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POSITIONING CONTROL OF PM STEPPER MOTOR BASED ON TYPE-2 FUZZY ROBUST CONTROL

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Abstract- Permanent Magnet (PM) stepper motors are widely used in accurate systems which are affected by external disturbances and parameters uncertainty. Also, an appropriate nonlinear controller is needed when the problem is to track the reference signal. In this paper, a robust adaptive controller is presented to control rotor angular position in stepper motors. The main idea to make a robust controller is to use an adaptive control system based on type-2 fuzzy sets. Finally, simulations are implemented to control stepper motor position in two cases: certain and uncertain equations. Simulation results show that proposed controller has a better performance in tracking and robustness compare to type-1 fuzzy controller.

Keywords: Adaptive Fuzzy Control, Uncertainty, Permanent Magnet Stepper Motor (PM Stepper Motor), Robust Control.

I. INTRODUCTION

Stepper motor is an electromechanical nonlinear motor which has been designed to rotate in specific angular position. Stepper motors require simple and cheap controllers for position and speed control. Therefore, these motors are very popular in industrial applications and are widely used in different industries. DC motors were used in the past for positioning systems. Permanent magnet stepper motors have become a popular alternative to the traditionally used brushed DC motors (BDCM) for many high performance motion control applications for several reasons: better reliability because of the elimination of mechanical brushes, better heat dissipation as there are no rotor windings, higher torque-to-inertia ratio, lower price and easy interfacing with digital systems. The shaft or spindle of a stepper motor rotates in discrete step increments when electrical command pulses are applied to it in the proper sequence. The motors rotation has several direct relationships to these applied input pulses. So, the changes in shaft position can generate oscillations or cause a long delay in the output (torque) which is related to selected controller.

Today, PM stepper motors are widely used in numerous motion control applications such as robotics, printers, and digital control circuits and so on. Recently, various methods have been introduced for rotor positioning control and determination of proper control signals in PM stepper motors. It is important that a nonlinear controller will be required due to nonlinear structure of PM stepper motors while output tracking problem is represented. In recent decades, adaptive algorithms have been applied to PM stepper motors more than before [1]. On the other hand, the other methods such as sliding-mode control [2, 3] and adaptive robust control have been developed specially for uncertain systems. [4-7]. Also, many of the papers focus on system diagnosis and control based on neural networks. Neural networks are capable in learning input-output mapping rules of nonlinear and complicated systems and they are very popular due to this capability [8-10]. The neural network is designed based on RBF model and trained for stepper motor diagnosis. In addition, combination of fuzzy systems and adaptive control is used to design controllers especially in canonical systems [13, 14]. This paper presents an adaptive robust controller for angular positioning control in PM stepper motors. In this paper, we use type-2 fuzzy systems in order to make a robust controller because type-2 fuzzy systems have the capability to cover and minimize all uncertainties of the model. Design of this controller has been described in [15] completely.

II. MATHEMATICAL MODEL OF PM STEPPER MOTOR

In this paper, PM stepper motor has a two-winding stator and a permanent magnet rotor. In fact, it is a two-phase stepper motor. The PM stepper motor operates on reaction between magnetic flux of the rotor and electromagnetic field in the stator. The strength of electromagnetic field in the stator is proportional to the amount of current sent to the stator windings and the number of turns in the windings. The mathematical model of PM stepper motor is given below:

$$\begin{aligned} i_a &= \frac{1}{L}(V_a - RI_a + k_m\omega\sin(N_r\theta_r)) \\ i_b &= \frac{1}{L}(V_b - RI_b + k_m\omega\sin(N_r\theta_r)) \\ \dot{\omega} &= \frac{1}{J}(-k_m I_a \sin(N_r\theta_r) + k_m I_b \cos(N_r\theta_r) - \beta\omega) \\ \dot{\theta}_r &= \omega \end{aligned} \tag{1}$$

where, I_a is the current in winding A, I_b is the current in winding B, ω is the angular velocity of the motor's shaft, θ_r is the angular displacement of the shaft, N_r is the number of rotor teeth, V_a is the voltage across winding A, V_b is the voltage across winding B, J is the rotor and load inertia, β is the viscous friction coefficient, L and R are the inductance and resistance, respectively, of the phase windings, k_m is the motor torque constant. Above equations include nonlinear factors which make difficulties for the controller design. Another method for describing the system model is called "DQ model" which can transform equation (1) to more simple equation due to better controller design. DQ model is obtained from the transfer matrix as follows [15]:

$$\begin{bmatrix} I_d \\ I_q \\ \omega \\ \theta_r \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta_r) & \sin(N_r\theta_r) & 0 & 0 \\ -\sin(N_r\theta_r) & \cos(N_r\theta_r) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ \omega \\ \theta_r \end{bmatrix} \tag{2}$$

Voltage transfer matrix will be obtained from:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta_r) & \sin(N_r\theta_r) \\ -\sin(N_r\theta_r) & \cos(N_r\theta_r) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} \tag{3}$$

where, V_d and I_d are the direct-axis (d-axis) voltage and current V_q and I_q are the quadrature-axis (q-axis) voltage and current. According to equations (2) and (3), equation (1) can be rewritten as follows with respect to the new variables:

$$\begin{aligned} i_d &= \frac{1}{L}(V_d - RI_d + N_r\omega LI_q) \\ i_q &= (V_q - RI_q + N_r\omega LI_d - k_m\omega) \\ \dot{\omega} &= \frac{1}{J}(k_m I_q - B\omega) \\ \dot{\theta}_r &= \omega \end{aligned} \tag{4}$$

are defined as:

$$\begin{aligned} & D_1 = \frac{R}{L}, D_2 = \frac{k_m}{L}, \\ & D_3 = \frac{k_m}{J} \\ & D_4 = \frac{\beta}{J}, D_5 = N_r \\ & u_q = \frac{V_q}{L}, u_d = \frac{V_d}{L} \end{aligned} \tag{5}$$

According to above factors, (4) can be written as:

$$\begin{aligned} \dot{\theta}_r &= \omega \\ \dot{\omega} &= D_3 I_q - D_4\omega \\ \dot{I}_q &= -D_1 I_q - D_5\omega I_d - D_2\omega + u_q \\ \dot{I}_d &= -D_1 I_d + D_5\omega I_q + u_d \end{aligned} \tag{6}$$

We obtain from (6) that:

$$\begin{aligned} I_q &= -\frac{\dot{I}_q}{D_1} - \frac{D_5}{D_1}\omega I_d - \frac{D_2\omega}{D_1} + \frac{1}{D_1}u_q \\ I_d &= -\frac{\dot{I}_d}{D_1} - \frac{D_5}{D_1}\omega I_q + \frac{1}{D_1}u_d \end{aligned} \tag{7}$$

Equations (8) and (9) are obtained by substituting (7) into (6):

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= F + \frac{D_3}{D_1}(u_q - \frac{D_5}{D_1}\omega u_d) \end{aligned} \tag{8}$$

$$\begin{aligned} F = & -\frac{D_3}{D_1}i_q - \frac{D_3 D_2}{D_1}\omega + \frac{D_3 D_5}{D_1^2}\omega i_d \\ & -\frac{D_3 D_5^2}{D_1^2}\omega^2 I_q - D_4\omega \end{aligned} \tag{9}$$

By defining $x = [x_1, x_2]^{\mathrm{T}} = [\theta_r, \dot{\theta}_r]^{\mathrm{T}}$, equation (8) can be written as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= F(x) + \frac{D_3}{D_1}(u_q - \frac{D_5 x_2}{D_1}u_d) \end{aligned} \tag{10}$$

Adaptive fuzzy controller design must provide two following objectives in the presence of uncertainty and without it:

- System output θ_r follows θ_d reference value
- Closed-loop system be stable and all closed-loop variables be bounded.

To design such a controller, the following assumptions will be considered:

1) Reference vector, $\theta_d = [\theta_d, \dot{\theta}_d]$, is defined such that $\|\ddot{\theta}_d\| < \theta_1$ and $\|\theta_d\| < \theta_0$; where, θ_0 and θ_1 are known positive constants. Note that we need to determine the sign of $g(x)$ in order to design the controller. The sign of x must be constant over all domain. ψ is defined to determine the sign of $g(x)$ and its value is chosen by designer. Also, u_q, u_d are considered as the inputs, and defined as follows:

$$u_q = \psi,\ u_d = \psi u_c \tag{11}$$

where u_c is control signal generated by control system. Equation (12) can be obtained from (10) and (11).

$$\frac{D_3}{D_1}(u_q - \frac{D_5 x_2}{D_1} u_d) = \frac{D_3}{D_1}(\psi - \frac{D_5 x_2}{D_1})u_c = g(x)u_c \quad (12)$$

2) It is assumed that $g(x)$ is a continuous function and its sign, is definite for $x \in \Omega$, where Ωx is controllability area. In our proposed method, ψ is chosen such that $g(x) > 0$. Therefore, we obtain:

$$\frac{D_3}{D_1}(\psi - \frac{D_2 x_2}{D_1}) > 0 \Rightarrow \psi - \frac{D_2 x_2}{D_1} > 0$$
$$\psi - \frac{N_r \omega L}{R} > 0 \Rightarrow \psi > \frac{N_r \omega L}{R} \quad (13)$$

Also, state space equation can be written as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x) + g(x)u_c \quad (14)$$

III. PROPOSED ADAPTIVE FUZZY SYSTEM

In previous section, the equations of PM stepper motor were transformed to canonical form. In this section, the objective is applying a feedback controller, $u = u(x,\theta)$, based on type-2 fuzzy system. Also, we present an adjustment rules to regulate the vector of parameters such that the system output achieves the desirable output [15]. However, it is desired that the system output achieves to the desirable output as much as possible, it is much better that the system output converges toward the desirable output asymptotically. In specific cases, it is assumed that there is an accessible set of fuzzy if-then rules which describes the input-output behavior of $f(x)$ and $g(x)$. These rules can be rewritten as a form of interval type-2 fuzzy rules [15] as follows:

$$\text{if } x_1 \text{ is } \bar{F}_1^r \text{ and...and } x_n \text{ is } \bar{F}_n^r \text{ then } f(X) \text{ is } \bar{C}^r \quad (15)$$

which describe $f(x)$ and

$$\text{if } x_1 \text{ is } \bar{G}_1^s \text{ and...and } x_n \text{ is } \bar{G}_n^s \text{ then } g(X) \text{ is } \bar{D}^s \quad (16)$$

which describe $g(x)$. If nonlinear functions, $f(x)$ and $g(x)$, are specified, then we can choose the control vector (u), to omit the nonlinear part and design a controller based on linear control theory. In specific cases, it is assumed that $e = y_m - y$, $e = (e, \dot{e}, ..., e^{(n-1)})^{\mathrm{T}}$, $K = (k_n, ..., k_1)^{\mathrm{T}}$ and K is determined in such a way that all roots of the characteristic equation $(s^n + k_1 s^{(n-1)} + ... + k_n)$ lie in the left-half S plane (left hand side of imaginary axis). Then, we can select the control rules as follows:

$$u^* = \frac{1}{g(x)}[-f(x) + y_m^{(n)} + k^T e] \quad (17)$$

The closed-loop system dynamic is obtained by substituting (17) into (14), as:

$$e^{(n)} + k_1 e^{(n-1)} + ... + k_n e = 0 \quad (18)$$

If the value of K is selected properly, the $\lim_{t\to\infty} e(t) \to 0$. It means that the system output converges to the desirable output, asymptotically. Equation (17) which is related to ideal controller cannot be used if $f(x)$ and $g(x)$ are unknown. Under these circumstances, only fuzzy if-then rules can be used to describe input-output behavior of $f(x)$ and $g(x)$ (Equations (15), (16)). Therefore, a reasonable idea is to replace $g(x)$ and $f(x)$ by fuzzy functions, $\hat{g}(x)$ and $\hat{f}(x)$, which have been obtained from equations (15) and (16), respectively. Equations (15) and (16) just provide approximate information about $f(x)$ and $g(x)$ functions.

Therefore, the fuzzy functions, $\hat{g}(x)$ and $\hat{f}(x)$, are not accurate enough to estimate $f(x)$ and $g(x)$. To improve the accuracy, it is recommended to release some parameters which change online during the operation in such a way that approximation accuracy improves after a period of time. Assume that $\theta_g \in R^{M_g}$ and $\theta_f \in R^{M_f}$ are free parameters in $\hat{g}(x)$ and $\hat{f}(x)$ functions, respectively. Hence, we have $\hat{f}(x) = \hat{f}(x,\theta)$ and $\hat{g}(x) = \hat{g}(x,\theta)$. In equation (17), by substituting and $f(x)$ and $g(x)$ with $\hat{f}(x,\theta)$ and $\hat{g}(x,\theta)$, the fuzzy controller can be presented as:

$$u = u_l = \frac{1}{\hat{g}(X,\theta)}[-\hat{f}(X,\theta) + y_m^{(n)} + K_e^T] \quad (19)$$

This type-2 fuzzy controller is called "certainty equivalent" [20]. After modeling $\hat{f}(x,\theta)$ and $\hat{g}(x,\theta)$ using type-2 fuzzy system (the details of which can be found in [15]), and defining parameters such as: $Q, P, \gamma_1, \gamma_2, b, \Lambda, \omega, \theta_g^*, \theta_f^*$ [20, section 23.2.3] and considering positive definite Lyapunov function as equation (20),

$$V = \frac{1}{2}e^T Pe + \frac{1}{2\gamma_1}(\theta_f - \theta_f^*)^T(\theta_f - \theta_f^*) + ...$$
$$\frac{1}{2\gamma_2}(\theta_g - \theta_g^*)^T(\theta_g - \theta_g^*) \quad (20)$$

Voltage-time derivative ($\dot{V}$) through closed-loop path (6) is obtained as follows:

$$\dot{V} = -\frac{1}{2}e^T Pe + e^T Pb\omega ...$$
$$+\frac{1}{\gamma_1}(\theta_f - \theta_f^*)^T[\dot{\theta}_f + \frac{1}{2}\gamma_1 e^T Pb(\xi_l(X) + \xi_l(X))] + \quad (21)$$
$$\frac{1}{\gamma_1}(\theta_g - \theta_g^*)^T[\dot{\theta}_g + \frac{1}{2}\gamma_2 e^T Pb(\eta_l(X) + \eta_r(X))u_l]$$

To minimize tracking error (e), adaptation rules must be chosen such that $\dot{V}$ becomes negative definite. $\frac{1}{2}e^T Pe$ is a negative term and we are able to choose fuzzy systems in such a way that minimum approximation error (w) becomes small. Therefore, a good strategy is to select the adjustment rule such that the last two terms of equation (21) become zero. Hence, the adaption rules can be written as follows:

$$\dot{\theta}_f = \frac{-\gamma_1}{2} e^T pb(\xi_l(X) + \xi_r(X)) \tag{22}$$

$$\dot{\theta}_g = \frac{-\gamma_2}{2} e^T pb(\eta_l(X) + \eta_r(X))u_l \tag{23}$$

Effective quantities in above equations have been presented in [13]. Moreover, indirect adaptive type-2 fuzzy control system has been shown in Figure 1.

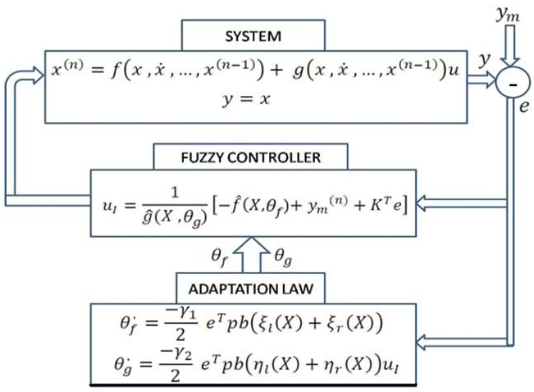

Figure 1. Indirect adaptive type-2 fuzzy control system

IV. SIMULATION RESULTS

In this section, we apply adaptive-fuzzy tracking control to canonical equation which obtained from section 2. Also, we compare the performance of type-1 and type-2 fuzzy systems in rotor angle tracking control. For this purpose, we need to use (14) in system block of block-diagram which has been shown in Figure 1. For the purpose of simulation, we use the following values for the system parameters, obtained from [14]:

$R = 10\ \Omega$, $L = 0.0011$ H, $K_D = 0$, $\beta = 0.001$ Nm.sec/rad

J=0.0000057 Kgm^2, $Nr = 50$, $Km = 0.113$ Nm/A

It must be noted that the friction torque has not been considered in (1). The purpose of control process is to have the rotor angular position follow the desirable path shown in Figure 2 with minimum error.

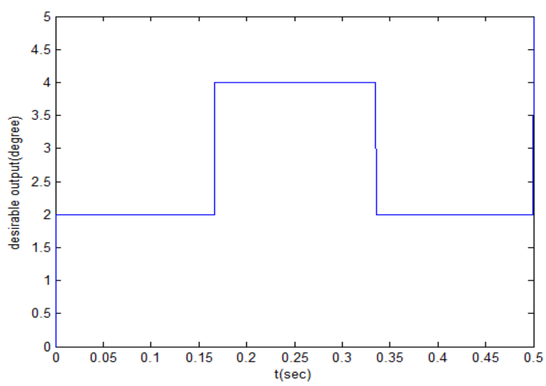

Figure 2. Desirable path for rotor angular position

To show the efficiency of proposed controller, in addition to certain model of the system, the experiments have been implemented in the presence of uncertainty. Moreover, controller parameters are the same as presented in [18] (see section 23.3). The membership functions for $\theta, \dot{\theta}$, have been shown in Figure 3. These type-2 fuzzy membership functions are triangular. For more information about type-2 fuzzy systems, see [15, 16].

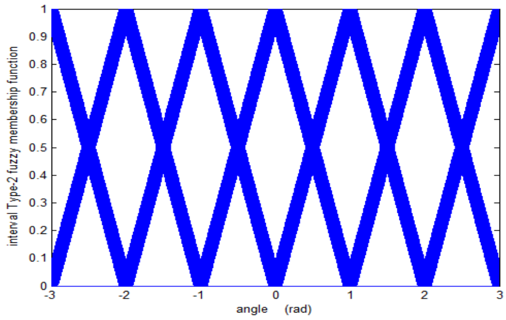

Figure 3. Type-2 fuzzy membership functions

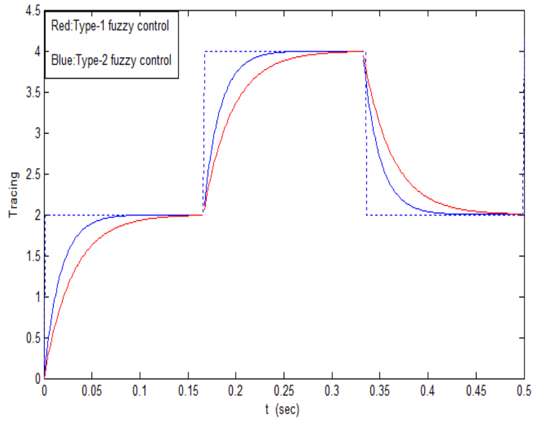

Figure 4. Type-1 Fuzzy Control (red), Type-2 Fuzzy Control (Blue)

Tracking performance for type-1 and type-2 fuzzy system, under certain model condition, is shown in Figure 4. It must be noted that type-1 fuzzy modeling is performed according to [20]. Also, model parameters are constant in both type-1 and type-2 fuzzy systems during simulation process.

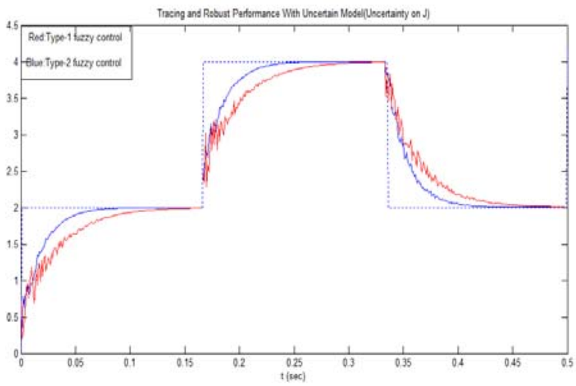

Figure 5. Tracing and robust performance with uncertain model (Uncertainty on J), Type-1 Fuzzy Control (Red), Type-2 Fuzzy Control (Blue)

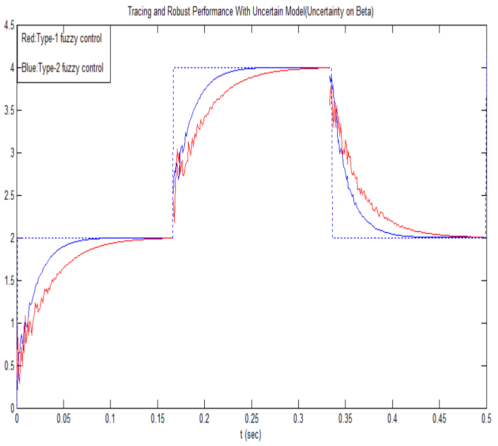

Figure 6. Tracing and robust performance with uncertain model (uncertainty on beta), Type-1 Fuzzy Control (Red), Type-2 Fuzzy Control (Blue)

It is observed from Figure 4 that the proposed type-2 fuzzy controller reaches to the desirable output faster than another controller. As shown in Figure 4, time constant of type-1 and proposed type-2 fuzzy controllers are 0.029 and 0.017, respectively. In addition, simulation results with the presence of system parameters uncertainty have been shown in Figures 5 and 6. It must be noted that uncertainty has been applied to rotor and load inertia (*J*) and viscous friction coefficient (β), separately. Also, a random noise with normal distribution around zero point has been applied to J and β in scale of 57×10^{-9} and 10^{-4}, respectively.

In Figures 5 and 6, uncertainty is applied to J and, respectively and tracking performance is compared for both adaptive type-1 and type-2 fuzzy controllers. It is obvious that proposed adaptive type-2 fuzzy controller has better performance in tracking control and also faster response rather than adaptive type-1 fuzzy controller. Moreover, proposed type-2 fuzzy controller is more robust against changes of the model. The reason is type-2 fuzzy modeling. Therefore, type-2 fuzzy logic systems are more robust against uncertainties.

V. CONCLUSIONS

According to simulation results and figures in previous section, it was clear that proposed adaptive type-2 fuzzy controller has a better tracking control and more robust response in comparison with type-1 fuzzy controller in the presence of both uncertainties and certainties because, type-2 fuzzy logic systems have better performance in the presence of system parameters uncertainties. In fact, using of indefinite (uncertain) membership functions and type-2 fuzzy methods to model nonlinear and indefinite functions, results in handling uncertainties much better than before. Therefore, the effect of uncertainties becomes minimum. Finally, it was shown that adaptive type-2 fuzzy controller provides more robust performance around operating point and the simulation results verified the main objective of the proposed controller which was accurate angular position control.

APPENDIX

Design of Indirect Adaptive Controller Based on Interval Type-2 Fuzzy System

Assume that there is a nonlinear system which can be presented with nth order differential equations as follows:

$$x^{(n)} = f(x,\dot{x},\ldots x^{(n-1)}) + g(x,\dot{x},\ldots x^{(n-1)})u \tag{23}$$

$$y = x \tag{24}$$

where f and g are uncertain, unknown, and nonlinear functions. $u\in R, y\in R$ are input and output of the process, respectively and $X=(x_1,\ldots,x_n)^{\mathrm{T}}\in R^n$ is a measurable state vector of the system. If $g(x)\neq 0$, it can be concluded that (23) is controllable. Here, the objective is to design a type-2 fuzzy feedback controller and present an adaptive rule to tune the parameter vector (θ) such that the system output (y) reaches to desirable output (y_m) as much as possible.

Due to design an indirect adaptive fuzzy controller, it is assumed that there is enough knowledge about control systems. Also, it is assumed that there is an accessible set of fuzzy if-then rules which can describe the input-output behavior of $g(x), f(x)$. These rules can be written as interval type-2 fuzzy rules as follows:

$$\text{If } x_1 \text{ is } F_1^r \text{ and} \ldots \text{and } x_n \text{ is } F_n^r \text{ then } \hat{f} \text{ is } \tilde{E}^r \tag{25}$$

$$\text{If } x_1 \text{ is } G_1^s \text{ and} \ldots \text{and } x_n \text{ is } G_n^s \text{ then } \hat{g} \text{ is } \tilde{H}^s \tag{26}$$

$s=1,2,,\ldots,M_g$, $r=1,2,\ldots,M_f$ which describe $f(x)$ and $g(x)$, respectively. If nonlinear functions, $f(x)$ and $g(x)$, are specified, then we can choose the control vector (u) to omit the nonlinear part and design a controller based on linear control theory.

In a specific case, it is assumed that $e=y_m-y, e=(e,\dot{e},\ldots,e^{(n-1)})^{\mathrm{T}}$ and $K=(k_n,\ldots,k_1)^{\mathrm{T}}$ which K is determined in such a way that all roots of characteristic equation (characteristic polynomial) lie in the left-half S plane (left hand side of imaginary axis). Then, we can select the control rules as follows:

$$u^* = \frac{1}{g(x)}[-f(x)+y_m(n)+K_e^T] \tag{27}$$

The closed-loop system dynamic is obtained by substituting (27) into (23), as:

$$e^{(n)}+k_1e^{(n-1)}+\ldots+k_ne=0 \tag{28}$$

If the value of K is selected properly, then $\lim_{t\to\infty} e(t)\to 0$. It means that the system's output converges to the desirable output, asymptotically. Equation (27) which is related to ideal controller cannot be used if $f(x)$ and $g(x)$ are unknown. Under these circumstances, only fuzzy if-then rules can be used to describe input-output behavior of $f(x)$ and $g(x)$ (Equations (25) and (26)). Therefore, a reasonable idea is to replace $f(x)$ and $g(x)$ in (27) by fuzzy functions,

$\hat{f}(x)$ and $\hat{g}(x)$, which have been obtained from (3) and (26), respectively. Equations (25) and (26) just provide approximate information about $f(x)$ and $g(x)$ functions. Therefore, the fuzzy functions, $\hat{f}(x)$ and $\hat{g}(x)$, are not accurate enough to estimate $f(x)$ and $g(x)$. To improve the accuracy, it is recommended to release some parameters which change online during the operation; in such a way that approximation accuracy improves after a period of time. Assume that $\theta_g \in R^{M_g}$ and $\theta_f \in R^{M_f}$ are free parameters in $f(x)$ and $g(x)$ functions, respectively. Hence, we have $\hat{f}(x) = \hat{f}(x,\theta)$ and $\hat{g}(x) = \hat{g}(x,\theta)$. In equation (27), by substituting $\hat{g}(x)$ and $\hat{f}(x)$ with $\hat{g}(x,\theta)$ and $\hat{f}(x,\theta)$, the fuzzy controller can be presented as:

$$u = u_I = \frac{1}{\hat{g}(X,\theta)}[-\hat{f}(X,\theta) + y_m^{(n)} + k^T e] \tag{29}$$

This type-2 fuzzy controller is called "certainty equivalent". Now consider interval type-2 fuzzy rules base with m rules as:

$$\text{If } x_1 \text{ is } F_1^i \text{ and} \dots \text{and } x_n \text{ is } F_n^i \text{ then } y \text{ is } \tilde{G}^r,\ \ i = 1,\dots,m \tag{30}$$

With considering product t-norm for combination of primary sets and after applying single fuzzy output, according to primary rules of fuzzy rules base which are related to designed fuzzy system, firing level is defined as:

$$F^i = \prod_{j=1}^{n} \mu_{\tilde{F}_j^i}(x_j) \quad , \quad i = 1,\dots,m \tag{31}$$

Because of applying type-2 fuzzy membership functions in fuzzy rules base, after applying single fuzzy input, F^i will be an interval type-1 fuzzy set. Then, (31) can be updated as follows:

$$F^i(x) = [\underline{f}^i(x), \bar{f}^i(x)] \tag{32}$$

where, $\underline{f}^i(x)$ are $\bar{f}^i(x)$ defined as:

$$\underline{f}^i(x) = \prod_{j=1}^{n} \bar{\mu}_{\tilde{F}_j^i}(x_j) \tag{33}$$

$$\bar{f}^i(x) = \prod_{j=1}^{n} \mu_{\underline{\tilde{F}}_j^i}(x_j) \tag{34}$$

$\mu_{\underline{\tilde{F}}_j^i}(x_j)$ and $\bar{\mu}_{\tilde{F}_j^i}(x_j)$ are the lower and upper bounding membership functions of $\mu_{\tilde{F}_j^i}(x_j)$, respectively.

The next step is the calculation of firing level corresponding to each rule. With considering product t-norm to calculate logic implication (entailment) of each rule, firing level corresponding to each rule will be the product of F^i and $\tilde{G}^i$. Since the obtained output of each rule is a type-2 fuzzy set before its firing, it must be transformed to a type-1 fuzzy set before deffuzification. In this paper, we use center-of-set (cos) type-reducer strategy [26] to obtain inference engine mapping from type-2 fuzzy rules base during the design of proposed controller.

The center-of-set type-reducer method acts such that $\tilde{G}^i$ in ith rules is replaced by its corresponding centroid which is a type-1 fuzzy set and finally, to compute inference engine mapping, it calculates a mean weight from these centroids where the weight corresponding to the centroid of *i*th rules will be F^i. Therefore, w^i that is corresponding to the centroid $\tilde{G}^i$, will be obtained as follows:

$$w^i = F^i(x) = \prod_{j=1}^{n} \mu_{\tilde{F}_j^i}(x_j), i = 1,2,\dots,m \tag{35}$$

If we show the centroid of $\tilde{G}^i$ by Y^i ($Y^i = C_{\tilde{G}^i}$), the inference engine mapping for the rules base of (30) will be as [27]:

$$Y_{\cos}(X) = \int_{y^1} \dots \int_{y^m} \int_{f^1} \dots \int_{f^m} 1 \Big/ \frac{\sum_{i=1}^{m} f^i y^i}{\sum_{i=1}^{m} f^i} = [Y_l, Y_r]\, st: \tag{36}$$

$$y^t \in \sup p(Y^t) = \sup p(C_{\tilde{G}^i})\ , i = 1,\dots,m$$

Also, the inference engine mapping to approximate $f(x)$ and $g(x)$ is obtained from (25), (26) as:

$$\hat{f}(X) = \int_{y^1} \dots \int_{y^{M_f}} \int_{f^1} \dots \int_{f^{M_f}} 1 \Big/ \frac{\sum_{i=1}^{M_f} f^i y^i}{\sum_{i=1}^{M_f} f^i} = [\hat{F}_l, \hat{F}_r]\ \ st: \tag{37}$$

$$y^i \in \sup p(Y^i) = \sup p(C_{\tilde{E}^i}),\ i = 1,\dots,M_f$$

$$\hat{g}(X) = \int_{y^1} \dots \int_{y^{M_g}} \int_{g^1} \dots \int_{g^{Mg}} 1 \Big/ \frac{\sum_{i=1}^{M_g} g^i y^i}{\sum_{i=1}^{M_g} g^i} = [\hat{G}_l, \hat{G}_r]\ \ st: \tag{38}$$

$$y^i \in \sup p(Y^i) = \sup p(C_{\tilde{H}^i}),\ i = 1,\dots,M_g$$

Above intervals must be calculated by Kernik-Mendel (KM) algorithm [38]. Now, assume that $y^i (i = 1,\dots,M_f)$ and $(i = 1,\dots,M_g)$ are free parameters gathered in $\theta_f \in R^{M_f}$ and $\theta_g \in R^{M_g}$, respectively. Now, we can rewrite (37) and (38) as:

$$\hat{f}(X,\theta_f)=\theta_f^T\xi(X)=$$
$$\int_{\theta_f^1}\cdots\int_{\theta_f^{M_f}}\int_{f^1}\cdots\int_{f^{M_f}}\frac{1}{\sum_{i=1}^{M_f}f^i\theta_f^i\Big/\sum_{i=1}^{M_f}f^i} \quad (39)$$

$$\hat{g}(X,\theta_g)=\theta_g^T\mu(X)=$$
$$\int_{\theta_g^1}\cdots\int_{\theta_g^{M_g}}\int_{g^1}\cdots\int_{g^{M_g}}\frac{1}{\sum_{i=1}^{M_g}g^i\theta_g^i\Big/\sum_{i=1}^{M_g}g^i} \quad (40)$$

where $\xi(X)$ and $\mu(X)$ are M_f and M_g dimensional vectors, respectively and their *i*th elements are calculated as:

$$\xi^i(x)=\frac{f^i}{\sum_{k=1}^{M_f}f^k} \qquad i=1,....,M_f \quad (41)$$

$$\eta^i(x)=\frac{g^i}{\sum_{k=1}^{M_g}g^k} \qquad i=1,....,M_g \quad (42)$$

After above calculations, two interval type-1 fuzzy sets, $\hat{g}(X,\theta_g)=[\hat{G}_l,\hat{G}_r]$ and $\hat{f}(X,\theta_f)=[\hat{F}_l,\hat{F}_r]$, are obtained. Since f and g are interval type-2 fuzzy sets, defuzzification stage [27] will be as:

$$\hat{f}(X,\theta_f)=\frac{\bar{F}_l+\bar{F}_r}{2},\hat{g}(X,\theta_g)=\frac{\bar{G}_l+\bar{G}_r}{2} \quad (43)$$

According to [27], $\hat{F}_l,\hat{F}_r,\hat{G}_l,\hat{G}_r$ can be written as:

$$\hat{F}_l=\theta_f^T\xi_l(X) \quad , \quad \hat{F}_r=\theta_f^T\xi_r(X) \quad (44)$$

$$\hat{G}_l=\theta_g^T\eta_l(X) \quad , \quad \hat{G}_r=\theta_g^T\eta_r(X) \quad (45)$$

where $\xi_l(X)$ and $\xi_r(X)$ are M_f dimensional vectors whose *i*th elements are obtained from:

$$\xi_l^i(X)=\frac{f_l^i}{\sum_{j=1}^{M_f}f_l^j},\xi_r^i(X)=\frac{f_r^i}{\sum_{j=1}^{M_f}f_r^j} \quad (46)$$

Similarly, $\eta_r(X)$ and $\eta_l(X)$ are M_g dimensional vectors whose *i*th elements are calculated from:

$$\eta_l^i(X)=\frac{g_l^i}{\sum_{j=1}^{M_g}g_l^j},\eta_r^i(X)=\frac{g_r^i}{\sum_{j=1}^{M_g}g_r^j} \quad (47)$$

The following equations are obtained by substituting (44) and (45) into (43):

$$\hat{f}(X,\theta_f)=\frac{1}{2}\theta_f^T(\xi_l(X)+\xi_r(X)) \quad (48)$$

$$\hat{g}(X,\theta_g)=\frac{1}{2}\theta_g^T(\eta_l(X)+\eta_r(X)) \quad (49)$$

By defining parameters and considering positive definite Lyapunov function as (50), voltage-time derivative ($\dot{V}$) through closed-loop path (28) is obtained as follows:

$$V=\frac{1}{2}e^TPe+\frac{1}{2\gamma_1}(\theta_f-\theta_f^*)^T(\theta_f-\theta_f^*)+...$$
$$\frac{1}{2\gamma_2}(\theta_g-\theta_g^*)^T(\theta_g-\theta_g^*) \quad (50)$$

$$\dot{V}=-\frac{1}{2}e^TPe+e^TPbw...$$
$$+\frac{1}{\gamma_1}(\theta_f-\theta_f^*)^T[\dot{\theta}_f+\frac{1}{2}\gamma_1e^TPb(\xi_l(X)+\xi_l(X))]+ \quad (51)$$
$$\frac{1}{\gamma_2}(\theta_g-\theta_g^*)^T[\dot{\theta}_g+\frac{1}{2}\gamma_1e^TPb(\eta_l(X)+\eta_l(X))u_I]$$

To minimize tracking error (*e*), adaptation rules must be chosen such that $\dot{V}$ becomes negative definite. $\frac{1}{2}e^TPe$ is a negative term and we are able to choose fuzzy systems in such a way that minimum approximation error (*w*) becomes small. Therefore, a good strategy is to select the adjustment rule such that the last two terms of (51) become zero. Hence, the adaption rules can be written as follows:

$$\dot{\theta}_f=\frac{-\gamma_1}{2}e^Tpb(\xi_l(X)+\xi_r(X)) \quad (52)$$

$$\dot{\theta}_g=\frac{-\gamma_2}{2}e^Tpb(\eta_l(X)+\eta_r(X))u_I \quad (53)$$

Indirect adaptive type-2 fuzzy control system has been shown in Figure 1, briefly.

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BIOGRAPHIES

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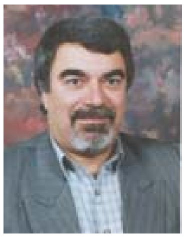

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