t]=p_0,\ \ p\frac{\partial p}{\partial r}\bigg|_{r=l(t)}=0 \quad (3)$$

$$p\frac{\partial p}{\partial r}\bigg|_{r=r_w}=\frac{Gp_{ta}}{\gamma_{ta}F(r_w)}\left(\frac{Gb}{gF(r_w)}+a\right)=B \quad (4)$$

In view of conditions in equations (3) and (4) from equation (2) we shall get

$$p^2(r,t)=p_0^2+\frac{2Br_w^2}{l(t)-r_w}\left(1-\ln\frac{r}{l(t)}-\frac{l(t)}{r}\right) \quad (5)$$

$l(t)$ is determined from the following integral ratio.

$$\frac{d}{dt}\int_{r_w}^{l(t)} F(r)p(r,t)dr = \frac{p_{ta}}{2mb\rho_{ta}}\left[\left[F(r)\left(-a+\sqrt{a^2+4b\frac{\rho_{ta}}{p_{ta}}p\frac{\partial p}{\partial r}}\right)\right|_{r_w}^{l(t)} + p_0F(l)\frac{dl}{dt}\right. \quad (6)$$

From here

$$t = \frac{1}{AF(r_w)}\left[\left[\int_{r_w}^{l(t)} F(r)\left[p_0 - p(r,t)dr\right]\right]\right|_{r_w}^{l(t)} \quad (7)$$

where

$$A = \frac{ap_{ta}}{2mb\rho_{ta}}\left(-1+\sqrt{1+4\frac{b}{a^2}\cdot\frac{\rho_{ta}}{p_{ta}}B}\right)$$

Let's substitute $p(r,t)$ from equation (5) into the expression (7) and expand the square root under integral in a series, where we take only two terms of expansion. Then we shall have

$$t = -\frac{Br_w^2}{p_0AF(r_w)}\cdot \left[\frac{1}{l(t)-r_w}\int_{r_w}^{l(t)} F(r)\left(1-\ln\frac{r}{l(t)}-\frac{l(t)}{r}\right)dr\right]\Bigg|_{r_w}^{l(t)} \quad (8)$$

Before investigating gas flow in the second stage, it is interesting to find out duration of the first stage. Setting various laws of variation for $F(r)$, we were convinced, that duration of the first stage is insignificant and does not exceed several minutes. The second stage of flow begins after $l(t)$ will reach impermeable boundary of a bed.

In the second stage distribution of pressure is defined by the formula

$$p^2(r,t) = p_c^2(t) + \frac{2Br_w^2}{R-r_w}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right) \quad (9)$$

where $p_c(t)$ is the contour pressure.

The integral ratio for definition of $p_c(t)$ takes the form

$$\frac{d}{dt}\int_{r_w}^{R} F(r)p(r,t)dr = -AF(r_w)$$

From here we determine

$$t-t_0 = -\frac{1}{AF(r_w)}\left[\int_{r_w}^{R} F(r)\cdot p(r,t)dr\right]\Bigg|_{p_0}^{p_c(t)} \quad (10)$$

where t_0 is the duration of the first stage of flow.

From the equation (10) we shall get

$$t-t_0 = \frac{1}{AF(r_w)}\left[\int_{r_w}^{R} F(r)\sqrt{p_0^2+\frac{2Br_w^2}{R-r_w}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right)}dr - -\int_{r_w}^{R} F(r)\sqrt{p_c^2(t)+\frac{2Br_w^2}{R-r_w}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right)}dr\right] \quad (11)$$

Calculations show that not tolerating poor accuracy, it is possible to disregard the second term of a radicand. Then the equation (11) will take the following type.

$$t-t_0 = \frac{1}{AF(r_w)}\left(p_0 - p_c(t)\right)\int_{r_w}^{R} F(r)dr \quad (12)$$

III. CALCULATIONS

The results of calculations according to the equation (12) even at sufficiently small bed pressure are in error less than 1% in comparison with the equation (11). Subject to equation (12) the expression (9) will be transformed as follows.

$$p^2(r,t) = \left[p_0 - \frac{AF(r_w)}{\int_{r_w}^{R} F(r)dr}(t-t_0)\right]^2 + + \frac{2Br_w^2}{R-r_w}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right) \quad (13)$$

Let's consider a case when constant pressure is given, i.e. in a well the following condition is fulfilled.

$$p(r,t)\big|_{r=r_w} = p_w \quad (14)$$

Based on the numerical research results, in this case duration of the first stage also does not exceed several minutes. This circumstance allows us to start directly with the second stage of flow. The distribution of pressure in a bed can be represented as follows:

$$p^2(r,t) = p_c^2(t) - \frac{p_c^2(t)-p_w^2}{1-\ln\frac{r_w}{R}-\frac{R}{r_w}}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right) \quad (15)$$

Therefore, the contour pressure $p_c(t)$ is defined from the following integral ratio.

$$\frac{d}{dt}\int_{r_0}^{R} F(r)p(r,t)dr = -E\left(-1+\sqrt{Mp_c^2(t)-D}\right) \quad (16)$$

So, after integration at $p_c(0) = p_0$ we shall obtain, that

$$t = -\frac{1}{E}\int_{p_0}^{p_c} \frac{d\left[\int_{r_w}^{R} F(r)\sqrt{p_c^2(t)-\frac{p_c^2(t)-p_w^2}{1-\ln\frac{r_w}{R}-\frac{R}{r_w}}\left(1-\ln\frac{r}{R}-\frac{R}{r}\right)}dr\right]}{-1+\sqrt{Mp_c^2(t)-D}} \quad (17)$$

where

$$E = \frac{ap_{ta}F(r_w)}{2mb\rho_{ta}}$$

$$M = -\frac{2b\rho_{ta}}{a^2p_{ta}}\cdot\frac{\frac{R}{r_w^2}-\frac{1}{r_w}}{1-\ln\frac{r_w}{R}-\frac{R}{r_w}}$$

$$D = -1+Mp_w^2$$

Similarly to the previous case we shall expand a square root under the second integral of equation (17) in a series with preservation of the first term only. Calculations have shown that the error in this connection makes less than 0.1%.

Then the equation (17) will become

$$t=-\frac{\int_{r_w}^{R}F(r)dr}{E}\cdot\int_{p_0}^{p_c}\frac{dp_c}{-1+\sqrt{Mp_c^2-D}} \qquad (18)$$

Opening integral we shall get

$$t=-\frac{\int_{r_w}^{R}F(r)dr}{E\sqrt{M}}\left[\ln\frac{p_c(t)\sqrt{M}+\sqrt{Mp_c^2(t)-D}}{p_0\sqrt{M}+\sqrt{Mp_0^2-D}}-\right. \qquad (19)$$

$$\left.-\frac{1}{\sqrt{1+D}}\ln\frac{\left(-1+\sqrt{Mp_c^1-D}\right)\left(D+p_c\sqrt{M(1+D)}+\sqrt{Mp_c^2(t)-D}\right)}{\left(-1+\sqrt{Mp_c^2(t)-D}\right)\left(D+p_0\sqrt{M(1+D)}+\sqrt{Mp_0^2-D}\right)}\right]$$

The mass gas flow rate, obtained from a well, is defined under the following formula:

$$Q=F\left(r_w\right)\frac{a}{2b}\left(-1+\sqrt{Mp_c^2(t)-D}\right) \qquad (20)$$

The offered solution provides sufficient accuracy both at the given discharge and at the given pressure that alongside with simplicity of calculation, enables to recommend it for practical calculations.

Let's compare results of calculations for cases of variable and constant thickness h_0 at identical volumes of a bed. We shall assume that a bed thickness varies according to the following equations.

$$h(r)=c_1\frac{r}{R}+c_2 \qquad (21)$$

$$h(r)=h\cos\frac{\pi r}{2R} \qquad (22)$$

and $h=\dfrac{h_0}{0,462}$.

For calculations we shall accept the following numerical data.

$m=0,2$

$\mu=0,0167$

$k=0,03,$

$\mu=0,0167,\ k=0,03,\ p_0$

$p_0=360\,ta,$

$\gamma_{ta}=0,8\,kg/m^3$;

$h_0=23,3\,m,$

$R=1000\,m,$

$r_w=0,1m,\ b=9\cdot10^8\,\dfrac{1}{m},\ c_1=20\,m,\ c_2=10\,m$

and $c_1=10\,m,\ c_2=30\,m.$

The value of *b* is taken from reference [2].

Let's note that at these numerical data duration of the first stage, calculated under the equation (8), makes accordingly 174.7; 141.6 and 136.5 [sec] at constant thickness, and also at linear and cosine-shaped change of thickness.

IV. CONCLUSIONS

Calculations show that inconstancy of thickness essentially influences contour pressure decline and well production rate. So, at linear change of thickness, when thickness is increased from a well to a contour of bed, the decline of contour pressure occurs weaker than at constant thickness, on the contrary, if thickness decreases in the direction of a contour of bed the contour pressure falls stronger. Besides, at cosine-shaped change of thickness the contour pressure declines stronger than at constant thickness.

At the same changes of thickness the well production rate is higher or lower than production rate, obtained at constant thickness of a bed.

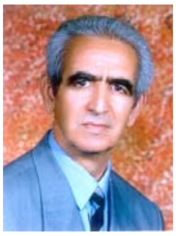
It is possible to show that at the given production rate the inconstancy of thickness of a bed essentially affects the distribution of pressure in a bed.

Thus, it is possible to conclude that change of thickness of a bed definitely influences well production rate and change of pressure in a bed.

REFERENCES

[1] Kh.M. Guliyev, "Manual on a Design Technique of Gas-Field Development", Part 2, BNIIGas, 1963.
[2] G.I. Barenblatt, "On Certain Approximate Methods in the Theory of One-Dimensional Unsteady Fluid Filtration at Elastic Control", Proceedings of the Academy of Sciences of the USSR, Tech. Sc. Depart., No. 9, 1954.

BIOGRAPHIES



Khalil Majid oglu Guliyev graduated from the mechanical-and-mathematical faculty of the Baku State University (Baku, Azerbaijan) on a specialty "Mechanical Mathematician". He worked as a junior and senior staff scientist at the research institute of oil engineering, as a teacher, senior teacher, assistant professor, head of the faculty, dean and rector at different universities. Currently, he is a professor doctor and author of 101 proceedings and journal papers. He has published 25 books, textbooks, manuals on higher and applied mathematics for students of universities. He has collected 18 methodical manuals, instructions for teachers of universities and 2 monographs where the basic results of my research activities.

He obtained inventors certificates for 4 inventions which were patents of England, France and Germany for two of inventions. His inventions have been received wide application in oil and gas industry of the former of the Soviet Union. The cost effectiveness of manufacturing application of his inventions accounts tens millions US dollars. He has two bronzes winner of All-Union Exhibition of the Soviet Union for his inventions. He was also the winner of competition of young scientists of the Soviet Union.

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