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ECONOMIC DISPATCH WITH PARTICLE SWARM OPTIMIZATION AND OPTIMAL POWER FLOW

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Abstract- As we know, the main subject of the economic dispatch is allocated for generator production. We must consider all limitations and economical mode and network operation. Power usages are organized in their efforts towards network utilization with high efficiency generation and low costs. Economic dispatch (ED) in large scale systems is completely complex non-linear problem and it is required by non-convex optimization. Global optimization methods can be named such as genetic algorithms (GA), particle swarm optimization algorithms (PSO), Taguchi algorithms and optimal power flow (OPF). In this paper, methods (PSO-OPF) are explained for ED with PSO and then for ED with OPF. Evolutionary strategies are used in recent decades due to their powerful search capabilities and their ability to influence the different types of cost functions, a growing number of researchers to solve problems by spreading economic partner, GA and PSO are implemented more in comparison with other methods and the number of articles that have been published in recent years. Heuristic optimization techniques are receiving great interests these days. Swarm intelligence (SI) is a type of heuristic optimization techniques.

Keywords: Economic Dispatch, Particle Swarm Optimization, Optimal Power Flow.

I. INTRODUCTION

In this paper the attention centers on economic dispatch with comparing the Particle Swarm Optimization (PSO) method and Optimal Power Flow (OPF) method. Economic dispatch problem has become a crucial task in the operation and planning of power system. Particle swarm optimization has been used to solve many optimization problems. In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group's previous best solution [13].

The original PSO described low is basically developed for continuous optimization problem. The objective of an Optimal Power Flow (OPF) method is to find steady state operation point which minimizes generation cost, losses, etc. or maximizes social welfare, load ability etc. Traditionally, classical optimization methods were used to effectively solve OPF. In recent years, Artificial Intelligence (AI) methods have been emerged which can solve highly complex OPF problems [10].

In section II, the concept of PSO is introduced. It presents the features and functions of PSO, and foundation so as to give a general picture of PSO. In section III, the economic dispatch with using PSO will be described and summarized for power systems respectively. In section IV the case study 1 and case study 2 which including 3 buses and 26 buses IEEE model is shown. The final results of the comparisons of algorithm are based on OPF, PSO are also presented. The conclusions are presented in section V.

II. FOUNDATIONS BY USING PSO METHOD

Particle swarm optimization (PSO) is a population based on computational technique inspired from the simulation of social behavior of flock of birds. PSO was originally designed and developed by Eberhart and Kennedy [16]. A newer version was introduced in 1998 by incorporating inertia weight.

In the group of the particles, the optimization problem is the same answers and they are scattered randomly in the search space. The position of these particles, which refers to their swarms, is collected from one another. The particles positions are updated by using their experiences and the experiences of neighboring particles. However PSO tries to find the optimal solution to the problem by moving the particles and evaluating the fitness of the new position. This update is done by the particle velocity vector [19].

The position vector and velocity vector of *i*th particle in a *d*-dimensional search space are expressed as follows [17, 18]:

$$X_i = (x_{i1}, x_{i2}, ..., x_{id}) \quad (1)$$

$$V_i = (v_{i1}, v_{i2}, ..., v_{id}) \quad (2)$$

The best previous position of a particle is recorded and displayed based on the evaluation function value as follows:

$$pbest = (p_{i1}, p_{i2}, ..., p_{id}) \quad (3)$$

If g as the particle has the best position in swarm in comparison with other particles then the situation is shown as below:

$$gbest = gbest_g = \left(p_{g1}, p_{g2}, ..., p_{gd}\right) \quad (4)$$

$$\left|gbest^k - gbest^{k-1}\right| < \varepsilon \quad (5)$$

Each particle tries to improve this position from personal best position (*pbest*) by using velocity and distance to global best position (*gbest*). Velocity and position of each particle in the current position will be applied for the particle position to fit in the next step, which is calculated by using the following formulas [17]:

$$\begin{aligned} v_{id}^{k+1} &= C[w \times v_{id}^{k} + c_1 \times \text{rand}_1(pbest_{id} - x_{id}) + \\ &+ c_2 \times \text{rand}_2(gbest_{gd} - x_{id})] \end{aligned} \quad (6)$$

$$x_{id}^{k+1} = x_{id} + v_{id}^{k+1} \quad (7)$$

$$c = \frac{Z}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|}, \quad \varphi = c_1 + c_2 \quad (8)$$

where c is constriction factor, w is inertia weight parameter, c_1 is cognitive coefficient, c_2 is social coefficient and $\text{rand}_1, \text{rand}_2$ are the random number between 1, 0.

The PSO parameters influence in optimization large amount of inertia weight parameter (w) has contributed to global search, while a small amount of it is the local identity. So, at the beginning of the search, we select large amount of inertia weight parameter and gradually decreases in the next iterations. So, inertia weight parameter is obtained by using the following equation:

$$w = (w_{\max} - w_{\min})\frac{(iter_{\max} - iter)}{iter_{\max}} + w_{\min} \quad (9)$$

where $iter$ is the number of current iterations and $iter_{\max}$ is the maximum number of iterations. Normally, parameter w can be changed between 0.4 and 0.9. Although the PSO method leads to acceptable answer by using w time variant, it is weak for global optimization.

Survey results indicate that the PSO method is optimal by setting parameters based on the nature and type of issue, a key factor in achieving accurate and efficient solution. On the one hand, if we choose large values for cognitive coefficient (c_1) in comparison with social coefficient (c_2), the particle trajectory is a large search space. In the other words, a relatively large amount of social coefficient leads the position of the particle in the premature local optimization.

The optimization methods based on PSO are:

• In the early stages of the search, the particles are emitted by the entire search space, without being trapped in local optimum points, and

• Next steps in the search, the particles are pushed towards the global optimum point,

accordingly, the optimal point to be achieved efficiently. In order to optimize, we use c_1 and c_2 time-varying acceleration.

The main idea is in the early stages of upgrades nationwide search, and then in the final stages of the search particles move in the direction of convergence towards the global optimal point. In this strategy, with the progress of the search process, cognitive coefficient (c_1) gradually declined, while the social coefficient (c_2) increases. With the large amount of momentum coefficient c_1 and a small amount of acceleration coefficient c_2 are allowed in search of the particles, rather than going to the local optimal point in their journey across the search space. On the other hand, the acceleration coefficient c_1 is small and c_2 is large that allows to the particles move to the global optimum point. c_1 and c_2 on the acceleration coefficient can be expressed:

$$c_1 = (c_{1f} - c_{1i})\frac{iter}{iter_{\max}} + c_{1i} \quad (10)$$

$$c_2 = (c_{2f} - c_{2i})\frac{iter}{iter_{\max}} + c_{2i} \quad (11)$$

where c_{1i} is initial cognitive coefficient:, c_{1f} is final cognitive coefficient, c_{2i} is initial social coefficient and c_{2f} is final social.

The original particle swarm optimization algorithm has undergone a number of changes since it was first proposed. Most of these changes affect the way the velocity of a particle is updated. In the following subsection, Discrete PSO briefly describes some of the most important developments [21].

Most particle swarm optimization algorithms are designed to search in continuous domains. However, there are a number of variants that operate in discrete spaces. The first variant proposed for discrete domains was the binary particle swarm optimization algorithm (Kennedy and Eberhart 1997). In this algorithm, a particle's position is discrete but its velocity is continuous. Velocities are updated as in the standard PSO algorithm, but positions are updated using the following rule [21]:

$$x_{id}^{k+1} = \begin{cases} 1 & \text{if } R < Sig\left(v_{id}^{k+1}\right) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$Sig(x) = \frac{1}{1 + e^{-x}} \quad (13)$$

Figure 1. Food searching of a swarm of birds mimetic the PSO

The basic idea of PSO based on food searching of a swarm of animals, such as fish flocking or birds swarm as depicted in Figure 1. Figure 2 shows the absolute convergence of particles to reach the spot [2, 11].

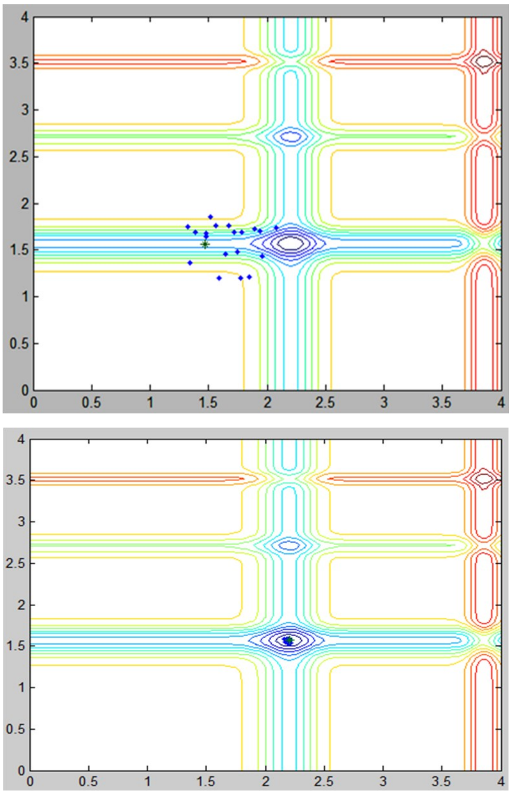

Figure 2. Convergence of particles

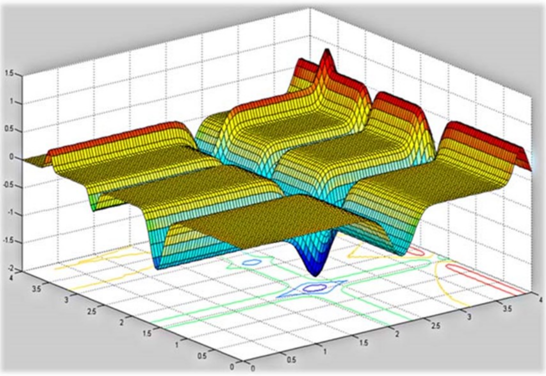
Figure 3. Example of the new particle position after many iterations

III. ED WITH PSO

As mentioned earlier, the PSO algorithm for solving the complex problems of ED, fuel costs, capacity limitations and other constraints will be considered. The PSO is a property of absolute convergence.

A. The ED Model [18, 24, 25]

1) Specification of the objective function

$$\min \ cost = \sum_{i}^{N_g} F_i(P_i) \tag{14}$$

where N_g is the number of units (generators), and F_i (P_i) is cost function for ith unit.

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{15}$$

where a_i, b_i, c_i are the coefficients of the cost of ith generator.

2) Limitations (Constraints):

a) Limitations for the operation of the unit:

$$P_i^{\min} \le P_i \le P_i^{\max} \quad , \quad i = 1, 2, ..., N_g \tag{16}$$

The $P_i^{\min}$ and $P_i^{\max}$ are minimum and maximum generation, respectively.

b) Balance of power:

$$\sum_{i=I}^{Ng} P_i = P_L + P_D \tag{17}$$

where P_L is the loss function and P_D is the load power losses can be obtained from matrix format (B matrix). The P_L can be obtained with loss matrix.

$$P_L = P^T BP + P^T B_0 + B_{00} \tag{18}$$

where B_0 and B_{00} are the coefficients of the loss matrix.

3) Constraint of power transition:

$$|Lf_i| \le Lf_i^{\max} \quad , \quad i = 1, 2, ..., N_L \tag{19}$$

where $Lf_i^{\max}$ is the maximum allowable power from transmission line based on MW and N_L is the transmission line.

4) Limitations on network stability:

$$|\delta_i - \delta_j| \le \delta_{ij}^{\max} \ , \ i, j = 1, 2, ...N_D \ , \ i \ne j \tag{20}$$

where δ_{ij} is the first voltage angle (load angle) at the bus of (i, j) and $\delta_{ij}^{\max}$ is the maximum voltage angle. The (i-j) are indicators of the line i-j and N_D is the number of bus that has limitation for network stability.

B. The Known Functions of PSO

The known functions PSO adopted here are four benchmark functions used by many researchers. They are the sphere, Griewank, Rastrigrin and Rosebrock functions [20]. The definition of the sphere function is

$$f(x_i) = \sum_{i=1}^{n} x_i^2 \tag{21}$$

where n is the dimension of the sphere. The n-dimensional Griewank function is defined as

$$f(x_i) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \frac{x_i}{\sqrt{i}} + 1 \tag{22}$$

The definition of the Rastrigrin function is

$$f(x_i) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10) \tag{23}$$

The definition of the Rosenbrock function is

$$f(x_i) = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2) \tag{24}$$

C. Multimodal Functions of PSO [14]

Multiple peak functions are often used to validate new algorithms. We can construct the following function with multiple peaks,

$$F(x,y) = \frac{\sin(x^2+y^2)}{\sqrt{x^2+y^2}} e^{-\lambda(x-y)^2} \quad , \quad \lambda>0 \tag{25}$$

where (x, y) is $[0, 5]\times[0, 5]$. Obviously, for a minimization problem, we can write it as

$$F(x,y) = -\frac{\sin(x^2+y^2)}{\sqrt{x^2+y^2}} e^{-\lambda(x-y)^2} \quad , \quad \lambda>0 \tag{26}$$

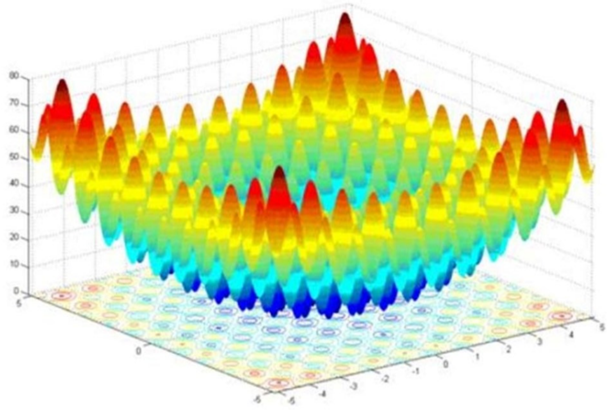

Figure 4. Example of multimodal optimization problem [14]

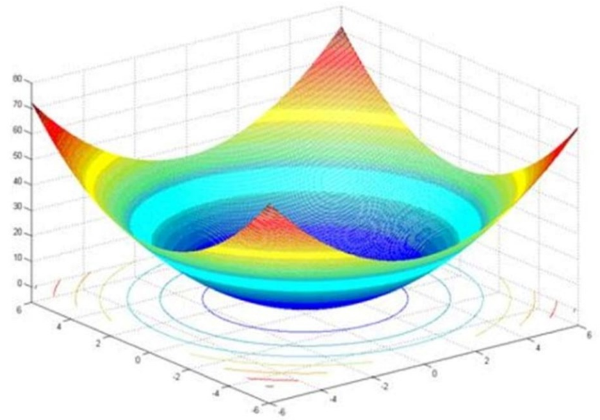

Figure 5. Example of unimodal optimization problem [14]

D. Implementation

The accelerated particle swarm optimization has been implemented by using Matlab. If we run the program, we will get the global optimum after about 200 evaluations of the objective function (for 20 particles and 10 iterations). The results are shown in Figure 2.

E. Limitations

For constrained optimization, there are many ways to implement the constraint equalities and inequalities. However, we will only discuss two approaches: direct Implementation and transform to unconstrained optimization. The new solutions are evaluated by using the standard PSO procedure. In this way, all of the new locations should be in the feasible region, and all infeasible solutions are not selected. There are other variations of particle swarm optimization, and PSO algorithms are often combined with other existing algorithms to produce new hybrid algorithms. In fact, it is still an active area of research with many new studies are being published each year.

F. The ED Explanation with OPF Method

The classical method such as traditional method, Newton-Raphson method and Decoupled method will be analyzed. These methods can be used online but the weak point is minimum local and is not global minimum. In all of mentioned methods, the aim of reducing losses is classic method focusing on value of angle load and voltage.

1) Real power system losses

$$P_L = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N_g} P_{gi} - \sum_{i=1}^{N} P_{li} \tag{27}$$

2) Real power of ith bus

$$P_i = \sum_{k=1}^{N} |V_i||V_k|[G_{ik}\cos(\delta_i-\delta_k) - B_{ik}\sin(\delta_i-\delta_k)] \tag{28}$$

3) Reactive power of ith bus

$$Q_i = \sum_{k=1}^{N} |V_i||V_k|[G_{ik}\sin(\delta_i-\delta_k) - B_{ik}\cos(\delta_i-\delta_k)] \tag{29}$$

4) The fuel cost function

$$F_{c,total} = \sum_{i=1}^{N_g} F_{ci} = \\ = \sum_{i=1}^{NG} \alpha_i (P_{gi})^2 + \beta_i P_{gi} + \gamma_i \text{ , unit of cost / hr} \tag{30}$$

5) The Lagrange function [3]

$$L(P_g, |V|, \delta) = \sum_{i=1}^{N_g} F(P_{gi}) + \sum_{i=1}^{N} \lambda P_i [P_i(|V|,\delta) - P_{gi} + P_{li}] + \\ + \sum_{i=N_g+1}^{N} \lambda Q_i [Q_i(|V|,\delta) - Q_{gi} + Q_{li}] \tag{31}$$

IV. CASE STUDIES

A. Case Study 1: 6-generator for 26-Bus [4]

Computation in this paper is based on a three-bus system with two generators and a system including of 26-bus and six generators, the circuit is shown in Figure 6 [4]. The Matlab software is applied for this paper. The cost functions are as follows:

$$F_{C1} = 0.007P_{g1}^2 + 7P_{g1} + 240$$

$$F_{C2} = 0.0095P_{g2}^2 + 10P_{g2} + 200$$

$$F_{C3} = 0.009P_{g3}^2 + 8.5P_{g3} + 200$$

$$F_{C4} = 0.009P_{g4}^2 + 11P_{g4} + 200$$

$$F_{C5} = 0.008P_{g5}^2 + 10.5P_{g5} + 220$$

$$F_{C26} = 0.0075P_{g26}^2 + 18P_{g26} + 190$$

Table 1. The results of the PSO method for IEEE network 26-bus

METHOD	PSO
P_D (MW)	1263
P_L (MW)	15.53
P_{g1} (MW)	472.552
P_{g2} (MW)	179.903
P_{g3} (MW)	201.453
P_{g4} (MW)	149.998
P_{g5} (MW)	196.496
P_{g26} (MW)	104.518
Total power (MW)	1304.92
Total cost (unit of cost /hr)	16492.92039

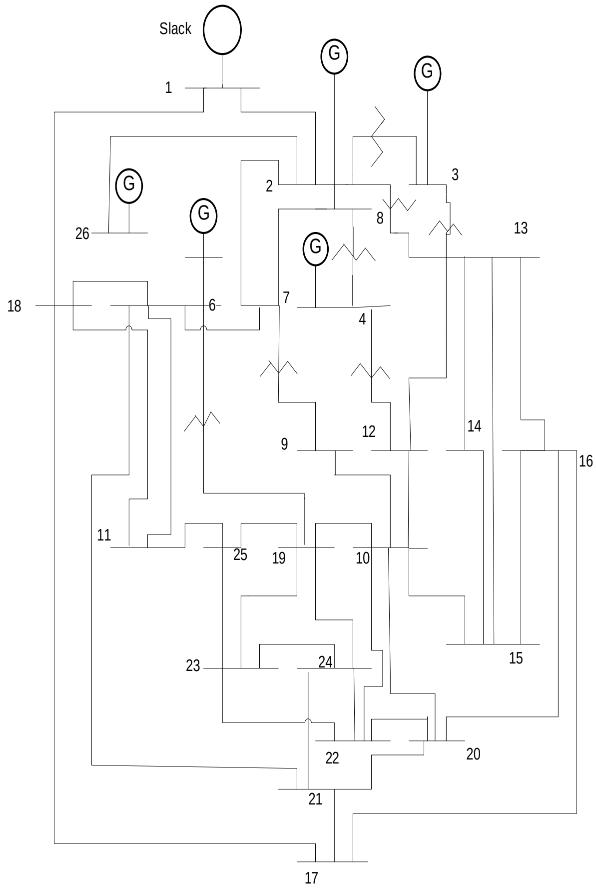

Figure 6. IEEE system 26-bus

B. Case Study 2: 2-generator for 3-Bus [1]

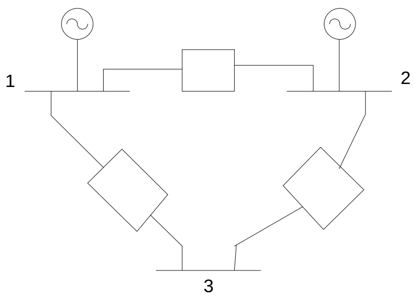

Figure 8. IEEE System 3-bus

Table 2. Line data of the system

Line no	From bus	To bus	Line impedance (p.u)	$B/2$ (p.u)
1	1	2	(0.05+j0.3)	j0.01
2	1	3	(0.05+j0.3)	j0.01
3	2	3	(0.05+j0.3)	j0.01

Table 3. Data bus of the system

Bus no	Bus type	V (p.u)	P_g (p.u)	Q_g (p.u)	P_d (p.u)	Q_d (p.u)
1	Slack	1.02	?	?	0.2	0
2	PV	1.01	?	?	0.1	0.15
3	PQ	?	0	0	0.25	0.1

The cost function for 3-bus network is as follows:

$$F_{C1} = 20P_{g1}^{\;2} + 175p_{g1} + 50$$

$$F_{C2} = 30P_{g2}^{\;2} + 180P_{g2} + 40$$

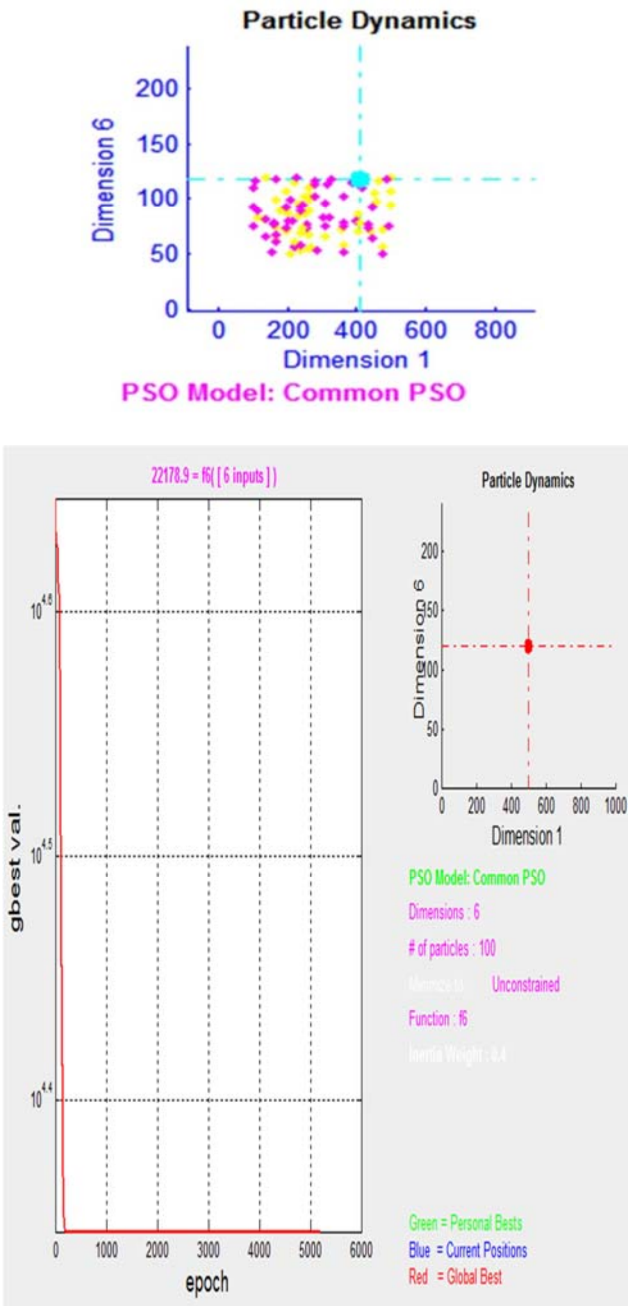

Figure 7. The final curve obtained from the convergence of particles in the system, the IEEE 26-bus

C. Optimal Power Flow Using Classical Methods [1, 6]

$$\left[\frac{\partial P_{g2}}{\partial \delta_2}\right]\left[1-\frac{\partial P_l}{\partial P_{g2}}\right] = -\left[\frac{\partial P_{g1}}{\partial \delta_2}+\frac{\partial P_3}{\partial \delta_2}\right] \quad (32)$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i V_K|\left[G_{ik}\sin\left(\delta_i-\delta_k\right) - B_{ik}\cos(\delta_i-\delta_k)\right] \quad (33)$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{i=1\\K\neq I}}^{N} |V_i V_K|\left[-G_{ik}\sin\left(\delta_i-\delta_k\right) + B_{ik}\cos(\delta_i-\delta_k)\right]$$

for $i = 2,3,\ldots,N$ (34)

for $k = 2,3,\ldots,N$

D. Newton-Raphson Method [1, 2]

i) Real power balance in the network

$$P_i\left(\left|V\right|,\delta\right)-P_{gi}-P_{li}=0\ ,\ i=1,2,3,....,N \quad (35)$$

where P_i is real power injection at ith bus and is a function of $\left|V\right|,\delta$. P_{gi},P_{li} are also the real power generation and loading.

$$i=(N_g+1),(N_g+2),...,N\quad ,\quad P_{gi}=0$$

ii) Reactive power balance in the network

$$Q_i\left(\left|V\right|,\delta\right)-Q_{gi}-Q_{li}=0\ ,\ i=(N_g+1),(N_G+2),....,N \quad (36)$$

where Q_i is reactive power injection at ith load bus and is also a function of $\left|V\right|,\delta$. The Q_{gi},Q_{li} are also the reactive power generation and loading.

iii) Limits on real power generation and reactive power generation

$$P_{gi_{\min}}\le P_{gi}\le P_{gi_{\max}}\ ,\ i=1,2,....,N_g \quad (37)$$

$$Q_{gi_{\min}}\le Q_{gi}\le Q_{gi_{\max}}\ ,\ i=1,2,....,N_g \quad (38)$$

Limits on the voltage angels

$$\delta_{i_{\min}}\le\delta_i\le\delta_{i_{\max}}\ ,\ i=1,2,....,N \quad (39)$$

Limits on the voltage magnitudes of load buses

$$\left|V_i\right|_{\min}\le\left|V_i\right|\le\left|V_i\right|_{\max}\ ,\ i=(N_g+1),(N_g+2),....,N \quad (40)$$

H.[Changein control variables] = J

where H is the Hessian matrix and J is the Jacobian matrix.

The control variables are

[ΔP_{g1}, ΔP_{g2}, $\Delta\delta_2$, $\Delta\delta_3$, $\Delta\lambda P_1$, $\Delta\lambda P_2$, $\Delta\lambda P_3$, $\Delta\lambda Q_1$, ΔV_3]

The tolerance in this iteration is given by

$$Tol=[\sum_{i=1}^{N_g}\left(\Delta P_{gi}^{\ 0}\right)^2+\sum_{i=2}^{N}\left(\Delta\delta_i^{\ 0}\right)^2+\sum_{i=1}^{N}\left(\Delta\lambda P_i^0\right)^2+ \\ +\sum_{i=N_g+1}^{N}\left(\Delta\lambda Q_i^{\ 0}\right)^2+\sum_{i=N_g+1}^{N}\left(\Delta V_i^0\right)^2]^{\frac{1}{2}}>\varepsilon \quad (41)$$

where $\varepsilon(=0.01)$ is the tolerance allowed for convergence. So whit these updated values of control variables obtained above, we have to go for next iteration, starting from calculation of elements of Jacobian matrix to obtain next set of control variables and this iterative process will continue till the tolerance becomes less than $\varepsilon(=0.01)$.

E. Fast Decoupled Method [5]

It is a well-known fact that in any practical power system, during operation in steady state condition there are strong interdependencies between real power (P) and bus voltage angle (δ) and reactive power (Q) and bus voltage magnitude ($|V|$),whereas couplings between P-$|V|$ and Q-δ are relatively weak. Therefore, the change in real power, specified at a bus is more dependent on the changes in bus voltage angle as well as the change in reactive power, specified at a bus is more dependent on the changes in bus voltage magnitude. The terms $\partial P/\partial V$ and $\partial Q/\partial\delta$ thus being small, they can be neglected in load flow computations.

$$\partial P/\partial V=0\quad ,\quad \partial Q/\partial\delta=0 \quad (42)$$

The simulation results for 3-bus systems by using the PSO algorithm and OPF method based on generator power and operating costs are shown in Table 4.

Table 4. Comparison results of the PSO and OPF for 3-bus

Methods	P_{g1} (MW)	P_{g2} (MW)	Total power (MW)	Cost F_{C1}	Cost F_{C2}	Total cost (unit of cost/hr)
PSO (P_D = 55 MW)	39.4534	18.001	57.4544	122.1541	73.372	195.5261
OPF (Classic)	38.625	16.415	55.04	120.5783	70.3562	190.9345
OPF (Newton-Raphson)	38.491	17.6972	56.1818	118.4189	72.7945	191.213
OPF (Fast decoupled)	37.481	17.705	55.186	118.4021	72.8107	191.2128

V. CONCLUSIONS

This paper purposes an application of population based PSO algorithm and OPF to solve the various ED problems. The particle swarm optimization is a new heuristic optimization method based on swarm intelligence. Comparison with the other algorithms, the method is very simple, easily completed and it needs fewer parameters, which made it fully developed. However, the research on the PSO is still at the beginning, a lot of problems are to be resolved. Particle swarm optimization has paid a lot of attention for solution of such problems, as it will not suffer from stuck into local optimal solution, dependability on initial variables, premature and slow convergence and curse of dimensionality in comparison to conventional optimization techniques, PSO has given an improved result within less computational time.

In the first part, ED is described by PSO method. This method is appropriate for solving nonlinear problems in complex networks of power system. The PSO is a property of absolute convergence (global minimum).The convergence speed of this method is preferred for solving non-convex nonlinear problems and particles of convergence as well. OPF method has been used in classical methods, which can lead to local minimum and but not global minimum.

By Comparison, we can say that each of these methods have strengths and weaknesses. Generally, In OPF method poor convergence may get stuck at local optimum, they can find only a single optimized solution in a single simulation run, they become too slow if number of variables are large and they are computationally expensive for solution of a large system.

The classical methods applied for online network and make good result (local minimum).But these results certainly do not prove that they are optimal solutions (global minimum).Although The results of the new methods such as pso are close to optimal (global minimum), They are used in offline network. However, PSO can be used to solve complex optimization problems, which are non-linear, and multi-model and improve the voltage profile, and to enhance voltage stability. The main merits of PSO are its fast convergence speed and it can be realized simply for less parameters need adjusting. However, Future research will cover this approach in more detail and try to find some theories as to why this might lead to a better solution.

NOMENCLATURES

X_i: Position vector
V_i: Velocity vector
pbest: The best previous position of a particle
gbest: The best position in swarm
C: Constriction factor
W: Inertia weight parameter
C_1: Cognitive coefficient
C_2: Social coefficient
C_{1i}: Initial cognitive coefficient
C_{2i}: Initial social Coefficient
C_{1f}: Final cognitive coefficient
C_{2f}: Final social Coefficient
$rand_1$, $rand_2$: Random number between 1, 0
N_g: is the number of units (generators),
$F_i(P_i)$: Cost function
P_L: Real losses power of system
P_D: Demand Load
B_0, B_{00}: Loss matrix coefficients
$\delta_{i,j}$: Voltage angle (load angle) at i, j bus
P_i, Q_i: Real and reactive powers of bus ith
L: The Lagrange function

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