# International Journal on <br> (IJTPE) 

Published by International Organization of IOTPE

# INVESTIGATION OF DYNAMIC MODELING OF DVR 

F.M. Shahir E. Babaei S. Ranjbar S. Torabzad<br>Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran<br>f-mohammadzadeh@iau-ahar.ac.ir, e-babaei@tabrizu.ac.ir, ranjbar.samira.11@gmail.com, saman_torabzade@yahoo.com


#### Abstract

The dynamic voltage restorer (DVR) is one of the flexible ac transmission system (FACTS) is vastly applied in power systems. The direct three-phase to single-phase device can be used effectively for mitigation of voltage sags and swells. This paper provides comprehensive development procedures and final forms of mathematical models of a new structure of DVR based on direct three-phase to single-phase converter. Through the presented modeling, the transient and steady states responses are analyzed. The validity of the presented modeling for DVR has been verified by simulation results using MATLAB software.


Keywords: DVR, FACTS, Dynamic Modeling, Steady State Modeling, Transient State Modeling.

## I. INTRODUCTION

Power quality has always been important for customer, but with increasing number of electronic loads and sensitive controllers to power quality, the subjects have attracted renewed interest in recent time [1-4]. DVR is a power electronic device to magnitude the voltage distortion in distribution systems. The basic operation of DVR is to inject a voltage of required magnitude, phase angle, and frequency in series with a distribution feeder to maintain the desired amplitude and waveform for load voltage even when voltage is unbalanced or distorted [5].

Many topologies and control methods have been presented for DVR. One of the best structures is based on direct three-phase to single-phase. Figure 1 shows the schematic diagram of the presented topology for DVR [6]. As shown in Figure 1, the presented DVR consist of two stages, converter side and injection side in each phase. This topology has more advance rather than conventional topologies. The presented topology is properly able to compensate unbalanced voltage sags and swells according to application of three independent three-phase to single-phase converters. Moreover, this structure does not need to energy-storage elements.

In this paper, it is tried to investigate and present a dynamic model for this type of DVR structure. First, a brief review is given for recommended DVR in [6]. Then, the mathematical modeling of this system is presented. Finally, the validity of presented theories is confirmed by simulation results.


Figure 1. Presented DVR in [6]

## II. REVIEW ON RECOMMENDED DVR [6]

As shown in Figure 1, in each phase of the DVR, a three-phase to single-phase direct converter, an filter ( $L_{k} C_{k}$ for $k=1,2,3$ ), a bypass switch ( $S_{b, i}$ for $i=a, b, c$ ), a resistance ( $R_{k}$ for $k=1,2,3$ ), and an injection transformer have been used. Also, each three-phase to single-phase direct converters consist of six power switch. Considering Figure 1 and assuming sinusoidal waveforms, these following equations can be obtained for phase ' $a$ ' of converter 1 , and phase ' $a$ ' of injection voltage [6]:
$v_{t, 1, a}(t)=v_{t, 1, m} \sin \omega t$
$v_{t, a}(t)=v_{t, m, a} \sin \left(\omega t+\phi_{i}\right)$
where $\phi_{i}$ is defined as follows [6]:
$\phi_{i}=\left\{\begin{array}{c}0^{\circ}, \text { for sag conditions } \\ 180^{\circ}, \text { for swell conditions }\end{array}\right.$
The relation between $v_{d c, k}(t), v_{t, k, h}(t)$, and $v_{t, i}(t)$ (for $k=1,2,3, h=a, b, c, i=a, b, c$ ) can be expressed by [6]:

$$
\begin{align*}
& v_{d c, k}(t)=m_{k} v_{t, k, h}(t) \text { for } k=1,2,3 \text { and } h=a, b, c  \tag{4}\\
& v_{t, i}(t)=m_{i} v_{d c, k}(t) \text { for } k=1,2,3 \text { and } i=a, b, c \tag{5}
\end{align*}
$$

## III. MATHEMATICAL MODELING

## A. Converter Side Modeling

The operation of input control parameters of the presented DVR in each converter and their mutually phase are investigated to evaluate its dynamic model. In order to achieve this aim, converter 1 and its mutually injection phase, ' $a$ ', are considered. Here, it is assumed that $C_{1}$, is divided into two equal parts and its middle point is named as n . In addition, $\xi_{1, a}$ and $\xi_{1, a}^{\prime}$ switches are bidirectional ones with $r_{S}$ resistance, which can be considered as the conduction losses of the switches and can be applied in losses analysis. In the following, it is assumed that $r_{1, a}=R_{1}+r_{t, a}$.

The control method of direct converters in presented DVR is based on pulse width modulation (PWM) technique. Applying PWM technique for converter 1, the following is valid [7-9]:
$S_{1, a}+S_{1, a}^{\prime}=1$
By applying the kirchhoff's voltage law in Figure 2, the following is obtained:
$\left(L_{1}+L_{t, a}\right) \frac{d i_{1, a}}{d t}+r_{1, a} i_{1, a}=v_{t, 1, a}-v_{1, a}$
Considering Figure 2, the following relation can be expressed:
$v_{1, a}=v_{F H}+v_{H n}$
where, $v_{F H}$ can be calculated as follows:
$v_{F H}=\left(i_{1, a} r_{s}+v_{d c, 1}\right) S_{1, a}+i_{1, a} r_{s} S_{1, a}^{\prime}$


Figure 2. Equivalent circuit of phase a of converter 1
Considering (8) to (9), and assuming $R_{1, a}=r_{s}+r_{1, a}$ and $L_{1, a}=L_{1}+L_{t, a}$, the following equation can be obtained:
$L_{1, a} \frac{d i_{1, a}}{d t}=-R_{1, a} i_{1, a}-\left(v_{d c, 1} S_{1, a}+v_{H n}\right)+v_{t, 1, a}$
According to Figure 2, the parameter $v_{H n}$ can be calculated as follows [10]:
$v_{H n}=-\left(\frac{v_{d c, 1}}{3}\right) \sum_{h=a, b, c} S_{1, h}$
If $S_{1, h}$ is expanded by Fourier discrete series [10], the following is valid:
$\bar{d}_{1, a}=\frac{m_{1}}{2} \cos \left(\omega t+\delta_{1}\right)+\frac{1}{2}$

Considering (11), $v_{1, a}$ can be expressed as follows:
$v_{1, a}=A_{1} \cos \delta_{1}=\frac{m_{1} v_{d c, 1}}{2} \cos \left(\omega t+\delta_{1}\right)$
Finally, considering the relation obtained for phase ' $a$ ' of converter 1 , and assuming same power switches in converter 1 , the following equation can be obtained:
$L_{1, a} \frac{d i_{1, a}}{d t}=-R_{1, a} i_{1, a}-A_{1} \cos \delta_{1}+v_{t, 1, a}$
The obtained mathematical model for phase ' $a$ ' can be acceptable for phase ' $b$ ' and ' $c$ ' with $120^{\circ}$ and $240^{\circ}$ phase difference, respectively. In addition, the obtained equations for converter 1 are acceptable for converters 2 and 3 .

## B. Injection Side Modeling

From (11) and (12), the average positive half switching function of $S_{b, a}$ can be calculated as follows:
$\bar{d}_{1, a}^{\prime}=\frac{m_{a}}{2} \cos \left(\omega t+\delta_{a}\right)+\frac{1}{2}$
Considering (13), (14), and (15), the injected voltage of phase 'a' of is given by:
$L_{1, a}^{\prime} \frac{d i_{1, a}^{\prime}}{d t}=-R_{1, a}^{\prime} i_{1, a}^{\prime}-A_{a}^{\prime} \cos \delta_{a}^{\prime}+v_{1, a}^{\prime}$
The obtained equation for phase ' $a$ ' is acceptable for phase ' $b$ ' and ' $c$ ' with $120^{\circ}$ and $240^{\circ}$ phase difference, respectively.

## C. Filter Modeling

In order to dynamic modeling of filter capacitor in converter 1 and phase ' $a$ ', the following equation can be considered:
$\frac{d v_{d c, 1}}{d t}=\frac{1}{C_{1}} i_{d c, 1}$
where $i_{d c, 1}$ is defined as follows:
$i_{d c, 1}=i_{1, d c}-i_{1, d c}^{\prime}=\sum_{h=a, b, c} i_{1, h} \bar{d}_{1, h}-i_{1, a}^{\prime} \bar{d}_{1, a}^{\prime}$
The obtained mathematical modeling for converter 1 and phase ' $a$ ' are acceptable for converter 2 and phase ' $b$ ', and converter 3 and phase ' $c$ ' with suitable phase difference, respectively.

## IV. STATE SPACE EQUATIONS DERIVATION

Considering same switches in presented DVR ( $R_{k, h}=R, R_{k, i}^{\prime}=R^{\prime}, L_{k, h}=L$, and $L_{k, i}^{\prime}=L^{\prime}$ ) and from (14), (16) and (17), a state space equation can be defined:

$$
\begin{equation*}
\dot{X}=A X+B U \tag{19}
\end{equation*}
$$

where,

$$
\begin{align*}
X= & {\left[i_{1, a} i_{1, b} i_{1, c}\right.} \\
i_{2, a} & i_{2, b} i_{2, c}  \tag{20}\\
i_{3, a} & i_{3, b}
\end{align*} i_{3, c} i_{i, a} i_{i, b} i_{i, c} .
$$

The non-zero arrays of $A$ and $B$ values have been presented in Appendix.

## V. STEADY STATE MODELING

Neglecting DVR losses and assuming that $v_{d c, k}(t)$ (for $k=1,2,3$ ) is constant, and applying KVL in investigated DVR, following equation can be obtained:
$V_{t, k, h}=j X_{k, h} I_{k, h}+V_{k, h} \quad($ for $h=a, b, c, k=1,2,3$ )
Thus, according to (22), the DVR steady state model is obtained as follows:

$$
\begin{aligned}
& V_{t, k, h}=\frac{M_{k} V_{t, k}}{2 \sqrt{2}} \angle \delta_{k, h} ; V_{t, k, i}^{\prime}=\frac{M_{k}^{\prime} V_{t, k}^{\prime}}{2 \sqrt{2}} \angle \delta_{k, i}^{\prime} ; \\
& Z_{k, h}=R_{k, h}+j \omega L_{k, h} ; Z_{k, i}^{\prime}=R_{k, i}^{\prime}+j \omega L_{k, i}^{\prime} \\
& \text { (for } h=a, b, c ; k=1,2,3 ; \text { and } i=1,2,3 \text { ) }
\end{aligned}
$$



Figure 3. Steady state model of converter 1 of phase 'a'
According to (23), the phase a of injection side of DVR can be shown in Figure 3. In Figure 3, it can be seen that If the voltage magnitude in power system does not change, the capacitor voltage will remain constant. The operation of converters depend to mutually phases.

## VI. DYNAMIC MODEL

Considering (19), the proposed dynamic model for DVR on dq0 axis is obtained as follows:
$\dot{x}_{d q 0}=A_{d q 0} x_{d q 0}+B_{d q 0} u_{d q 0}$
where $x_{d q 0}$ and $u_{d q 0}$ are as follows and the matrices $A_{d q 0}$ and $B_{d q 0}$ have been explained in Appendix.
$x_{d q 0}=\left[\begin{array}{llllllll}i_{1, d} & i_{1, q} & i_{2, d} & i_{2, q} & i_{3, d} & i_{3, q} & i_{i, d} & i_{i, q}\end{array} v_{d c, 1} v_{d c, 2} v_{d c, 3}\right]^{T}$ (25)
$u_{d q 0}=\left[\begin{array}{lllllll}v_{t, 1, d} & v_{t, 1, q} & v_{t, 2, d} & v_{t, 2, q} & v_{t, 3, d} & v_{t, 3, q} & v_{i, d} \\ v_{i, q}\end{array}\right]^{T}$

## VII. SMALL SIGNAL DYNAMIC MODEL

The small signal model of DVR, (24) is linearized by making an approximation around the system operation point and is expressed as follows:
$\Delta \dot{x}_{d q 0}=A_{d q 0}^{\prime} \Delta x_{d q 0}+B_{d q 0}^{\prime} \Delta u_{d q 0}$

The non-zero arrays of $A_{d q 0}^{\prime}$ and $B_{d q 0}^{\prime}$ have been presented in appendix. The small signal model presented in (27) can be easily applied in a wide range of system dynamic investigations such as low frequency electromechanical modes.

## VIII. SIMULATION RESULTS

In this section, the effects of variation of each DVR parameters are investigated and simulated in MATLAB software. Here, the DVR parameters are considered as Table 1. It is assumed that the system is symmetric.

## A. Transient State Investigation

In this subsection, the dynamic stability of DVR is investigated through critical eigenvalues analysis. In order to select optimum signal, the transient behavior of input signals in converter 1 of phase ' $a$ ' are analyzed. According to simulation results that are shown in Figure $4, \delta_{1}, m_{1}$, and $m_{a}$ do not improve the transient state stability parameters and do not affect on the dynamic stability of presented DVR. The $\delta_{a}$ improve the system's critical eigenvalues and transient state stability parameters in converter 1 of phase ' $a$ '.

Table 1. Power system parameters

| Three-phase <br> ac source | Rated voltage | $240 \mathrm{~V} \times 1.03$ |
| :---: | :---: | :---: |
|  | Frequency | 50 Hz |
|  | Short circuit level | 100 MVA |
|  | Base voltage | 240 V |
| Three-phase <br> load | $X_{g} / R_{g}$ | 7 |
|  | Nominal vlotage | 480 kV |
|  | Frequency | 50 Hz |
|  |  |  |
| lines | Active power | 1000 W |
|  | Reactive power | 100 VAR |
|  | Inductance per unit length | $0.01273 \Omega / \mathrm{km}$ |
|  | Capacitance per unit length | $12.74 \mathrm{nF} / \mathrm{km}$ |
|  | Length | 25 km |

## B. Steady State Investigation

In order to study the steady state, the power flow of DVR is analyzed. The studied power system is shown in Figure 5. In this research, it is tried to increase the power flow with controlling parameters of DVR. Figure 6 shows the impact of DVR on the receiving-end active $\left(P_{L}\right)$ and receiving-end reactive $\left(Q_{L}\right)$ powers with and without DVR. According to simulation results shown in Figure 6, sending range of $P_{L}$ and $Q_{L}$ are improved using DVR.

## IX. CONCLUSIONS

In this paper, a generalized mathematical model was presented for the presented DVR in [4]. The dynamic model of steady and transient states were presented. The presented modeling can be applied in power flow calculations, system's eigenvalues condition assess, and dynamic and transient state stability analysis. Steady and transient states, and small signal equations were derived. In additional, the effects of each DVR parameters
variations were simulated. According to simulation results in steady state, DVR helps to improve active and reactive power flows in power system. In transient state evaluation, it can be mentioned that $\delta_{i}$ (for $i=a, b, c$ ) variations improve small signal stability in each converters.


Figure 4. Critical eigenvalues variation; (a) $\delta_{1}$ and $m_{1}$; (b) $\delta_{a}$ and $m_{a}$


Figure 5. Power system with DVR

(a)

(b)

Figure 6. Power flow analysis; (a) active power; (b) reactive power

## APPENDIX

The non-zero arrays of matrix $A$, in (19) are:
$a_{e, e}=-\frac{R}{L} \quad$ for $\quad e=1,2,3,4,5,6,7,8,9$;
$a_{f, f}=-\frac{R^{\prime}}{L^{\prime}} \quad$ for $f=10,11,12 ; a_{1,13}=k_{1} \cos \delta_{1}$;
$a_{2,13}=k_{1} \cos \left(\delta_{1}-120^{\circ}\right) ; a_{3,13}=k_{1} \cos \left(\delta_{1}-240^{\circ}\right)$;
$a_{4,14}=k_{2} \cos \delta_{2} ; a_{5,14}=k_{2} \cos \left(\delta_{2}-120^{\circ}\right)$;
$a_{6,14}=k_{2} \cos \left(\delta_{2}-240^{\circ}\right) ; a_{7,15}=k_{3} \cos \delta_{3}$;
$a_{8,15}=k_{3} \cos \left(\delta_{3}-120^{\circ}\right) ; a_{9,15}=k_{3} \cos \left(\delta_{3}-240^{\circ}\right)$;
$a_{10,13}=k_{4} \cos \delta_{a} ; a_{11,14}=k_{2} \cos \delta_{b} ; a_{12,15}=k_{17} \cos \delta_{c} ;$
$a_{13,1}=k_{5} \cos \delta_{1} ; a_{13,2}=k_{5} \cos \left(\delta_{1}-120^{\circ}\right) ;$
$a_{13,3}=k_{5} \cos \left(\delta_{1}-240^{\circ}\right) ; a_{14,4}=k_{6} \cos \delta_{2}$;
$a_{14,5}=k_{6} \cos \left(\delta_{2}-120^{\circ}\right) ; a_{14,6}=k_{6} \cos \left(\delta_{2}-240^{\circ}\right)$;
$a_{15,7}=k_{7} \cos \delta_{3} ; a_{15,8}=k_{7} \cos \left(\delta_{3}-120^{\circ}\right)$;
$a_{15,9}=k_{7} \cos \left(\delta_{3}-240^{\circ}\right) ; a_{13,10}=k_{8} \cos \delta_{a} ;$
$a_{14,11}=k_{9} \cos \delta_{b} ; a_{15,12}=k_{10} \cos \delta_{c}$
The non-zero arrays of matrix $B$, in (19) are:
$b_{j, j}=\frac{1}{L}$ for $j=1,2,3,4,5,6,7,8,9 ; b_{l, l}=\frac{1}{L^{\prime}}$ for $l=10,11,12$
The non-zero arrays of matrix $A_{d q o}$, in (24) are:
$a_{r, r}=-\frac{R}{L}$ for $r=1,2,3,4,5,6 ; a_{s, s}=-\frac{R^{\prime}}{L^{\prime}}$ for $s=7,8$;
$a_{1,2}=a_{3,4}=a_{5,6}=a_{7,8}=-a_{2,1}=-a_{4,3}=-a_{6,5}=-a_{7,8}=\omega_{0} ;$
$a_{1,9}=k_{1} \cos \delta_{1} ; a_{2,9}=k_{1} \sin \delta_{1} ; a_{3,10}=k_{2} \cos \delta_{2}$;
$a_{4,10}=k_{2} \sin \delta_{2} ; a_{5,11}=k_{3} \cos \delta_{3} ; a_{6,11}=k_{3} \sin \delta_{3} ;$
$a_{7,9}=k_{4} \cos \delta_{a} ; a_{8,10}=k_{16} \sin \delta_{b} ; a_{9,1}=1.5 k_{5} \cos \delta_{1} ;$
$a_{9,2}=1.5 k_{5} \sin \delta_{1} ; a_{10,3}=1.5 k_{6} \cos \delta_{2} ; a_{10,4}=1.5 k_{6} \sin \delta_{2}$;
$a_{11,5}=1.5 k_{7} \cos \delta_{3} ; a_{11,6}=1.5 k_{7} \sin \delta_{3} ; a_{9,7}=1.5 k_{8} \cos \delta_{a} ;$
$a_{10,8}=1.5 k_{9} \sin \delta_{b}$;
The non-zero arrays of matrix $B_{d q o}$, in (24) are:
$b_{p, p}=\frac{1}{L}$ for $p=1,2,3,4,5,6 ; b_{o, o}=\frac{1}{L^{\prime}}$ for $o=7,8$
The non-zero arrays of matrix $A_{d q o}^{\prime}$, in (27) are:
$a_{m, m}=-\frac{R}{L} \omega$ for $m=1,2,3,4,5,6 ; a_{n, n}=-\frac{R^{\prime}}{L^{\prime}} \omega$ for $n=7,8$;
$a_{1,2}=a_{3,4}=a_{5,6}=a_{7,8}=-a_{2,1}=-a_{4,3}=-a_{6,5}=-a_{7,8}=\omega ;$
$a_{1,9}=k_{1} \omega_{0} \cos \delta_{1} ; a_{2,9}=k_{1} \omega_{0} \sin \delta_{1} ; a_{3,10}=k_{2} \omega_{0} \cos \delta_{2} ;$
$a_{4,10}=k_{2} \omega_{0} \sin \delta_{2} ; a_{5,11}=k_{3} \omega_{0} \cos \delta_{3} ; a_{6,11}=k_{3} \omega_{0} \sin \delta_{3} ;$
$a_{7,9}=k_{4} \omega_{0} \cos \delta_{a} ; a_{8,10}=k_{16} \omega_{0} \sin \delta_{b} ; a_{9,1}=k_{5} \omega_{0} \cos \delta_{1} ;$
$a_{9,2}=k_{5} \omega_{0} \sin \delta_{1} ; a_{10,3}=k_{6} \omega_{0} \cos \delta_{2} ; a_{10,4}=k_{6} \omega_{0} \sin \delta_{2} ;$
$a_{11,5}=k_{7} \omega_{0} \cos \delta_{3} ; a_{11,6}=k_{7} \omega_{0} \sin \delta_{3} ; a_{9,7}=k_{8} \omega_{0} \cos \delta_{a} ;$
$a_{10,8}=k_{9} \omega_{0} \sin \delta_{b} ;$

The non-zero arrays of matrix $B_{d q o}^{\prime}$, in (27) are:
$b_{1,1}=-k_{11} v_{d c, 1} \cos \delta_{1} ; b_{1,2}=k_{11} m_{1} \sin \delta_{1} ;$
$b_{1,5}=-k_{11} v_{d c, 2} \cos \delta_{2} ; b_{1,6}=k_{11} m_{2} \sin \delta_{2} ;$
$b_{1,9}=-k_{11} v_{d c, 3} \cos \delta_{3} ; b_{1,10}=k_{11} m_{3} \sin \delta_{3} ;$
$b_{2,1}=-k_{11} v_{d c, 1} \sin \delta_{1} ; b_{2,2}=-k_{11} m_{1} \cos \delta_{1} ;$
$b_{2,5}=-k_{11} v_{d c, 2} \sin \delta_{2} ; b_{2,6}=-k_{11} m_{2} \cos \delta_{2} ;$
$b_{1,9}=-k_{11} v_{d c, 3} \cos \delta_{3} ; b_{1,10}=-k_{11} m_{3} \sin \delta_{3} ;$
$b_{3,3}=k_{12} v_{d c, 1} \cos \delta_{a} ; b_{3,4}=-k_{12} m_{a} \sin \delta_{a} ;$
$b_{3,7}=k_{12} v_{d c, 2} \cos \delta_{b} ; b_{3,8}=-k_{12} m_{b} \sin \delta_{b} ;$
$b_{3,11}=k_{12} v_{d c, 3} \cos \delta_{c} ; b_{3,12}=-k_{12} m_{c} \sin \delta_{c} ;$
$b_{4,3}=k_{12} v_{d c, 1} \sin \delta_{a} ; b_{4,4}=k_{12} m_{a} \cos \delta_{a} ;$
$b_{4,7}=k_{12} v_{d c, 2} \sin \delta_{b} ; b_{4,8}=k_{12} m_{b} \cos \delta_{b}$;
$b_{4,11}=k_{12} v_{d c, 3} \sin \delta_{c} ; b_{4,12}=k_{12} m_{c} \cos \delta_{c} ;$
$b_{5,1}=k_{13}\left(i_{1, d} \cos \delta_{1}+i_{1, q} \sin \delta_{1}\right)$;
$b_{5,2}=-k_{13} m_{1}\left(i_{1, d} \sin \delta_{1}-i_{1, q} \cos \delta_{1}\right)$;
$b_{5,3}=-k_{13}\left(i_{1, d}^{\prime} \cos \delta_{a}+i_{1, q}^{\prime} \sin \delta_{a}\right)$;
$b_{5,4}=k_{13} m_{a}\left(i_{1, q}^{\prime} \sin \delta_{a}-i_{1, q}^{\prime} \cos \delta_{a}\right) ;$
$b_{5,5}=k_{14}\left(i_{2, d} \cos \delta_{2}+i_{2, q} \sin \delta_{2}\right)$;
$b_{5,6}=-k_{14} m_{2}\left(i_{2, d} \sin \delta_{2}-i_{2, q} \cos \delta_{2}\right)$;
$b_{5,7}=-k_{14}\left(i_{2, d}^{\prime} \cos \delta_{b}+i_{2, q}^{\prime} \sin \delta_{b}\right)$;
$b_{5,8}=k_{14} m_{b}\left(i_{2, q}^{\prime} \sin \delta_{b}-i_{2, q}^{\prime} \cos \delta_{b}\right)$;
$b_{5,9}=k_{15}\left(i_{3, d} \cos \delta_{3}+i_{3, q} \sin \delta_{3}\right)$;
$b_{5,10}=-k_{15} m_{3}\left(i_{3, d} \sin \delta_{3}-i_{3, q} \cos \delta_{3}\right)$;
$b_{5,11}=-k_{15}\left(i_{3, d}^{\prime} \cos \delta_{c}+i_{3, q}^{\prime} \sin \delta_{c}\right)$;
$b_{5,12}=k_{15} m_{c}\left(i_{3, q}^{\prime} \sin \delta_{c}-i_{3, q}^{\prime} \cos \delta_{c}\right)$
$k_{1}=-\frac{m_{1}}{2 L} ; k_{2}=-\frac{m_{2}}{2 L} ; k_{3}=-\frac{m_{3}}{2 L} ; k_{4}=\frac{m_{a}}{2 L^{\prime}} ; k_{5}=\frac{m_{1}}{2 C_{1}} ;$
$k_{6}=\frac{m_{2}}{2 C_{2}} ; \quad k_{7}=\frac{m_{3}}{2 C_{3}} ; \quad k_{8}=-\frac{m_{a}}{2 C_{1}} ; \quad k_{9}=-\frac{m_{b}}{2 C_{2}} ;$
$k_{10}=-\frac{m_{c}}{2 C_{3}} ; \quad k_{11}=\frac{\omega}{2 L} ; \quad k_{12}=\frac{\omega}{2 L^{\prime}} ; \quad k_{13}=\frac{\omega}{2 C_{1}} ;$
$k_{14}=\frac{\omega}{2 C_{2}} ; k_{15}=\frac{\omega}{2 C_{3}} ; k_{16}=\frac{m_{b}}{2 L^{\prime}} ; k_{17}=\frac{m_{c}}{2 L^{\prime}}$

## NOMENCLATURES

$i$ : Index to show the phase of injected voltage
$h$ : Index to show the phase of voltage in converter
$k$ : Index to show the converter number
$L_{k}$ for $k=1,2,3$ : Filter inductance in converter $k$
$C_{k}$ for $k=1,2,3$ : Filter capacitance in converter $k$
$S_{b, i}$ for $i=a, b, c:$ Bypass switch in phase $i$
$R_{k}$ for $k=1,2,3:$ Resistance in converter $k$
$v_{t, k, h}$ for $k=1,2,3, h=a, b, c$ : Input terminal voltage of phase $h$ in converter $k$
$v_{t, m}$ : Maximum value of input terminal
$v_{t, i}$ for $i=a, b, c$ : Output terminal voltage of phase $i$
$v_{t, m, i}$ for $i=a, b, c$ : Maximum value of output terminal voltage of phase $i$
$\phi_{i}$ for $i=a, b, c$ : Phase angle of injected voltage in phase $i$ $v_{d c, k}$ for $k=1,2,3$ : Output voltage of filter capacitor in converter $k$
$m_{i}$ for $i=a, b, c:$ Transfer ratio of phase $i$
$m_{k}$ for $k=1,2,3:$ Transfer ratio of converter $k$
$S_{k, h}$ for $k=1,2,3, h=a, b, c$ : Switching function of positive half-cycle
$S_{k, h}^{\prime}$ for $k=1,2,3, h=a, b, c$ : Switching function of negative half-cycle
$L_{k, h}$ for $k=1,2,3, h=a, b, c$ : Inductance in phase $h$ of converter $k$ in converter side
$r_{k, h}$ for $k=1,2,3, h=a, b, c$ : Resistance in phase $h$ of converter $k$ in converter side
$v_{k, h}$ for $k=1,2,3, h=a, b, c$ : Voltage of phase $h$ in converter $k$
$v_{F H}$ : Voltage drop in switch
$v_{H n}$ : Middle point voltage of filter to null
$r_{s}$ : Switch resistance
$R_{k, h}$ for $k=1,2,3, h=a, b, c$ : Equivalent resistance in phase $k$ of converter $h$ in converter side
$\bar{d}_{k, h}$ for $k=1,2,3, h=a, b, c$ : Switching function for positive half cycle of phase $h$ in converter $k$
$\bar{d}_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Switching function for positive half cycle of phase $i$ in converter $k$
$\delta_{k}$ for $k=1,2,3:$ Phase angle of converter $k$
$\delta_{i}^{\prime}$ for $i=a, b, c:$ Phase angle of phase $i$
$A_{k}$ for $k=1,2,3:$ Voltage amplitude in converter $k$
$A_{i}^{\prime}$ for $i=a, b, c$ : Voltage amplitude in phase $i$
$R_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Equivalent resistance in phase $i$ of converter $h$ in injection side
$v_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Voltage of phase $i$ in converter $k$
$i_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Current of phase $i$ in converter $k$
$i_{d c, k}$ for $k=1,2,3: \mathrm{DC}$ current in converter $k$
$i_{k, d c}$ for $k=1,2,3$ : DC current passing through converter $k$ in converter side
$i_{k, d c}^{\prime}$ for $k=1,2,3$ : DC current passing through converter $k$ in injection side
$R$ : Equivalent resistance in converter side
$R^{\prime}$ : Equivalent resistance in injection side
$L$ : Equivalent inductance in converter side
$L^{\prime}$ : Equivalent inductance in injection side
$A$ : State matrix
$B$ : Input matrix
$X$ : State vector
$U$ : Input vector
$L_{t, h}$ : Inductance of transformer in phase $h$
$i_{k, h}$ : Current of phase $h$ in converter $k$
$L_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Inductance in phase $i$ of converter $k$ in injection side
$r_{t, h}$ for $h=a, b, c$ : Resistance of transformer in phase $h$
$V_{t, k, h}$ for $k=1,2,3, h=a, b, c$ : Steady state input terminal voltage of phase $h$ in converter $k$
$X_{k, h}$ for $k=1,2,3, h=a, b, c$ : Reactance of phase $h$ in converter $k$
$I_{k, h}$ for $k=1,2,3, h=a, b, c$ : Steady state current of phase $h$ in converter $k$
$V_{k, h}$ for $k=1,2,3, h=a, b, c$ : Steady state voltage of phase $h$ in converter $k$
$V_{t, k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Steady state output terminal voltage of phase $i$ in converter $k$
$X_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Reactance of phase $i$ in converter $k$
$I_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Steady state current of phase $i$ in converter $k$
$V_{k, i}^{\prime}$ for $k=1,2,3, i=a, b, c$ : Steady state voltage of phase $i$ in converter $k$

## REFERENCES

[1] P. Kundor, "Power System Stability and Control", McGraw Hill, New York, 1994.
[2] F.M. Shahir, E. Babaei, "Evaluating the Dynamic Stability of Power System using UPFC Based on Indirect Matrix Converter", Proc. ICECT, pp. 254-258, 2012.
[3] F.M. Shahir, E. Babaei, "Evaluation of Power System Stability by UPFC via Two Shunt Voltage Source Converters and a Series Capacitor", Proc. ICEE, pp. 318-323, 2012.
[4] F.M. Shahir, E. Babaei, "Assessment of Power System Stability by UPFC with Two Shunt Voltage Source Converters and a Series Capacitor", Journal of ERR, 2013.
[5] M. Moazen, E. Babaei, "Power System Dynamic Model Including Dynamic Voltage Restorer," Proc. ICECT, pp. 1-6, 2011.
[6] E. Babaei, M.F. Kangarlu, M. Sabahi, "Mitigation of Voltage Disturbances Using Dynamic Voltage Restorer Based on Direct Converters", IEEE Transactions on Power. Del., Vol. 25, No. 4, pp. 2676-2683, Oct. 2010.
[7] F.M. Shahir, E. Babaei, "Dynamic Modeling of UPFC by Two Shunt Voltage Source Converters and a Series Capacitor", Proc. ICECT, pp. 248-253, 2012.
[8] F.M. Shahir, S. Ranjbar, E. Babaei, "Dynamic Modeling of UPFC Based on Indirect Matrix Converter," Proc. IICPE, pp. 1-6, 2012.
[9] R.H. Adware, P.P. Jagtap, J.B. Helonde, "Power System Oscillations Damping Using UPFC Damping Controller", Proc. ICETET, pp. 340-344, 2010.
[10] B. Mwinyiwiwa, Z. Wolanski, B.T. Ooi, "Current Equalization in SPWM FACTS Controllers at Lowest Switching Rates", IEEE Trans. on Power Electron., Vol. 14, No. 5, pp. 900-905, Sep. 1999.

## BIOGRAPHIES



Farzad Mohammadzadeh Shahir was born in Tabriz, Iran, in 1985. He received the B.Sc. degree in Electronic Engineering from Islamic Mianeh Brach, Islamic Azad University, Mianeh, Iran, and the M.S. degree in Electrical Engineering from Ahar Branch, Islamic Azad University, Ahar, Iran, in 2008 and 2011, respectively. In 2004, he joined the Iran Tractor Manufacturing Company, Tabriz, Iran, where he has been an Electrical Engineering over there. His current research interests include power system dynamic and power electronic converters.


Ebrahim Babaei was born in Ahar, Iran in 1970. He received his B.Sc. and M.Sc. in Electrical Engineering from the Department of Engineering, University of Tabriz, Tabriz, Iran, in 1992 and 2001, respectively, graduating with first class honors. He received his Ph.D. in Electrical Engineering from the Department of Electrical and Computer Engineering, University of Tabriz, in 2007. In 2004, he joined the Faculty of Electrical and Computer Engineering, University of Tabriz. He was an Assistant Professor from 2007 to 2011 and has been an Associate Professor since 2011. He is the author of more than 200 journal and conference papers. His current research interests include the analysis and control of power electronic converters, matrix converters, multilevel converters, FACTS devices, power system transients, and power system dynamics.


Samira Ranjbar was born in Tabriz, Iran, in 1987. She received the B.Sc. degree in Computer Engineering from Tabriz Branch, Islamic Azad University, Tabriz, Iran in 2011. Her current research interests include power system dynamic and power electronic converters.


Saman Torabzad was born in Tabriz, Iran, in 1982. He received the B.Sc. degree in Electronics Engineering from University of Tabriz, Tabriz, Iran in 2008, and the M.Sc. degree in Electrical Power Engineering from the same university in 2011. He is presently the CEO of Urum Fan \& Niroo Co., Iran where he is engaged in the development and design of electrical facilities, managing electrical competency group and leading electrical design activities to bring new technologies to production lines.

