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OPTIMAL MULTISTAGE SCHEDULING OF PMU PLACEMENT FOR POWER SYSTEM OBSERVABILITY

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Abstract- The standards of power system monitoring, control and protection are elevated by Phasor Measurement Units (PMUs) as an auspicious tool. It is the purpose of this paper to determine the optimal number and locations of PMUs so as to make the system measurement model observable and also estimate the power system state. The proposed modelling approach which models PMU placement as an integer linear programming problems, is extended and generalized to satisfy various aspects of optimal phasor measurement unit (PMU) placement problem. Cases with and without zero injection measurements are considered. The optimal PMU placement problem can be achieved by using linear constraints. This implies that optimal PMU placement problem with zero injection busses can now be solved by standard ILP solvers. Subsequently, we also develop a simple and an effective methodology to handle single PMU outage as well as single line outage in the system. The robust performance of the proposed approach is confirmed on the IEEE 9, 14 and 57-bus test systems. The results obtained in this paper are compared with those published before in literature.

Keywords: Integer Linear Programming, Phasor Measurement Unit, Observability, Optimal Placement, Zero Injection Measurement.

I. INTRODUCTION

A power system state estimator uses information from various measurement monitoring systems to estimate the states of a power system. Until recently, it was not possible to measure the phase angle of the bus voltage in real time due to the technical difficulties in synchronizing measurements from distant locations. The advent of phasor measurement units (PMUs) all obviated this problem by synchronizing the voltage and current waveforms at widely dispersed locations with respect to a Global Positioning System (GPS) clock [1-3]. A suitable methodology is needed to determine the optimal locations of PMUs in a power system. In addition to its ability to measure voltage and current phasors, a state-of-the-art PMU may include other features such as protective actions.

The scope of the present paper is limited to the use of PMUs for state estimation. A power system is called completely observable only when all of its states can be uniquely determined [4, 5]. Beyond the number of PMUs required to make a system observable, a good PMU placement algorithm must also consider following additional issues;

1) Loss of a PMU or communication line;
2) Modelling of zero injection busses;
3) Phasing of PMU placement.

In recent years, there has been significant research activity on the problem of finding the minimum number of PMUs and their optimal locations. In [6], a simulated annealing method is used to find the optimal PMU locations. There is, however, a possibility that a placement set that can make the system observable be overlooked by this search process. In [7], a bisecting search method is implemented to find the minimum number of PMUs to make the system observable. The simulated annealing method is used to randomly choose the placement sets to test for observability at each step of the bisecting search. In [8-10], a genetic algorithm is used to find the optimal PMU locations. The minimum number of PMUs needed to make the system observable is found by using a bus-ranking methodology.

A methodology for PMU placement for voltage stability analysis in power system is developed in [11]. Reference [12] introduced a strategic PMU placement algorithm to improve the bad data processing capability of state estimation by taking advantage of PMU technology. In [13] and [14], the authors propose an exhaustive search-based methodology to determine the minimum number and optimal locations of PMUs for complete observability of the power system. Although the method gives the global optimal solution to the PMU placement problem, it becomes computationally intensive for large systems. An algorithm based on integer programming was successfully implemented to find the optimum number and the locations of the installed PMUs of the power system [15, 16].

The significant aim of this paper is to find the optimal number and locations of PMUs to make the system topologically observable. An integer linear programming (ILP) approach is used to determine the optimal number and locations of PMUs. The PMU placement methodology guarantees the system observability during normal operating conditions as well as single branch outages and single PMU outages. The proposed scheme is applied to the IEEE 9, 14 and 57-bus systems to verify its robust performance. A comparative analysis is done with those published before in literature.

II. INTEGER LINEAR PROGRAMMING FOR PMU PHASING

A PMU placed at bus *i* will measure the voltage phasor of bus *i* and a predetermined number of phasor currents of outgoing branches of that bus. The number of the measured current phasors depends on the number of PMU channels made available.

In this paper, it is assumed that a PMU placed at bus *i* will measure all current phasors of the branches connected to that bus, in addition to the voltage phasor of bus *i*. Therefore, with the absence of any conventional measurements in the system, bus i will be observable if at least one PMU is placed within the set formed by bus *i* and all buses incident to it.

Therefore, the objective of optimal PMU placement problem o is to determine the minimum number of PMUs so as to preserve the system observability. This objective can be formulated as:

$$\min \sum_{i \in I} x_i \tag{1}$$

$$f_i = \sum_{j \in I} a_{ij} x_j \quad , \quad \forall i \in I \tag{2}$$

$$f_i \geq 1 \quad , \quad \forall i \in I \tag{3}$$

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if buses } i \text{ and j are connected} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where x_i is a binary decision variable associated with bus *i*; *I* is the set of buses; f_i is the observability function related to bus *i*; a_{ij} is a binary decision variable associated with bus *i* and *j*.

In order to clearly explain the above formulation of integer linear programming for the case of full observability, the IEEE 9 bus system is used to illustrate it as shown in Figure 1. By rewriting the Equation (1) the following relations can be obtained:

$$OF: \ \min x_1 + x_2 + ... + x_9 \tag{5}$$

Subject to bus observability constraints defined as follows:

Bus - 1: $x_1 + x_4 \geq 1$ (6)

Bus - 2: $x_2 + x_8 \geq 1$ (7)

Bus - 3: $x_3 + x_6 \geq 1$ (8)

Bus - 4: $x_1 + x_4 + x_5 + x_9 \geq 1$ (9)

Bus - 5: $x_4 + x_5 + x_6 \geq 1$ (10)

Bus - 6: $x_3 + x_5 + x_6 + x_7 \geq 1$ (11)

Bus - 7: $x_6 + x_7 + x_8 \geq 1$ (12)

Bus - 8: $x_2 + x_7 + x_8 + x_9 \geq 1$ (13)

Bus - 9: $x_4 + x_8 + x_9 \geq 1$ (14)

The objective function in (5) is the total number of PMUs required for complete system observability, which has to be minimized. Solution of problem (5)-(14) shows that for full system observability, a minimum of three PMUs are required at busses 4, 6 and 8. As a matter of fact, the set of busses where the PMUs have to be installed correspond to a dominating set of the network. Therefore, minimum PMU placement problem maps to smallest dominating set problem on the network.

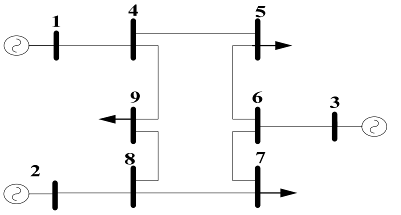

Figure 1. IEEE 9-bus system

III. A SINGLE PMU BEING OUT OF USE

The proposed PMU placement method is designed to maintain complete observability even in the case of the outage of any single PMU. In general, a bus is observed by only one PMU by using a direct or a pseudo measurement. The exceptions are cases, such as double lines between buses, where a bus may be observed more than once by the same PMU. A minimum redundancy level of one, therefore, ensures complete system observability, in general, for any single PMU outage. In order to increase the reliability of system monitoring, each bus should be observed by at least two PMUs. This ascertains that a PMU outage will not lead to loss of observability. With reference to the system of Figure 1, the minimum PMU placement problem for this system can be formulated as follows:

$$f_i + \sum_{j \in I} a_{ij} y_{ij} \geq 2 \quad , \quad \forall i \in I \tag{15}$$

This relation states that all of the buses will be observable in the case of a single PMU outage. When the second term of the equation 23 is equal to zero, it means that the bus *i* has not been observable by the influence of zero injection. Hence, constrain $f_i \geq 2$ makes the system be observable via two PMUs. However, if this term is equal to 1, it means that the bus *i* has been observable by the influence of zero injection. By rewriting this equation the following relations can be obtained:

$$OF: \ \min x_1 + x_2 + ... + x_9 \tag{16}$$

Subject to bus observability constraints defined as:

Bus - 1: $x_1 + x_4 \geq 2$ (17)

Bus - 2: $x_2 + x_8 \geq 2$ (18)

Bus - 3: $x_3 + x_6 \geq 2$ (19)

Bus - 4: $x_1 + x_4 + x_5 + x_9 \geq 2$ (20)

Bus - 5: $x_4 + x_5 + x_6 \geq 2$ (21)

Bus - 6: $x_3 + x_5 + x_6 + x_7 \geq 2$ (22)

Bus - 7: $x_6 + x_7 + x_8 \geq 2$ (23)

Bus - 8: $x_2 + x_7 + x_8 + x_9 \geq 2$ (24)

Bus - 9: $x_4 + x_8 + x_9 \geq 2$ (25)

The objective function in (16) is the total number of PMUs required for complete system observability, which has to be minimized. Solution of problem (16)-(25) shows that for full system observability, a minimum of four PMUs are required at busses 4, 5, 7 and 8 as depicted in Figure 2. If each one of these PMUs is out of use, the system observability will be guaranteed by the three left PMUs. Buses 4, 5, 6, 7, 8 and 9 will be observable by themselves our connected buses. Bus 3 will be observable via the zero injections installed in 6. Since the observability of bus 6 is resistance against the PMU outage, the observability of bus 3 will also be resistance. The similar observation can be found for buses 1 and 2.

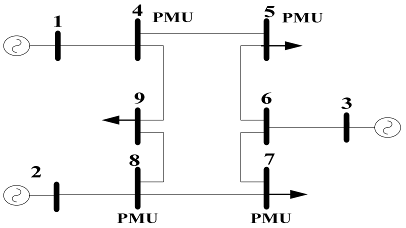

Figure 2. The observability of IEEE 9-bus system in the case of single PMU loss

IV. MODELLING OF ZERO INJECTION

Zero injection corresponds to the transhipment nodes in the system. At zero injection busses, no current is injected in to the system. If zero injection busses are also modelled in the PMU placement problem, the total number of PMUs can further be reduced. To understand this issue, consider the IEEE 9-bus system (Figure 1). In this system, zero injection busses are: {4, 6, 8}. Using the zero injection, the connected buses made observable, hence, the topology of system can be changed as it has been shown in Figure 3. Considering the busses {4, 6, 8} as zero injection busses and transmitting these buses to the one of the connected busses, {1, 3, 2}, and after changing the topology, the equation can be written as follow:

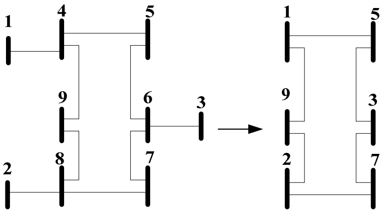

Figure 3. Transferring the zero injection buses

$$OF:\ \min x_1 + x_2 + x_3 + x_5 + x_7 + x_9 \tag{26}$$

Subject to bus observability constraints defined as:

Bus - 1: $x_1 + x_5 + x_9 \geq 1$ (27)

Bus - 2: $x_2 + x_7 + x_9 \geq 1$ (28)

Bus - 3: $x_3 + x_5 + x_7 \geq 1$ (29)

Bus - 5: $x_1 + x_3 + x_5 \geq 1$ (30)

Bus - 7: $x_2 + x_3 + x_7 \geq 1$ (31)

Bus - 9: $x_1 + x_2 + x_9 \geq 1$ (32)

The objective function in (26) is the total number of PMUs required for complete system observability, which has to be minimized. Solution of problem (26)-(32) shows that for full system observability, a minimum of two PMUs are required at busses 5 and 8 as depicted in Figure 4. As it obvious from the above equations, there is no need to write the zero injection bus and they can be omitted from the equations. In addition, number of PMUs decrease in this case.

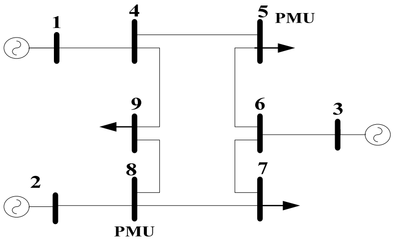

Figure 4. The observability of IEEE 9-bus system in the case of zero injection

V. A SINGLE PMU LOSS WITH MODELLING OF ZERO INJECTION

Similar to what stated in Sections IV and V, With reference to the system of Figure 1, the minimum PMU placement problem for this system can be formulated as follows:

$$OF:\ \min x_1 + x_2 + x_3 + x_5 + x_7 + x_9 \tag{33}$$

Subject to bus observability constraints defined as follows:

Bus - 1: $x_1 + x_5 + x_9 \geq 2$ (34)

Bus - 2: $x_2 + x_7 + x_9 \geq 2$ (35)

Bus - 3: $x_3 + x_5 + x_7 \geq 2$ (36)

Bus - 5: $x_1 + x_3 + x_5 \geq 2$ (37)

Bus - 7: $x_2 + x_3 + x_7 \geq 2$ (38)

Bus - 9: $x_1 + x_2 + x_9 \geq 2$ (39)

The objective function in (33) is the total number of PMUs required for complete system observability, which has to be minimized. Solution of problem (33)-(39) shows that for full system observability, a minimum of four PMUs are required at busses 1, 2, 5 and 7. As it is observed in this case, number of PMUs is reduced in compared with case without zero injection measurements

VI. A SINGLE TRANSMISSION LINE BEING OUT OF USE

The minimum PMU placement problem via the outage of a single line, With reference to the system of Figure 1, can be formulated as follows:

$$f_i^k = \sum_{j \in I} a_{ij}^k u_j + \sum_{j \in I} a_{ij}^k z_j y_{ij}^k \quad , \quad \forall k \in K \tag{40}$$

$$\sum_{i \in I} a_{ij}^k y_{ij}^k = z_j \quad , \quad \forall k \in K \tag{41}$$

$$f_i^k \geq 1 \quad , \quad \forall i \in I \quad , \quad \forall k \in K \tag{42}$$

These relations have been defined due to the outage of line *k*. The y_{ij} is a binary decision variable which is defined for all the zero injection buses.

The Z_j is a binary parameter which is equal 1, if bus j is a zero injection bus, otherwise is 0. The number of auxiliary binary variables for a non-zero injection bus is equal to the number of zero injection buses connected to it. In addition, the number of auxiliary binary variables for a zero injection bus is equal to the number of zero injection buses connected to it plus 1. The proposed procedure is repeated for $k \in K$ so as to model the outage of all lines. K is the set of transmission lines and the network connection new parameter is defined as follow:

$$a_{ij}^{k} = \begin{cases} 0 & \text{if line } k \text{ is between } i \text{ and } j \\ 1 & \text{otherwise} \end{cases} \tag{43}$$

If the PMU placement pattern needs to be resistance in response to the outage of two or more lines, the proposed procedure can be generalized. The arguable point here is the presence of parallel lines in the network. As an example, if there are two parallel lines between two considered buses, via the outage of a line the other line will be obviously in use in the network. Therefore, in the term of observability, there is still a connection between two buses and no change has happened in the network topology. As a result, these two lines will be eliminated from the set of transmission lines in the above formulation. The proposed model is applied on the 9-bus system shown in Figure 1.

For full system observability, a minimum of four PMUs are required at busses 1, 2, 3 and 6 as depicted in figure 5. The buses 1, 2, 3 and 4 have no sensibility to the network topology because of being observable by their own PMUs. Bus 4 is observable via the PMU installed in the bus 1. But in the case of the line 1-4 outage, it will be observable owing to zero injection. Bus 5 is observable via the PMU installed in the bus 6 and in the case of the line 6-5 outage it will be observable owing to zero injection installed in the bus 4. Bus 7 is in the same condition as bus 5. Bus 9 will be observable via the zero injections installed in 8 and 4 owing to the outage of lines 9-4 or 9-8, respectively.

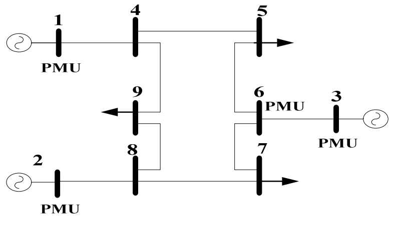

Figure 5. The observability of IEEE 9-bus system in the case of single line loss

VII. CASE STUDY

The proposed PMU placement method is applied to the IEEE 14-bus and 57-bus systems [11] as shown in figures 6 and 7, respectively. It is remarkable to mention that the ILP formulation for optimal PMU placement is able to combine different above- mentioned stages. This 57-bus system consist of 80 transmission lines, 15 zero injection buses and the maximum number of lines connected to one bus is 6. It should be cited that the single diagram of this network has been shown with 78 transmission lines. It is due to the fact that the double lines between buses 4 and 18 as well as buses 24 and 25 have been shown with a single transmission line. In most of the references these two double parallel lines have been considered as a single line. In order to make a comparison with other references each one of double lines has been assumed as a single transmission line. The simulation is carried out using the proposed integer linear programming algorithm for various scenarios as follow:

Scenario 1: Normal topology
Scenario 2: Modelling of zero injection
Scenario 3: Loss of a single PMU
Scenario 4: Loss of a single line
Scenario 5: Loss of a single PMU or Loss of a single line

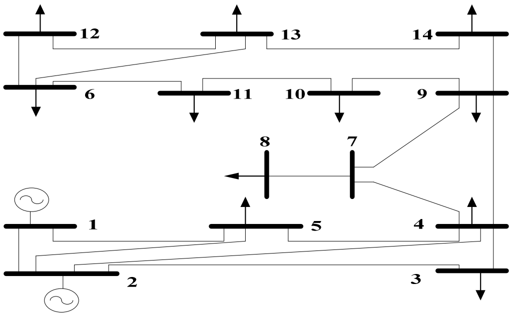

Figure 6. IEEE 14-bus test system

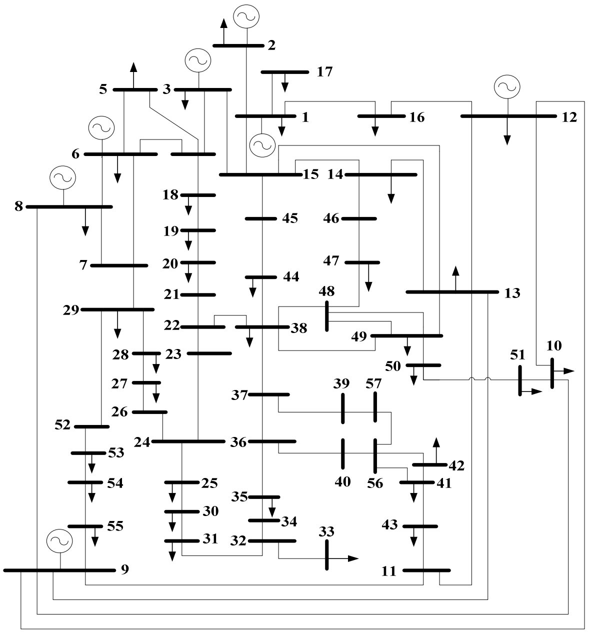

Figure 7. IEEE 57-bus test system

Table 1. Zero injection buses for different systems

System	Bus number
14-bus	7
57-bus	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48

Table 2. Optimal number and locations of PMUs for the 14-bus test system for different system configurations

Scenario	Minimum number of PMUs	Optimal PMU locations
1	4	2, 6, 7, 9
2	3	2, 6, 9
3	9	1, 2, 4, 6, 7, 8, 9, 10, 3
4	9	1, 2, 4, 6, 7, 8, 9, 10, 13
5	10	1, 2, 4, 6, 7, 8, 9, 10, 13,3

Table 3. Optimal number and locations of PMUs for the 57-bus test system for different system configurations

Scenario	Minimum numbr of PMUs	Optimal PMU locations
1	17	1, 4, 6, 9, 15, 20, 24, 28, 30, 32, 36, 38, 41, 46, 50, 53, 57
2	11	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56
3	26	1,2, 4, 6, 9, 12, 14, 19, 20, 24, 25, 27, 29, 30, 32, 33, 38, 41, 44, 46, 50,51, 53, 54, 56
4	19	1, 2, 6, 12, 14, 19, 21, 27, 29, 30, 32, 33, 41, 44, 49, 51, 53, 55, 56
5	26	1, 2, 4, 6, 9, 12, 14, 19, 20, 24, 25, 27, 29, 30, 32, 33, 36, 38, 41, 44, 46, 50, 51, 53, 54, 56

Table 4. CPU time required to find the optimal PMU locations

System	Computational time (Sec)	
	Scenario 1	Scenario 5
14-bus	2.96	9.78
57-bus	3.85	4.14

Table 5. Comparative analysis of proposed approach with other methods

S.No	GA [17]	IGA [18]	TS [19]	PSO [20]	SA [21]	IP [22]	ILP [23]	IQP [24]	P.M
1	-	-	-	-	-	-	17	17	17
2	12	11	13	11	11	12	14	-	11
3	-	-	-	-	-	-	29	-	26

Zero injection busses considered for various systems have been tabulated in Table 1. The minimum number and locations of PMUs for the 14-bus and 57-bus test systems have been brought out in Table 2 and 3, respectively. It is observed that the number of PMUs almost doubles if the system observability is to be maintained after single PMU loss. Considering zero injections with no PMU outage scenario, the number of PMUs required for system observability reduces, at most, by the number of zero injection busses (Table 2 and 3, column 2 , row-1 and 2). Obviously, the impact of zero injection buses is depends on the network topology and number of buses. Similarly, considering zero injections while maintaining system observability for a single PMU outage, the number of PMUs required reduces, at most, by twice the number of zero injection busses (Table 2 and 3, column 2 , row-3 and 4).

The similar result can be found for 14-bus system. As it can be seen from the tables, scenario 3 to 5 requires more PMUs for system observability owing to its resistance against the different circumstances. The result of PMUs placement in scenario 3 and 5 are the same. However, the PMU installed in bus 39 of state 3 has been transferred to the bus 36 in scenario 5. Simulation result on different IEEE standard systems have shown that the number of PMUs required for system observability in scenario 5 are more than scenario 3 and 4 as well as PMUs required in Scenario 3 are more than scenario 4. Although these results are true in most of the cases, but may not be generalized to all power systems. Table 4 shows the computational time requirements to find the optimal PMU locations for the case studies presented in this work. The simulations are carried out on an Intel Xeon 3.4-GHz CPU with 2-GB RAM.

According to the IEEE standard systems published in different papers, finding all of the simulation states for optimal PMU Placement may not be possible. Therefore, only some scenarios (S.No) are discussed for 57-bus test system as tabulated in Table 5. As it can be seen from the table, the minimum number of PMUs needed to make the system observable under normal operating conditions using proposed method (P.M) is the same as found in [21] and [22]. In the case of zero injection, the minimum number of PMUs is decreased from 12 [15, 20], 13 [17], 14 [21] to 11 using proposed method. The same number of PMUs can be obtained for other references compared with proposed technique. Additionally, the proposed scheme decreases the number of PMUs from 29 [21] to 26 in the case of single PMU outage.

A desirable property of a measurement placement scheme is to avoid critical measurements, the outage of which makes the system unobservable. The optimal PMU placement considering single-line or PMU outages, as own in the simulation results in this paper, improves the reability of the state estimator by eliminating the occurrence of any critical measurements. Into the bargain, the results show that the proposed technique is able to find optimal answers to the problem and its capability is more tangible in the case of large scale and complex mode (Scenario 3 to 5).

VIII. CONCLUSIONS

In this paper, a generalized integer linear programming formulation is proposed for determining the optimal number and locations of PMUs required making the entire power system observable. The proposed scheme considers situations with and without zero injection measurements, the possibility of single PMU loss and single transmission line loss into the decision strategy of the optimal PMU allocation problem. The proposed technique is easy in implementation. Considering single PMU loss as well as single line loss, a new concept is developed which keeps the entire system observable. The generalized formulation paves an efficient way for future research in PMU placement and related topics. The developed scheme is applied to different IEEE power systems (9-bus, 14-bus and 57-bus) and results are compared with those published in literature with very good agreement. The simulation Results demonstrate that: 1) optimal phasing can be computed efficiently; 2) developed scheme of modelling zero injection constraints improve computational performance; 3) proposed technique can be used in practice.

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