

# FURTHER RESULTS ON DMC SENSITIVITY TO THE *m* PARAMETER

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**Abstract-** In this paper, authors add further results to previous analysis on the sensibility of the two main error indexes, i.e., the Mean Squared Error (*mse*) and the overshoot of the output over the reference ( $M_p$ ), when the control horizon is varied in a concrete Model Predictive Control (MPC) algorithm named Dynamic Matrix Control (DMC). Besides, we also study the sensibility of the objective function J of the DMC controller to that variation. To carry out this analysis 840 experiments have been performed using a plant that has shown to be unstable when a PID controller tuned by means of the Ziegler-Nichols method is used, showing that the value of the error indexes also varies with a determined pattern beyond to the previous analyzed range of control horizon parameter m.

**Keywords:** Error Indexes, Control Horizon, Model Predictive Control, Dynamic Matrix Control.

# I. INTRODUCTION

Model Predictive Control (MPC) is a wide set of advanced control techniques devoted to deal with complex systems. This type of advanced controllers has been used and compared with classic PID controllers [4, 13], showing a good performance. In the literature, they have been used for a wide variety of applications, such as energy management [1], signal processing applications [9], multi-robot systems implementation [5, 6] and motor control [10], among others. Besides, they have shown their suitability for being implemented by means of neural networks [7].

One of the most popular of these algorithms is Dynamic Matrix Control (DMC), and the main objective of this paper is to analyze the sensitivity of two error indexes and the objective function of the predictive controller under the effect of different control horizon values going beyond the range of parameters used in previous works. Excepting our previous works, we have neither studied nor found in the literature any study about the influence of the control horizon m on these performance indexes.

Probably, this circumstance arises because it is usually supposed a fixed implementation of the predictive controller, which is defined by a concrete m value. However, it is a factor to consider in the case that the engineer or practitioner can choose or influence the concrete implementation of the controller to design.

The paper is structured as follows. In the second section, we recall some classic references about MPC and DMC, where fundamental explanations of MPC and DMC will be found, and the importance and the role of the m parameter in such control scheme will be exposed. Besides, a reference to a previous work where the controlled system and its working point are detailed.

In the third section, we describe briefly the error indexes we have used to describe the performance of the DMC controllers and the mathematical definition of its objective function. The fourth section gives the experimental design we have carried out. The fifth section exposes the results obtained on the *mse* index, and the sixth on the  $M_p$  respectively, while the seventh section shows the behavior of the objective function J. Finally, the last section provides our conclusions.

# **II. PREVIOUS RELATED WORKS**

In this section, we are going to give a brief background and to recall some classic references of the literature where a good background on MPC and DMC can be found. MPC is an advanced control technique used to deal with systems that are not controllable using classic control schemas. In fact, it is not a concrete technique, it is a set of algorithms with several common characteristics.

There is a world model that is used to predict the system output from the actual instant until p samples, an objective function J that must be minimized and a control law that minimizes that objective function by m control actions, and a  $\lambda$  parameter that defines the embodiment of the controller. For a deep insight about MPC and DMC see [2, 3, 8, 11, 12]. Finally, with regard to the system that has been used to carry out the experiments.

The main part of the argumentation on its utilization has been intentionally omitted due to space issues. Its detailed description, the determination of the working point can be found in [4]. At this point, we only describe its dynamics through Equation (1) and its response while controlled by means of a Ziegler-Nichols PID controller in Figure 1, which shows clearly that its response is unstable.

$$H(z) = \frac{1}{z - 0.5} \tag{1}$$

### III. ERROR INDEXES AND OBJECTIVE FUNCTION DEFINITION

In this section, we describe the two error indexes that are going to be monitorized along the experimentation, paying attention to the measure of the error of the controller, as well as the objective function of the controller. On one hand, we will use the Mean Squared Error (*mse*) between the reference and the system output. On the other hand, we will also measure the overshoot of the output over the reference ( $M_p$ ), taking place usually on the first rising of the output signal. This last performance index is graphically represented in Figure 2.

With regard to the objective function definition, it can be found in the classic literature referenced in section III. At this point, we recall it by means of Equation (2):

$$J = \sum_{j=1}^{p} \left[ \hat{y}(t+j \mid t) - w(t+j) \right]$$
(2)

where,  $\hat{y}(t+j|t)$  is the prediction of the output at time (t+j) predicted at *t* and w(t+j) is the desired reference at time (t+j).

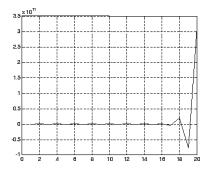


Figure 1. Unstable response of the closed loop system

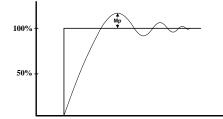


Figure 2. Graphical representation of the error indexes

#### **IV. EXPERIMENTAL DESIGN**

As stated before, the main objective of this paper is to analyze the error indexes and objective function sensitivity to the control horizon parameter m. In this section, we are going to describe the experimental design that we have carried out to asses that effect. A combination of a number of values of p, m and  $\lambda$  has been taken. The values that have been involved are m = 1, m = 3, m = 5, m = 7, m = 10, m = 15 and m = 20. The value for the prediction horizon varies from p = 1 to p = 20 with a step of one unit. Finally, the values of the  $\lambda$  parameter are 10<sup>-3</sup>, 10<sup>-2</sup>, 10<sup>-1</sup>, 1, 10<sup>1</sup> and 10<sup>2</sup>. This results in a Cartesian product of 840 experiments.

# V. SENSITIVITY OF mse INDEX

In this section, we are going to study the sensibility of the *mse* index under controlling action of DMC controllers with different control horizon *m* values and the remaining experimental conditions described in section III. As we can see in Figures 3 to 9, with increasing values of the control horizon *m*, the value of the *mse* index becomes smaller. Although in each sample time only the first control signal is taken into account by the DMC controller, (i.e., the remaining *m*-1 control signals are discarded), the fact of having a control horizon of *m* sampling times helps to the controller to have a more complete dynamic matrix *G*, and to obtain a more accurate control signal  $\Delta u$ .

This is a reasonable result because with a larger control horizon, the prediction made along the prediction horizon will be more accurate, and this will influence on  $\Delta u$ . In this way, there is an important interdependence between the control horizon m and the prediction horizon p, because there is only a significant descent of the *mse* value for those values of p equal of less than m. This tendency is observed in all the analyzed cases.

Finally, the parameter  $\lambda$  also plays a relevant role, because the larger values of the *mse* index is always reached with the higher values of that parameter, damped in some cases by an adequate value of the prediction horizon *p*. The only exception can be seen with m = 1, where the value of the *mse* index goes from 0 to 0.35 and the value of  $\lambda$  is irrelevant for the most area of the figure. On the other hand, the value of the  $\lambda$  parameter shows to be very relevant in the case of m = 20.

#### VI. SENSITIVITY OF M<sub>p</sub> INDEX

In this section, we are going to study the sensibility of the  $M_p$  index under the controlling action of DMC controllers with different control horizon m values and the remaining experimental conditions described in section III. As we can see in Figures 10 to 16, with increasing values of the control horizon *m*, the value of the  $M_p$  index becomes smaller only for small values of m, because as m becomes larger, the value of  $M_p$  index increases again. This is a different behavior with respect to the mse index, because in that case, increasing values of m always have the effect of reducing the value of the mse index. The  $M_p$  index value reduces until m = 5, when again its value increases near to 20%.

That tendency follows because an overshoot of 30% is reached with m = 10 and more than 35% with m = 20. In fact, these increasing shapes appear only with very small values of the prediction horizon p, because when it reaches moderate values (even smaller than m in all cases), the value of the  $M_p$  index decreases for the most part of the values of the  $\lambda$  parameter.

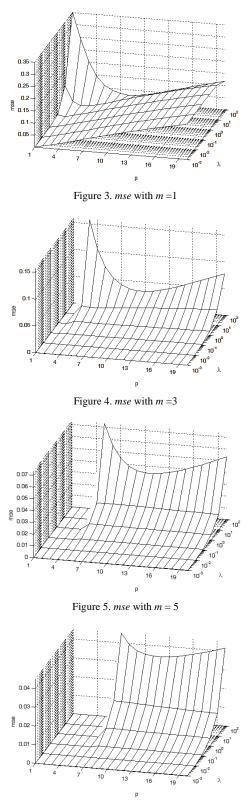
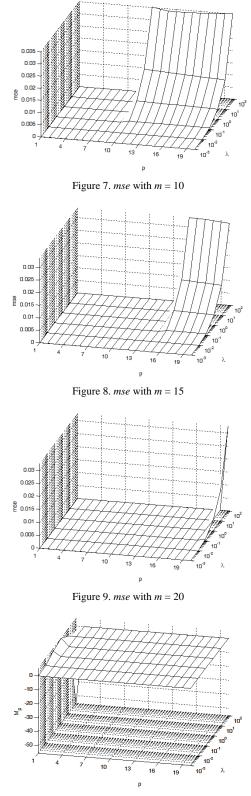
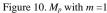


Figure 6. *mse* with m = 7

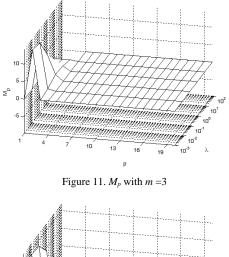
This means that the speed of the actuators and the embodiment of the controller do not pay an important role except to those values. That are so large in combination with punctual values of the prediction horizon p makes the  $M_p$  index very large in its absolute value, reaching a value of -50% due to the slow reaction of the controller.





# VII. SENSITIVITY OF OBJECTIVE FUNCTION

In this section, we are going to study the sensibility of the objective function J under the controlling action of DMC controllers with different control horizon m values and the remaining experimental conditions described in section III, excepting the simulation for m = 1 due to obvious reasons. In this case, the analysis is particularly simple. In the mathematical expression of the objective function given by the Equation [2] can be seen that there are essentially three elements dependence.



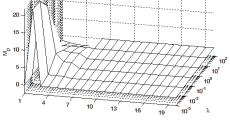


Figure 12.  $M_p$  with m = 5

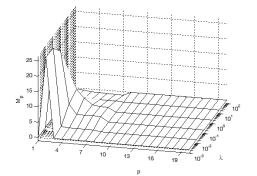


Figure 13.  $M_p$  with m = 7

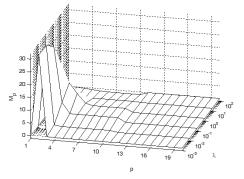


Figure 14.  $M_p$  with m = 10

On one side there is the prediction horizon p, on the other hand there is the prediction of the plant output  $\hat{y}(0)$  at  $\hat{y}(t+j)$  predicted at t, and finally there is the reference signal to follow.

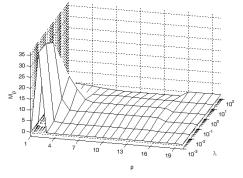


Figure 15.  $M_p$  with m = 15

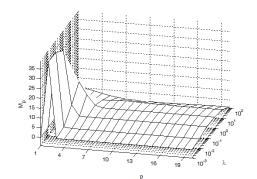


Figure 16.  $M_p$  with m = 20

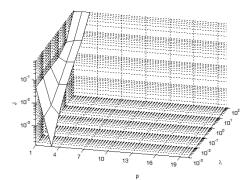


Figure 17. J with m = 3

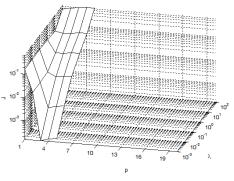


Figure 18. J with m = 5

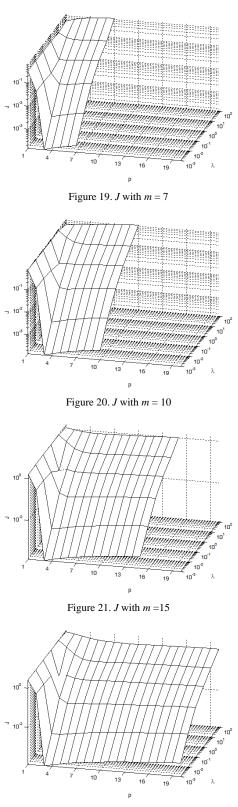


Figure 22. J with m = 20

In conclusion, there is no influence on the value obtained by the objective function J by the horizon control, as we can be seen in Figures 17 to 22.

#### **VIII. CONCLUSIONS**

In this paper, we have given a short introduction on the scope and the application fields of MPC and DMC, giving

a brief background and referencing some relevant sources of information regarding this. We have motivated the study of the effect of the control horizon parameter m on error indexes and we have stated that as objective.

We have enumerated the two error indexes that have been used and recalled objective function mathematical expression. The experimental design that has been carried out has been specified, resulting in a total of 840 experiments. The last three sections explain the results that have been reached with a number of figures, discussing the behavior of each error index and objective function independently. As summary, we can conclude that the control horizon parameter is relevant regarding error of DMC controller.

### NOMENCLATURES

G: The dynamic matrix of the DMC controller

 $\lambda$ : The parameter of the DMC controller related to its embodiment

*J*: The objective function to minimize by the controller *m*: The control horizon

 $M_p$ : The overshoot (performance index)

*mse*: The mean squared error (performance index)

*t*: The time instant subtitle

*p*: The prediction horizon

u(t): The whole input of the controlled system at  $t \Delta u(t)$ : The output of the DMC controller at t

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