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APPLICATION OF FUZZY-GENETIC ALGORITHM FOR SOLVING AN OPEN TRANSPORTATION

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Abstract- The paper considers the problem of solving an open transportation problem in conditions when demand exceeds supply. The transportation problem is considered to have fuzzy objective, fuzzy constraints, and fuzzy decision variables. To solve this problem without adding any fictitious supplier is impossible. The proposed paper describes development of a new method that would provide a better and realistic solution to the transportation problem considering fuzzy conditions and that would be free of other complications of classical methods.

Keywords: Transportation Problem, Optimization Method, Fuzzy Conditions, Genetic Algorithm.

I. INTRODUCTION

A number of technical and economical problems related to planning and scheduling can be solved effectively by mathematical programming methods and in particular by linear programming methods. One of important problems which are traditionally solved by linear programming methods is the transportation problem. For example, there exist methods based on linear programming that allow optimal distribution of transportation resources for closed transportation problems to obtain maximum profit and minimize expenses by effectively assigning consumers to suppliers.

Some of the related methods belong, for example, Hitchcock method, Modified Distribution Method, Method of Potentials, Simplex Method and others. It should be noted that these methods are especially effective when volumes of supply and consumption are balanced. Unfortunately, these methods are not effective for solving open transportation problems. To allow solving of open transportation problems, in which consumption exceeds supply, the fictitious suppliers are added as suggested in [2, 4] and other works. A disadvantage of this approach is that some "unlucky" consumers will not be provided with required cargo (i.e. those consumers which would get them assigned to fictitious suppliers).

The suggested approach finds a near-optimal supplier-consumer solution. The given data, including constraints and preferences are described by fuzzy values and relationships and the optimization is done by genetic algorithm maximizing the fitness function of which most effectively combine all necessary objectives, constraints satisfaction, and preferences. The computer simulation is done on specific examples, which show high efficiency of the proposed method.

II. STATEMENT OF PROBLEM

At the decision of various problems of transport process the general performance of data selection and representation are necessary. For performance of these functions the special system software is created.

The increase in mobility, flexibility of information system concerns to advantages of the developed database. Thus integrity, consistency of the data is easily supervised. It represents the unified data set shared by participants of transport process. Its problem includes storage of the data required for management in a uniform place, exception of redundancy of the data, and minimization of probability of preservation of the inconsistent data. The program is a part of a database provides software for creation, loadings of inquiry, updating of the data and dialogue with a database. In the developed database input of the information is made on a condition of car in settlement base daily.

III. THE STATEMENT OF THE PROBLEM

In the most general form, the open transportation problem for assigning consumers to suppliers in conditions when the total volume of supplied cargo is lower than demanded cargo is formulated as follows. There are n supply source points (A_1, A_2, ..., A_n) and m consumer delivery points (B_1, B_2, ..., B_m). The supplied cargo volumes a_i of i-th supplier and demanded cargo volumes b_j of j-th consumer are known. Transportation expenses to deliver a unit (one ton) of cargo from a supplier i to a consumer j are also known. It is required to determine a transportation plan that would minimize the overall transportation expenses. The transportation process should be organized so that all cargo is transferred.

The objective for an optimal matching the consumers and suppliers is expressed mathematically as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} Z_{ij}X_{ij} \rightarrow \min \quad (1)$$

subject to the constraints:

$$\sum_{j=1}^{m} X_{ij} = a_i \ , \ i = 1,\ldots,n \quad (2)$$

$$\sum_{i=1}^{n} X_{ij} \le b_j \ , \ j = 1,\ldots,m \quad (3)$$

$$X_{ij} \ge 0 \ , \ i = 1,\ldots,n \ ; \ j = 1,\ldots,m \quad (4)$$

$$\sum_{i=1}^{n} a_i < \sum_{j=1}^{m} b_j \quad (5)$$

The problem is in identifying what consumers and how much cargo would receive when having minimized the overall transportation expenditure. We use the following denotations:
n - number of suppliers;
m - number of consumers;
a_i - volume of cargo with i-th supplier, i= 1, …, n;
b_j - cargo requirement of j-th supplier, j= 1, …, m;
Z_{ij} - expenditures related to delivery of 1 ton of cargo from i-th supplier to j-th consumer;
X_{ij} - volume of cargo to deliver from i-th supplier to j-th consumer, these are decision variables we need to determine.

In the considered model, the objective is to minimize the objective function (1) describing overall expenses for delivery of cargo from suppliers to consumers subject to constraints (2) and (3). The equations in system (2) reflect constraints on volumes of cargo that can be transported from each source point, whereas the equations in system (3) describe constraints on volumes of cargo that should be delivered to destination locations under specified tariffs Z_{ij}. The constraint (4) ensures the variables are not negative, and (5) reflects that demand is likely to be higher than supply.
Any collection of variables X_{ij} $(i = 1,\ldots,n \ ; \ j = 1,\ldots,m)$, satisfying constraints (1)-(5) can be considered a possible transportation plan. As an optimality criterion, in this transportation problem, we consider the physical work in tons-hours. The objective function is the sum of transportation tariffs multiplied by transported cargo volumes in tons. The transportation and distribution plan with minimal transportation expenses will be considered an optimal plan. Application of linear programming with fuzzy variables is considered in [5, 6, 7].

IV. SOLUTION METHOD

The considered problem is the problem with fuzzy objective, fuzzy constraints, and fuzzy variables. It is impossible or very difficult to solve this problem by traditional methods. Search for near-optimal solution is done on the basis of genetic algorithm and the method of optimization of distributed resources developed.

In this paper, the application of genetic algorithm for solving an open transportation problem is demonstrated on a specific example, the results of which have shown high efficiency of this approach for solving fuzzy linear programming problems. The source information about supply and delivery volumes is represented in form of fuzzy vectors:

$$\tilde{a}_i = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \quad (6)$$

$$\tilde{b}_j = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_m) \quad (7)$$

$$\tilde{x}_{ij} = (\tilde{x}_{11}, \tilde{x}_{12}, \ldots, \tilde{x}_{nm}) \quad (8)$$

Let's consider the following example of matching of consumers and suppliers, in which fuzzy data are used. Without loss of generality, but for the sake of simplicity, let's assume that $\tilde{a}_i$, $i = 1,\ldots,n$; $\tilde{b}_j$, $j = 1,\ldots,m$ and $\tilde{x}_{ij}$, $i = 1,\ldots,n$; $j = 1,\ldots,m$ are described by triangular fuzzy numbers in Table 1. The graphical representation of fuzzy number is shown in Figure 1. The expenditures related to the delivery of 1 ton of cargo from i-th supplier and j-th consumer are as Table 2.

Table 1. Triangular fuzzy numbers

Volume of delivery	Volume of consumption
$\tilde{a}_1 = [10,15,20]$	$\tilde{b}_1 = [35,40,45]$
$\tilde{a}_2 = [20,35,40]$	$\tilde{b}_2 = [32,40,42]$
$\tilde{a}_3 = [35,40,45]$	$\tilde{b}_3 = [50,80,85]$
$\tilde{a}_4 = [60,70,80]$	$\tilde{b}_4 = [20,40,50]$
	$\tilde{b}_5 = [5,10,15]$

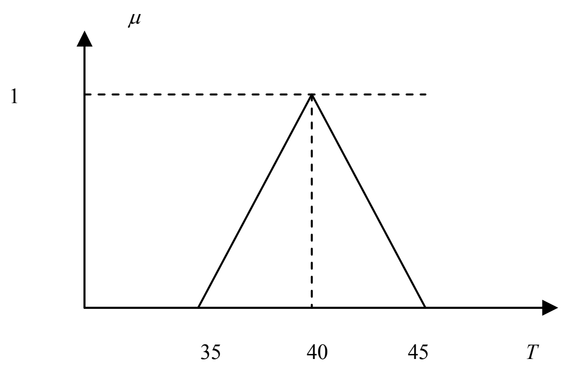

Figure 1. Graphical representation of fuzzy number

Table 2. Triangular fuzzy numbers

$z_{11} = 8$	$z_{12} = 12$	$z_{13} = 15$	$z_{14} = 23$
$z_{21} = 7$	$z_{22} = 10$	$z_{23} = 14$	$z_{24} = 11$
$z_{31} = 9$	$z_{32} = 11$	$z_{33} = 19$	$z_{34} = 14$
$z_{41} = 16$	$z_{42} = 14$	$z_{43} = 16$	$z_{44} = 18$
$z_{51} = 17$	$z_{52} = 20$	$z_{53} = 19$	$z_{54} = 20$

Let's denote the quantity of cargo transported from supplier i to consumer j as X_{ij}, then the constraints can be written as follows:

$$\begin{aligned} X_{11} + X_{12} + X_{13} + X_{14} &= \tilde{b}_1 \\ X_{21} + X_{22} + X_{23} + X_{24} &= \tilde{b}_2 \\ X_{31} + X_{32} + X_{33} + X_{34} &= \tilde{b}_3 \\ X_{41} + X_{42} + X_{43} + X_{44} &= \tilde{b}_4 \\ X_{51} + X_{52} + X_{53} + X_{54} &= \tilde{b}_5 \end{aligned} \quad (9)$$

$$\begin{aligned} &X_{11}+X_{21}+X_{31}+X_{41}+X_{51}=\tilde{a}_1\\ &X_{12}+X_{22}+X_{32}+X_{42}+X_{52}=\tilde{a}_2\\ &X_{13}+X_{23}+X_{33}+X_{43}+X_{53}=\tilde{a}_{13}\\ &X_{14}+X_{24}+X_{34}+X_{44}+X_{54}=\tilde{a}_4 \end{aligned} \quad (10)$$

A fuzzy solution can be obtained without the necessity to include a fictitious supplier (as it was in crisp case). In the considered example, when matching the suppliers and consumers, the demand exceeds the supply by 50 tons in average.

It is necessary to determine such values of X_{ij}, which would satisfy the conditions (9) and (10), and would lead overall transportation expenditure to a minimal value, i.e.

$$\begin{aligned} &P=8X_{11}+12X_{12}+15X_{13}+23X_{14}+7X_{21}+\\ &+10X_{22}+14X_{23}+11X_{24}+9X_{31}+11X_{32}+\\ &+9X_{33}+14X_{12}+16X_{41}+14X_{42}+16X_{43}+\\ &+18X_{44}+17X_{51}+20X_{52}+19X_{53}+20X_{54} \end{aligned} \quad (11)$$

The search for near-optimal solution in this open transportation problem is done by genetic algorithm [1]. The genetic algorithm for a specific function works as follows:

1. A genome structure is constructed to hold the solution as a bit string. A fitness function is constructed.
2. A population of k genomes is generated
3. Genetic operators crossover and mutation are applied to the genomes in the population to create new genomes.
4. Select better genomes based on value of fitness function.
5. Leave in the population L better genomes and generate (K-L) new ones. Continue the process while the fitness function stops to improve.
6. If the number of iterations is greater than Nu procede to step 7, else wise go to step 3.
7. Retrieve the solution from the best genome.

V. THE APPLICATION OF GENETIC ALGORITHM TO THE TRANSPORTATION PROBLEM

The transportation problem is described on the basis of Equation (1) defining the objective and Equations (2)-(5) defining constraints. The solution can be described in form of matrix with elements as X_{ij} in Table 3.

Table 3. Matrix elements of X_{ij}

X_{11}	X_{12}	X_{13}	X_{14}
X_{21}	X_{22}	X_{23}	X_{24}
X_{31}	X_{32}	X_{33}	X_{34}
X_{41}	X_{42}	X_{43}	X_{44}
X_{51}	X_{52}	X_{53}	X_{54}

Table 3 describes the case with 5 suppliers (i=1,...,5: first index) and 4 consumers (j=1,...,4: second index). For application of genetic algorithm (GA), we need to transform every its potential solution into a structure describing a genome in the population of many other structurally similar genomes. The structure of genome is most frequently represented as a continuous bit string. The length of this bit string depends on complexity of problem (number of parameters the values of which we need to optimize) and the required accuracy of solution.

To evaluate the fitness of a genome (a solution) GA utilize a fitness function. The higher its value, the better is the genome and consequently, the solution. The genomes for solutions that do not satisfy the constraints are assigned a minimum value of fitness function (e.g. 0). For the transportation problem, the fitness function F of an arbitrary solution $X^*=\{X_{ij}^*\}$ can be constructed as:

$$F=\begin{cases} 0, \text{ in the case of an infringement of the} \\ \quad \text{crisp constraints on the solution} \\ P_{\max}-P(X)+\varepsilon(1-\phi(X)), \text{ in other cases} \end{cases} \quad (12)$$

where $P(X)$ if the value of expenses related to the solution $X=\{X_{ij}\}$, $P_{\max}$ is the maximum possible value of $P(X)$, $\phi(X)$ is the degree of accomplishment of fuzzy constraints, ε is the balancing coefficient, required to balance effect of either component of the fitness function. The genome for the considered problem can be constructed as Table 4.

Table 4. Construction of genome

X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	…	X_{43}	X_{44}	X_{51}	X_{52}	X_{53}	X_{54}

As it was discussed above, each value is transformed from decimal to binary, for example, the value of X_{11}=19 will be represented as X_{11}=00010011. Thus, the genome will be represented as a bit string like the one that Genome k:

00010011, 00111001, 00......10, 10010001, 11100011

The number of such genomes (similar in structure, but different in content) is called a population size and is taken usually between 50-100 for average complexity problems and computers: k=1,...,K. The search for better solutions is done by GA by evaluating fitness function values of the new genomes generated during the evolution process. The new genomes are generated by applying specific genetic operators: crossover, mutation, selection, etc.

The quality of the solution found by GA is evaluated by the value of fitness function by better genome. To retrieve this solution, the genome with higher fitness is converted back to the table form. The GA generations are done iteratively and the process is stopped after we see that fitness function does further decrease significantly and required solution accuracy has been reached.

The solutions to the considered problem obtained by application of genetic algorithm for case 5 are shown in Table 5. Crisp solutions to the cases 1-6 are given in Table 6. The analysis results are presented in Table 7.

From the given analysis, it follows that acceptable cases are cases 4 and 5 with average expenses on transportation of 1 ton of cargo equal to 12.84 and 12.98 Manats(Az)/ton, respectively. Regarding the expenses for delivery the case 4 seems to be better, however, it is worse than case 5 for volumes and demand satisfaction coefficients.

The results of cases 4 and 5 obtained by GA were further optimized by the method developed by author. The obtained results are represented in Table 8. The objective function takes the following value:

$$P=14\times 8+25\times 15+1\times 7+1\times 14+38\times 11+$$
$$+31\times 11++24\times 14+4\times 14+9\times 16+8\times 18+$$
$$5\times 19=112+375+7+14+418+341+336+$$
$$56+144++144+95=2042 \quad (13)$$

Table 5. Solution of genetic algorithm for case 5

Decision x_{ij} variables	Fuzzy values x_{ij}	Defuzzified (crisp) values x_{ij}
x_{11}	[-1.00,8.99,18.99]	9
x_{12}	[6.48,6.48,6.48]	6
x_{13}	[6.46,17.46,28.46]	17
x_{14}	[0.04,0.04,0.04]	0
x_{15}	[-13.44,1.55,16.55]	2
x_{21}	[-11.65,3.35,18.35]	3
x_{22}	[-12.88,2.12,17.12]	2
x_{23}	[25.39,30.39,35.39]	30
x_{24}	[-5.62,4.38,14.38]	4
x_{25}	[14.79,25.78,36.79]	26
x_{31}	[-4.3,0.7,5.7]	1
x_{32}	[17.23,31.23,45.23]	31
x_{33}	[-14.9,0.1,15.1]	0
x_{34}	[14.54,0.46,15.46]	0
x_{35}	[2.5,18.5,34.5]	19
x_{41}	[7.48, 7.48, 7.48,]	7
x_{42}	[-4.99,0.0007,5]	0
x_{43}	[0.016, 0.016, 0.016,]	0
x_{44}	[-8.33,1.67,11.67]	2
x_{45}	[-12.05,2.95,17.95]	3

Table 6. Crisp solutions to the cases 1-6

Assigning a the supplier to consumer	Distribution case					
	1	2	3	4	5	6
A_1B_1	2	0	1	1	9	1
A_1B_2	10	17	9	9	6	9
A_1B_3	24	9	9	7	17	26
A_1B_4	4	3	0	0	0	1
A_1B_5	5	0	1	6	2	4
A_2B_1	2	5	15	13	3	8
A_2B_2	2	4	3	9	2	3
A_2B_3	19	18	17	18	30	18
A_2B_4	1	9	8	9	4	8
A_2B_5	11	7	1	6	26	7
A_3B_1	2	2	3	3	1	1
A_3B_2	29	44	45	45	31	41
A_3B_3	7	1	0	0	0	1
A_3B_4	10	0	5	1	0	7
A_3B_5	13	16	18	12	19	5
A_4B_1	10	0	1	0	7	7
A_4B_2	5	1	0	2	0	0
A_4B_3	1	0	0	1	0	1
A_4B_4	4	4	4	2	2	3
A_4B_5	3	0	0	0	3	1
Volume of demand and supply	164	140	140	144	162	152

Then we test for optimality and do optimization on the basis of the method [8] developed by the author. The suggested optimization method require less time for producing a near-optimal solution when compared with other methods such as Method of Potentials. The final result of case 4 after further optimization is shown in Table 9. The objective function value for the 5-th case is equal:

$$P=17\times 15+11\times 14+35\times 11+18\times 9+$$
$$+17\times 11++28\times 14+13\times 14+5\times 9=1812 \quad (14)$$

Average expenditure for delivery of one ton is $Z_{cp}=P/Q$=1812/144=12.58 Manats(Az)/ton (15)

The final result of case 5 after the optimization is presented in Table 10. The objective function of case 5 is equal to:

$$P=15\times 8+7\times 12+10\times 15+37\times 11+$$
$$+28\times 11++34\times 14+26\times 16+5\times 19=2056 \quad (16)$$

Table 7. Analysis results to the cases 1-6

Distribution case	Satisfaction degrees		Cargo volume	Transportation expenditure	Expenditure for one ton of cargo
	for supplier	for consumer			
1	0.9625	0.9598	164	2335	14.23
2	0.9	1	140	1889	13.49
3	0.903	1	140	1859	13.27
4	0.9076	0.9063	144	1849	12.84
5	1	1	162	2104	12.98
6	0.95	1	152	2035	13.39

Table 8. Results of cases 4 and 5 obtained by GA

Considered cases	Cargo volume	Transportation expenditure		Average expenditure for one ton of cargo		Difference
		Using GA	After further optimization	Using GA	After further optimization	
4	144	1849	1812	12.84	12.58	0.26
5	162	2104	2056	12.98	12.69	0.29

Table 9. Result of case 4 after further optimization

Consumers	Suppliers				Demand in cargo
	A_1	A_2	A_3	A_4	
B_1	8	12	15 17	23	17
B_2	7	10	14 11	11 35	46
B_3	9 18	11 17	19	14 28	63
B_4	16	14 13	16	18	13
B_5	17	20	19 5	20	5
Availability of cargo in tons	18	30	33	63	144

Table 10. Result of case 5 after further optimization

Consumers	Suppliers				Require ment in cargo
	A_1	A_2	A_3	A_4	
B_1	8 15	12 7	15 10	23	32
B_2	7	10	14	11 37	37
B_3	9	11 28	19	14 34	62
B_4	16	14	16 26	18	26
B_5	17	20	19 5	20	5
Availability of cargo in tons	15	35	41	71	162

The average expense for transportation of 1 ton of cargo is equal to:

$Z_{cp}=P/Q$=2056/162=12.69 Manats(Az)/ton (17)

For the optimal case the objective function is 26 units less than just after the application of genetic algorithm.

VI. CONCLUSIONS

It should be noted that fuzzy-genetic approach applied to solution of open transportation problem, demonstrated on concrete examples, has shown its high efficiency. The used genetic algorithm allowed to efficiently solution of problem described by fuzzy objective function and constraints, which would be very difficult or almost impossible by classical methods. Computational experiments proved the efficiency of the developed approach.

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