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MODEL REFERENCE ADAPTIVE CONTROL OF PEMFC WITH DC/DC CONVERTER

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Abstract- Maintaining a constant voltage in Proton Exchange Membrane Fuel Cell (PEMFC) has always attracted the attention of many researchers and many articles have been published on this issue. In this paper a state space model of the set made up of a PEMFC and a DC/DC converter is studied, which correlates changes in the fuel cell output voltage to changes in the load current and converter duty cycle. Model reference adaptive control (MRAC) strategy is used to fix cell voltage regardless of the varying system parameters. The model and the controller are simulated in MATLAB/Simulink software and results are compared with a PID controller.

Keywords: PEMFC, DC/DC Converter, Model Reference Adaptive Control (MRAC), Voltage Control, State Space Model, Simulation.

I. INTRODUCTION

Fuel cells are set to become the power source of the future due to the fact that fossil fuel sources are limited and using them as power sources would result in many negative consequences like environmental pollutions. Polymer Electrolyte Membrane (PEM) fuel cells are the most popular type of fuel cells and traditionally use hydrogen as the fuel. The interest in PEM fuel cells has increased during the past decade especially for ground vehicle applications [1-3].

A PEM fuel cell produces electrochemical power when a hydrogen-rich gas passes through the anode and an oxygen (or air) rich gas passes through the cathode, with an electrolyte between the anode and cathode which allows the exchange of electrical charge (ions). The dissociation of hydrogen molecules produces the flow of ions through the electrolyte and an electrical current through an external circuit (Figure 1) [4].

DC/DC converters are used in applications where an average output voltage is required, which can be higher or lower than the input voltage. This is achieved by governing the times in which the converter's main switch conducts or does not conduct usually to a constant frequency [4].

In this work, a state space model of a fuel cell-DC/DC converter system, considered as the plant to be controlled, is presented. This model describes the relationship between different electrical variables and is valid for any operating point of the cell. That is, the model describes the behavior of the system for any point of the fuel cell polarization curve.

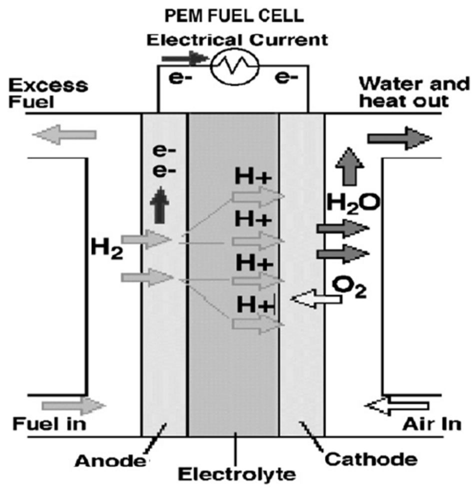

Figure 1. Diagram of PEM fuel cell

When the system parameters are determined a controller with fixed parameters can be designed. Feedback gain controller, PID control, optimal LQ control represent some examples of the controllers used in literature for the smart structures development. Another possibility is to design a controller which does not require explicitly defined system parameters. For this purpose a model reference adaptive control (MRAC) algorithm can be used.

The idea of the MRAC is based upon the existence of the reference model, specified by the designer, which reflects the desired behaviour of the controlled structure. The controller is designed in such a manner that the output of the controlled structure should track the output of the reference model [5, 6].

This paper is organized as follows: Section 2 reviews the considered PEMFC's model. Section 3, introduces Model Reference Adaptive Control (MRAC). In Section 4, the simulation results that validate the developments are shown. Finally, conclusion is discussed in Section 5.

II. THEORETICAL MODEL OF A PEM FUEL CELL

In order to control fuel cell voltage, first, it is necessary to consider the dynamic model. In recent years, many papers have been written by researchers about dynamic modeling of PEMFC [7, 8, 9, 10]. In this paper a model that has been previously presented by the reference [4] is used. This model is a theoretical model of a proton exchange membrane (PEM) fuel cell and DC/DC converter as a plant. Figure 2 shows the diagram of the system (fuel cell and DC/DC converter).

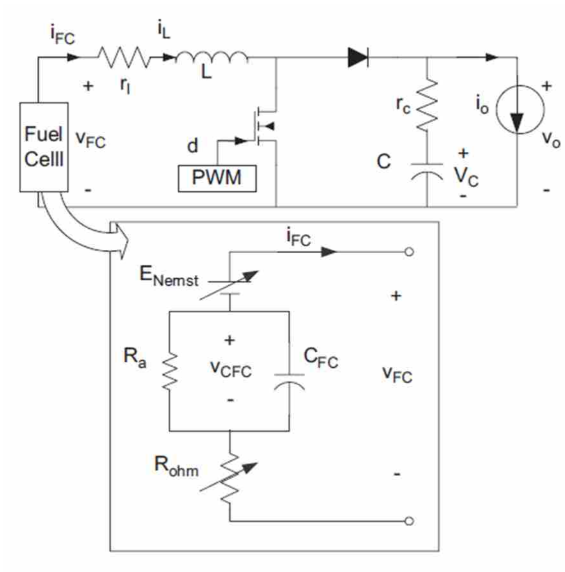

Figure 2. Complete diagram of the fuel cell DC/DC converter

In fuel cells, the movement of electrons through the external circuit and of protons through the membrane for a single cell generates a voltage difference between the cell terminals. This voltage can be defined by Equation (1), [8, 11]:

$$V_{cell} = E_{Nernst} - V_{act} - V_{ohm} - V_{conc} \quad (1)$$

where E_{Nernst} is the cell thermodynamic potential drop. In this model, E_{Nernst} is calculated from the Nernst equation taking into account temperature changes with respect to reference standard temperature. This voltage can be calculated from the Nernst equation [8] given as:

$$E_{Nernst} = \frac{\Delta G}{2F} + \frac{\Delta S}{2F}\left(T - T_{ref}\right) + \frac{RT}{2F}\left[\ln\left(P_{H2}\right) + \frac{1}{2}\ln\left(P_{O2}\right)\right] \quad (2)$$

where ΔG is the change in the free Gibbs energy of the reaction (J/mol), F is the constant of Faraday (96485.309 C/mol), ΔS is the change of entropy of the reaction (J/mol), R is the universal constant of the gases (83.143 J/mol/K) and P_{H2} and P_{O2} are the partial pressures of hydrogen and oxygen (atm), respectively. Variables of T and T_{ref} denote the cell operating temperature and the reference temperature (K), respectively.

The second term of Equation (1), V_{act} is the activation over potential which can be calculated by:

$$V_{act} = -\left(\xi_1 + \xi_2 T + \xi_3 T \ln\left(C_{O2}\right) + \xi_4 T \ln\left(I_{FC}\right)\right) \quad (3)$$

where I_{FC} is the static current passing through the cell and $\xi_1, \xi_2, \xi_3, \xi_4$ represent the experimental coefficients depending on each type of cell. Oxygen concentration (C_{O2}) in the interface between the cathode and the catalyst (mol/cm^3) is given by [12]:

$$C_{O2} = \frac{P_{O2}}{5.08 \times 10^6 \exp(-498/T)} \quad (4)$$

In practice, the third addend of Equation (3) has a very low value as compared with the rest of addends. Therefore, it can be rejected. The third term of Equation (1) is the ohmic voltage drop, V_{ohm}. This term represents the voltage drop due to resistance to the transfer of electrons through the electrodes and to the transfer of protons through the membrane. The expression of the voltage drop due to ohmic losses is:

$$V_{ohm} = I_{FC}\left(R_M + R_C\right) \quad (5)$$

where R_C represents the resistivity to the transfer of electrons through the electrodes. It is usually considered with a constant value.

The equivalent resistance of the membrane (R_M) is calculated as:

$$R_M = \frac{\rho_M l}{A} \quad (6)$$

where ρ_M is the membrane-specific resistivity to the flow of protons (Ωcm), A is the cell active area (cm^2) and l is the membrane thickness (cm). For Nafion membranes, the specific resistivity is given as [13]:

$$\rho_M = \frac{181.6\left[1 + 0.03\left(I_{FC}/A\right) + 0.062\left(T/303\right)^2\left(I_{FC}/A\right)^{2.5}\right]}{\left[\varphi - 0.634 - 3\left(I_{FC}/A\right)\right]\exp\left(4.18\left(T - 303/T\right)\right)} \quad (7)$$

The parameter φ is an adjustable parameter with a maximum value of 23. This parameter depends on the membrane fabrication process and is a function of the relative humidity and the stoichiometric rate of the gas in the anode. Under ideal humidity conditions (100%), this parameter may have a value ranging from 14 to 20.

The last addend of Equation (1) is the term corresponding to concentration voltage drop, V_{conc}. This drop is mainly due to the reactive concentration excess near the catalyst surfaces. This voltage drop can be known from:

$$V_{conc} = -B\ln\left(1 - \frac{J}{J_{max}}\right) \quad (8)$$

where B is a parameter that depends on the type of cell and J represents the current density passing through the cell at each moment (A/cm^2) and is defined as:

$$J = I_{FC}/A \quad (9)$$

Under normal operating conditions, a single cell produces approximately 1.2 V. For use in energy generating systems, where a relatively high power is required, several cells are connected in series, forming a stack that can supply electrical power of the order of some kilowatts. For a stack formed by N cells, the voltage between its terminals is obtained from:

$$v_{FC} = N\, v_{cell} \quad (10)$$

The state space equations of the fuel cell and DC/DC converter are as (11), where r_l and r_c are losses in the inductor and the capacitor respectively. The variable d takes a value between 0 and 1, matching the converters duty cycle [14] and $d' = (1-d)$:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \\ \dot{v}_{CFC} \end{bmatrix} = \begin{bmatrix} -\frac{r_l}{L} - \frac{r_C D'}{L} + \frac{D_{FC}}{L} & -\frac{D'}{L} & \frac{C_{FC}}{L} \\ \frac{D'}{C} & 0 & 0 \\ B_{FC} & 0 & A_{FC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \\ v_{CFC} \end{bmatrix} + \begin{bmatrix} \frac{r_C D'}{L} & \frac{1}{L}\left(r_C\left(I_o - I_L\right) - v_C\right) \\ -\frac{1}{C} & \frac{I_L}{C} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_o \\ d' \end{bmatrix} \quad (11)$$

In this model the state variables are (i_L, v_C, v_{CFC}) and input variables are (i_o, d'), whereas the control of the fuel cell output voltage (v_C) is required, then the system output matrix is $C = [0\ 1\ 0]$. The load is generically modeled as a current source. Value resulting from each of the parameters is shown in Table 1. Jacobean matrixes of PEMFC are obtained from [4].

III. MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

MRAC is a wide class of adaptive control schemes. In MRAC, the desired plant behavior is described by a reference model which is driven by a reference input. The control law $C(s, \theta_c^*)$ is then developed such that the closed-loop plant has a response equal to response of reference model. This response matching guarantees that the plant will behave like the reference model for any reference input signal. Let us now consider the *n*th-order plant:

$$\dot{x} = Ax + Bu \ , \ x \in R^n \quad (12)$$

where $A \in R^{n\times n}$, $B \in R^{n\times q}$, are unknown constant matrices and (A, B) is controllable. The control objective is to choose the input vector $u \in R^q$ such that all signals in the closed-loop plant are bounded and the plant state x follows the state $x_m \in R^n$ of a reference model. The reference model is described by:

$$\dot{x}_m = A_m x_m + B_m r \quad (13)$$

where $A_m \in R^{n\times n}$ is a stable matrix, $B_m \in R^{n\times q}$, and $r \in R^q$ is a bounded reference input vector. The reference model and input r are chosen such that $x_m(t)$ represents a desired trajectory that x has to follow.

If the matrices A, B were known, we could apply the control law:

$$u = -K^* x + L^* r \quad (14)$$

and obtain the closed-loop plant:

$$\dot{x} = \left(A - BK^*\right)x + BL^* r \quad (15)$$

hence, if $K^* \in R^{q\times n}$ and $L^* \in R^{q\times q}$ are chosen to satisfy the algebraic equations:

$$A - BK^* = A_m \ , \ BL^* = B_m \quad (16)$$

then the transfer matrix of the closed-loop plant is the same as that of the reference model and $x(t) \to x_m(t)$ exponentially fast for any bounded reference input signal $r(t)$. We should note that in general, no K^*, L^* may exist to satisfy the matching condition (16) for the given matrices A, B, A_m, B_m, indicating that the control law (14) may not have enough structural flexibility to meet the control objective. In some cases, if the structure of A, B is known, A_m, B_m may be designed so that (16) has a solution for K^*, L^*.

Let us assume that K^*, L^* in (16) exist, i.e., that there is sufficient structural flexibility to meet the control objective, and propose the control law:

$$u = -K(t)x + L(t)r \quad (17)$$

where $K(t), L(t)$ are estimates of K^*, L^*, respectively, to be generated by an appropriate adaptive law. By adding and subtracting the desired input term, namely, $-B\left(K^* x - L^* r\right)$ in the plant equation, and using (16):

$$\dot{x} = A_m x + B_m r + B\left(K^* x - L^* r + u\right) \quad (18)$$

Defining the tracking error $e = x - x_m$ and subtracting (13) from (18), we obtain the tracking error equation:

$$\dot{e} = A_m e + B\left(K^* x - L^* r + u\right) \quad (19)$$

which is in the form of the B-SSPM. The estimation model is given by:

$$\dot{\hat{e}} = A_m \hat{e} + B\left(Kx - Lr + u\right) \ , \ \hat{e}(0) = 0 \quad (20)$$

Due to the control law $u = -K(t)x + L(t)r$, the signal $\hat{e}(t) = 0 \ \forall t \ge 0$ and the estimation error $\varepsilon = e - \hat{e} = e$. Therefore, the estimation model is not needed, and the tracking error e is used as the estimation error.

We can show that the tracking error $e = x - x_m$ and parameter error $\tilde{K} = K - K^*$, $\tilde{L} = L - L^*$ satisfy equation:

$$\dot{e} = A_m e + B\left(-\tilde{K}x + \tilde{L}r\right) \quad (21)$$

which also depends on the unknown matrix B. In the scalar case we manage to get away with the unknown B by assuming that its sign is known. Let us assume that L^* is either positive definite or negative definite $\Gamma^{-1} = L^* \operatorname{sgn}(l)$, where $l = 1$ if L^* is positive definite and $l = -1$ if L^* is negative definite. Then $B = B_m L^{*-1}$ and (20) becomes:

$$\dot{e} = A_m e + B_m L^{*-1}\left(-\tilde{K}x + \tilde{L}r\right) \quad (22)$$

We propose the following Lyapunov function candidate:

$$V\left(e,\tilde{K},\tilde{L}\right)=e^{T}Pe+tr\left(\tilde{K}^{T}\Gamma\tilde{K}+\tilde{L}^{T}\Gamma\tilde{L}\right) \quad (23)$$

where $P=P^{T}>0$ satisfies the Lyapunov equation:

$$PA_{m}+A_{m}^{T}P=-Q \quad (24)$$

for some $Q=Q^{T}>0$. Then:

$$\dot{V}=-e^{T}Qe+2e^{T}PB_{m}L^{*-1}\left(-\tilde{K}x+\tilde{L}r\right)+2\text{tr}\left(\tilde{K}^{T}\Gamma\dot{\tilde{K}}+\tilde{L}^{T}\Gamma\dot{\tilde{L}}\right) \quad (25)$$

Now:

$$e^{T}PB_{m}L^{*-1}eKx=\text{tr}\left(x^{T}\tilde{K}^{T}\Gamma B_{m}^{T}Pe\right)\text{sgn}\left(l\right)= \\ =\text{tr}\left(\tilde{K}^{T}\Gamma B_{m}^{T}Pex^{T}\right)\text{sgn}\left(l\right) \quad (26)$$

and:

$$e^{T}PB_{m}L^{*-1}\tilde{L}r=\text{tr}\left(\tilde{L}^{T}\Gamma B_{m}^{T}Per^{T}\right)\text{sgn}\left(l\right) \quad (27)$$

Therefore, for:

$$\dot{\tilde{K}}=\dot{K}=B_{m}^{T}Pex^{T}\,\text{sgn}\left(l\right)\ ,\ \dot{\tilde{L}}=\dot{L}=-B_{m}^{T}Per^{T}\,\text{sgn}\left(l\right) \quad (28)$$

we have:

$$\dot{V}=-e^{T}Qe\leq 0 \quad (29)$$

From the properties of V , $\dot{V}$ we establish, as in the scalar case, that $K\left(t\right),L\left(t\right),e\left(t\right)$ are bounded and that $e\left(t\right)\rightarrow 0$ as $t\rightarrow\infty$. The adaptive control scheme developed is given by (17) and (28). The matrix $B_{m}^{T}P$ acts as an adaptive gain matrix, where P is obtained by solving the Lyapunov equation $PA_{m}+A_{m}^{T}P=-Q$ for some arbitrary $Q=Q^{T}>0$. Different choices of Q will not affect boundedness and the asymptotic behavior of the scheme, but they will affect the transient response. The assumption that the unknown L^{*} in the matching equation $BL^{*}=B_{m}$ is either positive or negative definite imposes an additional restriction on the structure and elements of B,B_{m}. Since B is unknown, this assumption may not be realistic in some applications.

IV. SIMULATION RESULTS

In order to investigate how well controller tracks the reference voltage when a change occurs in the plant parameters, a sinusoidal change in the membrane resistance, R_{m}, is applied to the plant while the membrane specific resistivity (ρ_{M}) changes in the same way because of the changes in the parameter φ. It is assumed that a $30\,\text{Hz}$ sinusoidal signal with the amplitude of $0.1\times R_{m}$ is added to R_{m}. In other word, $R_{m}=50^{\Omega}+5^{\Omega}\sin\left(2\pi\times 30t\right)$. Figures 3-8 show the simulation results when R_{m} changes as described before. The reference voltage is set to 78 V. As it can be seen, the model reference adaptive controller adapts itself to the new condition and fixes the voltage at 78 V (Figure 3)and errors are shown in Figures 4-6 which have converged to zero. The PID controller is unable to maintain a constant voltage (Figure 7). The output voltage without controller is shown in Figure 8.

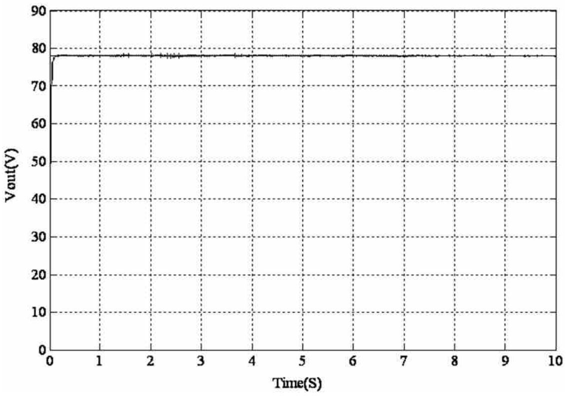

Figure 3. Output voltage with model reference adaptive controller

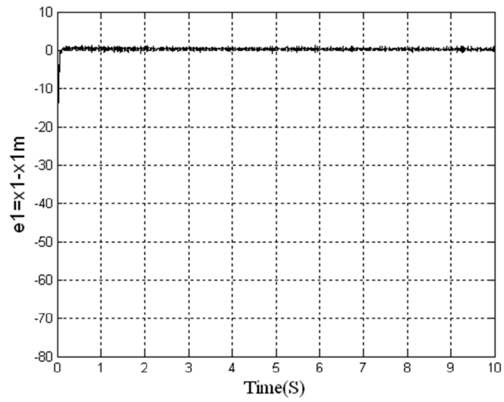

Figure 4. $e_{1}=x_{1}-x_{1m}$

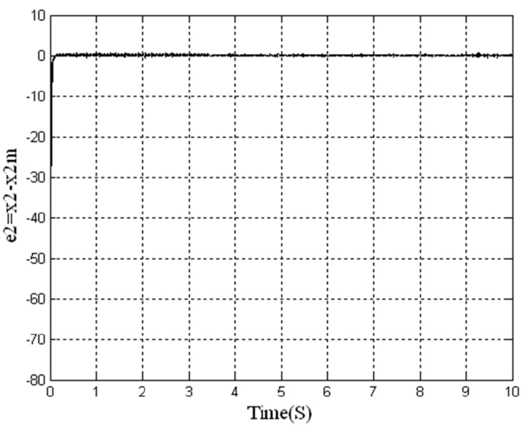

Figure 5. $e_{2}=x_{2}-x_{2m}$

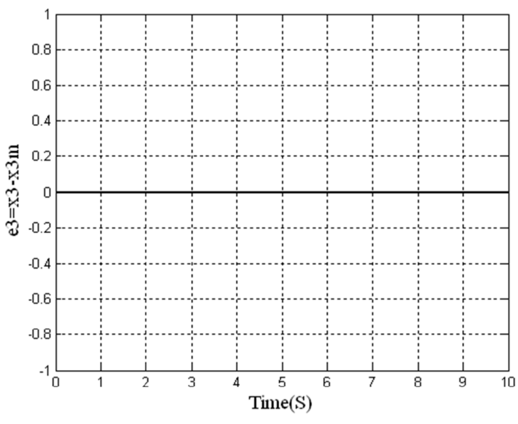

Figure 6. $e_{3}=x_{3}-x_{3m}$

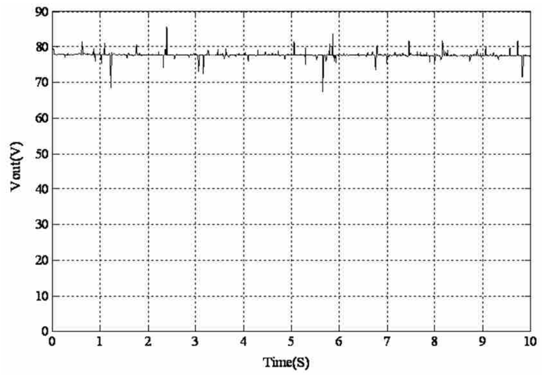

Figure 7. Output voltage with PID controller

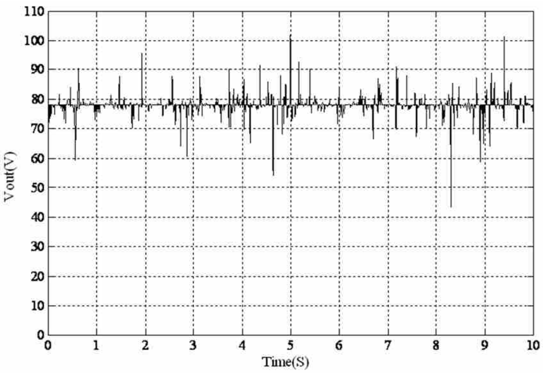

Figure 8. Output voltage without controller

V. CONCLUSIONS

This paper presented a state space model of the set made up of a proton exchange membrane fuel cell (PEMFC) and a DC/DC converter. All dynamic equations for activation, concentration, and ohmic over potentials were presented. It became clear that the set output voltage changed with changes in membrane resistance, R_m. The model reference adaptive controller (MRAC) showed that it was able to maintain a constant voltage even when a parameter of the plant was changing. In order to maintain a framework for comparison, a PID controller was also designed. Simulation results revealed that the model reference adaptive controller has a faster response than the PID controller.

APPENDIX

Values Used for Parameters and Jacobean Matrixes of PEMFC

Table1 shows the Values used for parameters and Jacobean matrixes of PEMFC.

Table 1. Values used for parameters and Jacobean matrixes of PEMFC

Parameters	Names	Values
A	Area	$40.6\,\text{cm}^2$
A_{FC}	Jacobean matrix	-29.7299
B	A parameter that depends on the type of cell	$40.6\ \text{cm}^2\text{V}$
B_{FC}	Jacobean matrix	0.1422
C	Capacitor	$390\mu\text{F}$
C_{FC}	Jacobean matrix	-33
D_{FC}	Jacobean matrix	$-0.1887-165\sin(60\pi t)$
I_i	Input current	$45.88\,\text{A}$
I_o	Output current	$25\,\text{A}$
I_{max}	Maximum current	$60\,\text{A}$
J_{max}	Current density	$1.42\,\text{A/cm}^2$
L	Inductor	$75\,\mu\text{H}$
l	Membrane thickness	$178\mu\text{m}$
N	Number of cells	33
P_{H2}	Partial pressures of hydrogen	$3\,\text{atm}$
P_{O2}	Partial pressures of oxygen	$1\,\text{atm}$
R_C	Resistivity to the transfer of electrons	0.0001Ω
r_C	losses in the capacitor	0.056Ω
r_L	losses in the inductor	$6.5\times10^{-3}\,\Omega$
T	Temperature	$338\,\text{K}$
ξ_1	Experimental coefficients	$-0.8\,\text{V}$
ξ_2	Experimental coefficients	$0.0036008\,\text{V/K}$
ξ_3	Experimental coefficients	$7.6\times10^{-5}\ \text{V/K}$
ξ_4	Experimental coefficients	$-1.35\times10^{-4}\ \text{V/K}$

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BIOGRAPHIES

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