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# CASCADED TRACKING CONTROL OF NONHOLONOMIC MOBILE ROBOT BASED ON NEURAL NETWORKS AND FEEDBACK LINEARIZATION

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**Abstract-** In this paper, the problem of trajectory tracking control of nonholonomic mobile robot is investigated via an intelligent cascaded control strategy. The vehicle inverse dynamic is represented and controlled utilizing neural networks and the kinematics is controlled by feedback linearization. Simulation results demonstrate the high efficiency of the proposed strategy.

**Keywords:** Nonholonomic Mobile Robot, Neural Network, Feedback Linearization, Inverse Dynamic.

# I. INTRODUCTION

Mobile robots are autonomous vehicles equipped to perform missions in variety of environments with fixed or uncertain conditions and are increasingly becoming the center of attention of engineers. The motion of nonholonomic mechanical systems, however, is restricted by their own kinematics so the control law could not be driven in a straightforward manner [1, 2].

The design objective in mobile robot motion control is to control both kinematics and dynamics of the vehicle. First step is to design a controller for kinematic model assuming that velocity tracking is perfect. Feedback linearization could be utilized to fulfill this objective. Even though the proposed method is efficient in controlling the vehicle, it neglects the dynamic model. Taking into account the dynamics is more realistic and different attempts are made to solve this problem [3, 4].

Second step in motion control is to control the dynamic model. A neural inverse dynamic is introduced for this mean. Neural networks are proper choices in the identification and control of dynamic systems. They can successfully be utilized for modeling nonlinear systems and also for implementing of nonlinear controllers [5-8].

In this paper, a cascaded feedback linearizing control strategy is developed to control the mobile robot. The strategy requires two control loops, the first of which is the inner control loop for dynamic model to control the linear and angular velocities of the robot. The desired position tracking is carried out via outer control loop of the kinematic model. Evolutionary algorithms can be used to improve the tracking results. In this paper, first a cost function consist of step response main factors is defined

and genetic algorithm (GA) is applied in order to minimize it. Paper is organized as follows: Section II is devoted to the mobile robot kinematics and dynamics, Section III deals with the applied controllers. Section IV gives a brief introduction of genetic algorithm. Section V contains simulation results and section VI concludes the paper.

#### II. NONHOLONOMIC MOBILE ROBOT

Figure 1 presents the structure of the robot. It has two driving wheels that are controlled independently by two motors. Mobile robot also has one passive orientable wheel placed in front of the vehicle. This wheel helps preventing the robot from possible tipping over while moving on a plane. As it is illustrated in the figure, (x, y) are the position coordinates in the world reference frame and the heading direction from the x-axis is shown by  $\theta$ .

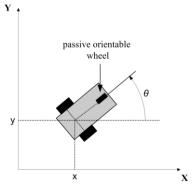


Figure 1. Mobile robot

The following equations of motion present the kinematics of the mobile robot [9],

$$\dot{q} = S(q)V \tag{1}$$

where, 
$$S(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix}$$
 and  $V = \begin{bmatrix} v \\ \omega \end{bmatrix}$ . The dynamics of

the mobile robot is,

$$M(q)\dot{V} + C(q,\dot{q})V + DV = \tau \tag{2}$$

where,  $q=(x, y, \theta)$  is the generalized coordination vector,  $V=[v \ \omega]^T$  is the vector of velocities including both linear

velocity v and angular velocity  $\omega$ ,  $\tau_1$  and  $\tau_2$  are the torques that are applied on left and right wheels and are presented by the vector  $\tau = [\tau_1 \tau_2]^T$  and M(q) is a 2×2 positive definite inertia matrix.

Centripetal and Coriolis forces are represented by the vector  $C(q, \dot{q})$  and D is a 2×2 diagonal positive definite matrix. The nonholonomic constraint, which prevents the robot from slipping sideways, is introduced by the following equation [10],

$$A(q)\dot{q} = \dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{3}$$

# III. APPLIED CONTROL STRATEGY

Controlling mobile robot undergoes two main stages,

the first of which is to control the dynamics, regarding linear and angular velocities as outputs. When the dynamics is controlled one can proceed into second stage which is to control the kinematics i.e. the position of the vehicle. So, taking into account both dynamics and kinematics, designing the controller structure requires two main loops:

- 1. The inner loop that is based on robot dynamic model and is utilized to control linear and angular velocities of mobile robot.
- 2. The outer loop is dependent on kinematics and helps control the position of the robot. Figure 2 depicts the block diagram of the described cascaded control structure.

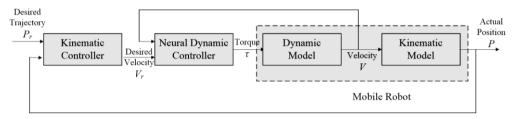


Figure 2. Block diagram of cascaded control structure

#### A. Dynamic Control of Mobile Robot

The main idea of the inverse dynamics modeling for mobile robot is to find a feedback control based on the dynamic model of the system that is presented in Equation (2) [1]. The vector  $\tau = [\tau_1 \tau_2]^T$  is regarded as the input to the system and the velocity vector  $V=[v \omega]^T$  is the output of the system. The design objective is to find the control input torque  $\tau$  in a way that,

$$\lim_{t \to \infty} ||V_r(t) - V(t)|| = 0 \tag{4}$$

where,  $V_r$  is the desired velocity vector and  $t_s < \infty$  is the reachability time. Different approaches are made toward the inverse dynamic modeling of the mobile robot, including fuzzy control, pole placement and Hurwitz equation [3, 4].

This paper takes benefits of neural networks to model the inverse dynamic of the system. Two major applications of neural networks are nonlinear systems identification and dynamic systems control [11]. These characteristics make them a proper choice for modeling the inverse dynamic [5]. Figure 3 shows the structure of the neural network that is used to model the inverse dynamics. The inputs of the neural network are the linear and angular velocities and the outputs of the network are the torques applied on the left and right wheels.

As it is presented in the figure, the neural structure has three layers. The data remains unchanged passing through the first layer. Second and third layer activation functions are 'tansig' and 'purlin' respectively. When the dynamic model is controlled, one can proceed into second step that is to design a controller for kinematics of the robot. Next section is devoted to this step.

# **B.** Kinematic Control of Mobile Robot

In this paper, feedback linearization approach is used to control the kinematics model of the robot. Mobile robot is not input-output linearizable using static feedback but is input-output linearizable based on dynamic state feedback.

Using feedback linearization problem for trajectory tracking, a feedback compensator should be considered of the form [12],

$$\dot{\xi} = f_{\xi}(q,\xi) + g_{\xi}(q,\xi)u$$

$$v = \alpha(q,\xi) + \beta(q,\xi)u$$
(5)

where,  $\xi$  is the 2×1 state vector and u is the 2×1 input vector, in a way that Equations (1) and (5) be equivalent to a linear system.

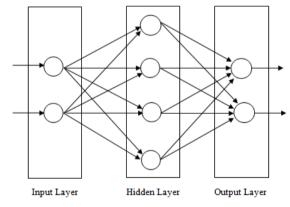


Figure 3. Neural network structure used to model of robot inverse dynamics

The problem of controlling kinematics model is followed based on the procedure developed by Oriole. The linearizing output vector is defined as  $\eta = [x, y]^T$  and according to Equation (1), differentiating  $\eta$  will result in [13],

$$\dot{\eta} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

According to the equation the angular velocity  $\omega$  has no effect on  $\dot{\eta}$ . An integrator is added to the linear velocity input as:

$$v = \xi, \ \dot{\xi} = a \Rightarrow \dot{\eta} = \xi \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$
 (7)

where,  $\xi$  is state variable, a stands for mobile robot linear acceleration. Differentiating Equation (7) will result in,

$$\ddot{\eta} = \dot{\xi} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} + \xi \dot{\theta} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\xi \sin \theta \\ \sin \theta & \xi \cos \theta \end{bmatrix} \begin{bmatrix} a \\ \omega \end{bmatrix}$$
 (8)
Under the assumption  $\xi \neq 0$  the matrix multiplying by

 $\begin{bmatrix} a \\ o \end{bmatrix}$  is nonsingular and Equation (8) can be rewritten as:

$$\begin{bmatrix} a \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \theta & -\xi \sin \theta \\ \sin \theta & \xi \cos \theta \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\ddot{\eta} = \begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = u$$
(9)

The resulting dynamic compensator is as follows:

$$\dot{v} = \dot{\xi} = u_1 \cos \theta + u_2 \sin \theta , \quad v = \xi$$

$$\omega = \frac{u_2 \cos \theta - u_1 \sin \theta}{\xi} \tag{10}$$

and the new coordinates is resulted as:

$$z_1 = x$$

$$z_2 = y$$

$$z_3 = \dot{x} = \xi \cos \theta \tag{11}$$

$$z_4 = \dot{y} = \xi \sin \theta$$

Based on Equation (11) the extended system kinematics is linearized and described by two chains of integrators,

$$\ddot{z}_1 = u_1 
\ddot{z}_2 = u_2$$
(12)

An exponentially stabilizing feedback for the trajectory tracking can be driven based on the decoupled and fully linear system of Equation (12),

$$u_{1} = \ddot{x}_{r} - K_{d1}(\dot{x} - \dot{x}_{r}) - K_{p1}(x - x_{r})$$

$$u_{2} = \ddot{y}_{r} - K_{d2}(\dot{y} - \dot{y}_{r}) - K_{p2}(y - y_{r})$$
(13)

Finally the related dynamic errors are resulted as:

$$\ddot{e}_{x}(t) + K_{d1}\dot{e}_{x}(t) + K_{p1}e_{x} = 0$$

$$\ddot{e}_{y}(t) + K_{d2}\dot{e}_{y}(t) + K_{p2}e_{y} = 0$$
(14)

where, x and y are the coordinates of the mobile robot,  $x_r$ and  $y_r$  are the desired coordinates and  $e_{x,y}$  is the error between actual and desired coordination.  $K_p$  and  $K_d$  are the gains which should be properly chosen so that the characteristic equations are Hurwitz.

Proper Gain selection is important in controlling process. A good performance indicator in the time domain should include the main step response factors which are the overshoot (Mp), rise time  $(t_r)$ , settling time  $(t_s)$ , and steady-state error ( $E_{ss}$ ). A cost function consisting of all these parameters is defined as,

$$f(k) = (1 - e^{-\beta})(Mp + E_{ss}) + e^{-\beta}(t_s - t_r)$$
(15)

where  $\beta$  is the weighting factor. Evolutionary algorithms are strong tools in optimizing cost functions. In this paper genetic algorithm is utilized to fulfill this aim.

#### IV. GENETIC ALGORITHM - A BRIEF REVIEW

Genetic Algorithm is the first evolutionary algorithm, developed by Jon Holland [14] the basis of which is placed on the natural process of evolution through reproduction. Being able to solve many nonlinear and large problems, GA has become so useful in a large variety of applications. This algorithm starts with a random numbers (population) each of which is called a chromosome.

Chromosomes' fitness are evaluated by the cost function. The cost values are then sorted. A specific amount of worst chromosomes-those with higher costsare ignored via elimination. At this stage the discarded chromosomes are replaced by new offsprings. The better chromosomes'-parents'-produce offspring chromosomes via pairing and crossover. The offsprings will help breeding new chromosomes on next iterations. This way, on each of the iterations the worst chromosomes will be replaced by better ones. This procedure will help the information flow throughout the population. After crossover, during the mutation stage a certain rate of the population, are randomly selected and substituted by other random values. Due to elitism, the best chromosome which has the lowest cost value will remain unchanged during the mutation.

After applying the mutation to the generation, the new population will be re-evaluated. As far as the stopping conditions are not satisfied the algorithm will keep on computing the cost function in new iterations. If the stopping criteria are satisfied the algorithm will terminate and the chromosome with the lowest cost value will be the desired parameters set [15, 16].

# V. SIMULATION RESULTS

In this section, the efficiency of the proposed control strategy is evaluated via simulation results. The matrix numerical values of kinematic and dynamics equations are as follows [17],

$$M(q) = \begin{bmatrix} 0.3749 & -0.0202 \\ -0.0202 & 0.3739 \end{bmatrix}, D = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0.135\dot{\theta} \\ -0.135\dot{\theta} & 0 \end{bmatrix}$$

The controller parameters are first chosen by trial and error in order to satisfy the tracking performance. These values are set to  $K_{p1} = K_{p2} = K_{d1} = K_{d2} = 2$ . As the first step, it is necessary to control the system dynamics. Neural inverse dynamic is utilized to control the linear and angular velocities. The hidden layer consists of four neurons and output layer has two neurons.

Hidden and output layer activation functions are 'tansig' and 'purlin' respectively. 5000 samples with sampling time of 0.001 s is fed to the network and feed forward back propagation is utilized to train the network in 1000 epochs. Simulation results are carried out to show the effectiveness of the proposed method. Figure 4 and Figure 5 present the linear velocity tracking results and the related tracking error respectively.

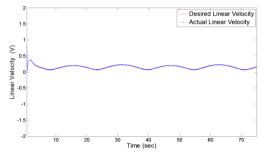


Figure 4. Linear velocity by applying the cascaded controller

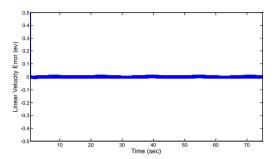


Figure 5. Linear velocity tracking error by applying cascaded controller

As the figures present, the utilized neural network is able to control the linear velocity and the desired value is tracked. Figure 6 and Figure 7 present the angular velocity tracking results and the related tracking error respectively.

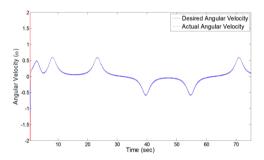


Figure 6. Angular velocity by applying the cascaded controller

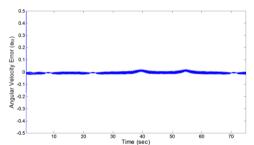


Figure 7. Angular velocity tracking error by applying cascaded controller

The proposed method is efficient in dynamic control of mobile robot since it forces the system to track the desired linear and angular velocities and related errors fall in an acceptable range. Second step is to control the system kinematics, i.e. the mobile robot position. The initial position of mobile robot is considered as (x, y) = (-0.5, 0.5). The vehicle is then commanded to track a desired trajectory. Figure 8 presents the trajectory tracking regarding the cascaded control strategy.

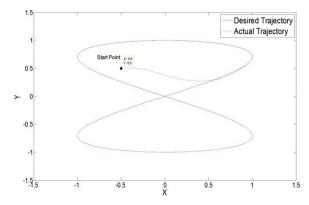


Figure 8. Trajectory tracking regarding cascaded controller

As the figures present, applying feedback linearization method force the vehicle to track the desired trajectory. So, the control approach is capable of controlling the nonholonomic mobile robot velocity and position in an effective way and the results show the ability of the intelligent cascaded controller.

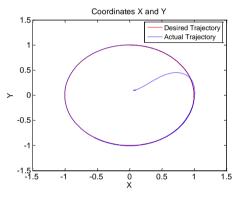


Figure 9. Trajectory tracking regarding cascaded controller for a circle path

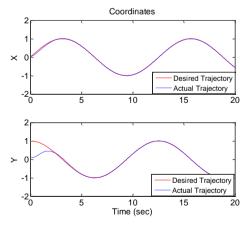


Figure 10. Each coordinate trajectory tracking regarding cascaded controller

In order to investigate the performance more precisely another reference trajectory is regarded for the vehicle. Figure 9 presents the x-y plane desired trajectory and controlled vehicle tracking result. Figure 10 presents each of the coordinates and figure 11 presents the resulted dynamic control, i.e. the linear and angular velocities tracking results.

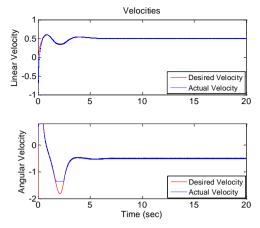


Figure 11. Linear and angular velocity tracking regarding cascaded controller

Utilizing evolutionary algorithms in order to optimize the cost function introduced in Equation (15) can help improve the tracking results. The GA initial population is set to 20 with 50 iterations. Mutation and selection rates are set to 0.4 and 0.5 respectively. Table 1 presents the obtained gain values.

Table 1. Optimization results

$K_{p1}$	$K_{d1}$	$K_{p2}$	$K_{d2}$
5.1217	3.834	4.9961	6.3457

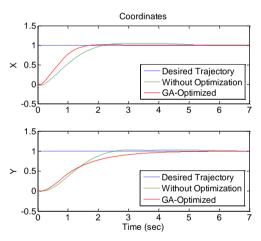


Figure 12. Step response for the vehicle coordinates with and without optimized controller gains

Figure 12 presents the desired coordinates  $x_r$  and  $y_r$  and the step responses of applying the controller with and without the optimized gains. Table 2 and Table 3 depict the main factors of the unit step response for the coordinates x and y regarding each controller. These factors are the rise time, overshoot, settling time and steady state error.

According to Table 2 and Table 3 GA-optimized controller shows a better performance, regarding almost all of the step response factors. Figure 13 presents the velocity tracking for the GA-optimized controller. The optimized controller is capable of controlling the system kinematics and the system dynamics are also well tracked.

Table 2. Step response factor for the first coordinate *X* 

	Steady State Error	Settling Time (sec)	Overshoot	Rising Time (sec)
Without Optimization	0.0037	4.898	5.3%	1.374
GA- Optimized	0	1.597	1.64%	1.016

Table 3. Step response factor for the first coordinate Y

	Steady State Error	Settling Time (sec)	Overshoot	Rising Time (sec)
Without Optimization	0.0004	5.498	3.32%	3.019
GA- Optimized	0.0023	4.617	0%	2.444

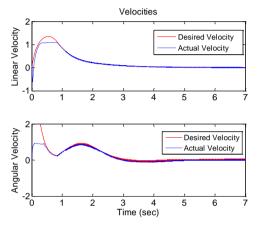


Figure 13. Dynamic control for the vehicle with optimized controller gains

# VI. CONCLUSIONS

A successful attempt has been made to develop a control method for a nonholonomic mobile robot. A model for the robot has been presented including the kinematics and dynamics of the vehicle. Two combined control algorithms were used to control this system via a cascaded structure, the first one of which was a neural network inverse dynamic controller designed to control the linear and angular velocities of the robot. Neural networks are strong candidates for system identification and control. Utilizing the neural network as the inverse dynamic of the vehicle, force it track the desired velocities. In the second step, a feedback linearization based controller was applied on the kinematics model of the system. The robot model is linearized based on dynamic state feedback. Linearizing the model and adjusting the controller parameters using GA make the robot track a desired trajectory. The cascaded controller has been applied on simulated system. The results demonstrate the high efficiency of the proposed method in controlling the robot. So this cascaded controller can be considered as a suitable method in controlling mobile robots.

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Nazila Nikdel received her B.Sc. and M.Sc. degrees in Control Engineering from University of Tabriz, Tabriz, Iran, in 2007 and 2010, respectively. She is currently pursuing Ph.D. degree in Control Engineering at the same university. She has been a

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