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APPLICATION OF SOME CONFIGURATIONS OF LLC IN TRANSMISSION AND DISTRIBUTION SYSTEMS

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Abstract- In this paper, three new configurations for Line to Line Compensators (LLC) is proposed. These compensators can control several parameters of network, like active power flow and unbalance currents through controlling line to line voltages. The other feature of presented topologies is the fewer components in comparison with conventional FACTS devices. A review on line to line compensation and the principles of operation of new structures including their control method are discussed. The results are verified through simulations on the transmission and distribution test networks using MATLAB/SIMULINK.

Keywords: FACTS controller, Line To Line Compensation, Voltage Frame, AC Chopper, Unbalance Load.

I. INTRODUCTION

Due to environment and energy problems, constructing new power generation facilities and transmission lines is very difficult [1]. A solution to this problem can be Flexible Alternating Current Transmission Systems (FACTS) by increasing the transferred power through the existing transmission lines [2, 3]. Consequently providing an extra flexibility to the system, these controllers gain great importance in the liberalized power markets. Modern FACTS devices can control and adjust some parameters of the network at the coupling point, such as active power flow, voltage magnitude or voltage phase [4]. The flexibility and the capability of FACTS devices depend on their structures. For example, series, shunt or hybrid FACTS controllers provide different features in the system. Also more abilities can be obtain using a power source or an energy storage device as well as FACTS devices [5, 6].

A recently introduced FACTS controller type is Line to Line Compensator (LLC) [7, 8]. In LLC, three controlled voltages are inserted between each of two phases of a three phase transmission line. This structure can be extended to distribution level, too. In LLC, a transformer with at least one delta winding in primary and/or secondary side is needed to produce and control line to line voltages.

In this study three new topologies based on LLC are proposed. These topologies are named as open delta LLC (V-LLC), Transformer V-LLC (TV-LLC) and Chopper V-LLC (CV-LLC). Computer software simulations in MATLAB/SIMULINK are used to verify the claims and calculations.

II. A REVIEW ON LLC

Figure 1 shows the configuration of LLC [7, 8]. In this FACTS controller, the compensated voltages are added to phase to phase voltages using Voltage Source Inverters (VSI) and injecting transformers. This topology needs a three phase transformer with at least one delta side i.e. $\Delta-\Delta$, $\Delta-Y$ or $Y-\Delta$ connection. In this figure a $\Delta-\Delta$ transformer bank with three single phase transformers and three single phase inverters are used to compose a LLC.

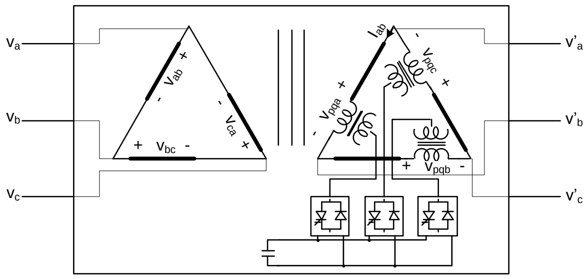

Figure 1. Structure of LLC [7, 8]

Changing V_{ab}, V_{bc} and V_{ca} adjust the phase voltages to new values. So, V_a' is the new phase-a to ground voltage which can be calculated from (1) [7, 8].

$$V_a' = V_a + (\frac{\sqrt{3}}{3}\angle -30)V_{pqa} \quad (1)$$

where,
V_a' is phase to ground voltage after LLC,
V_a is phase to ground voltage before LLC,
V_{pqa} is inserted voltage between phases a and b.

The turn ratio of $\Delta-\Delta$ three phase transformer bank is assumed unit. This assumption does not affect the generality of argument. Equation (1) comes from phasor diagram in Figure 2 [7, 8].

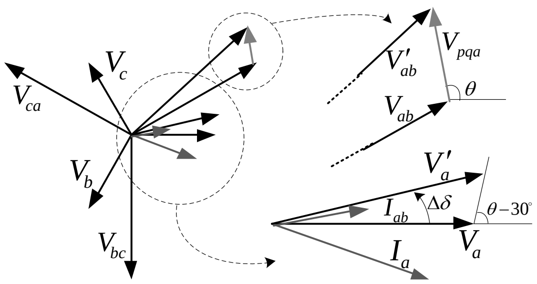

Figure 2. Phasor diagram of LLC [8]

If LLC is used in a Single Machine Infinite Bus (SMIB) network [9], the Equations (2-4) can be written. The active power of each phase of transmission line without LLC is [2, 7]:

$$P = \frac{V^2}{X}\sin\delta \tag{2}$$

where,
P is active power of line,
V is sending and receiving ends rms phase voltages,
X is reactance of transmission line,
δ is phase difference between sending and receiving end voltages.

With LLC the active power is changed to [7]:

$$\hat{P} = \frac{V^2}{X}\cos(\frac{\delta}{2})\left[2\sin(\frac{\delta}{2}) + K\right] \tag{3}$$

where,

$$K = \frac{V_{pq}/\sqrt{3}}{V} \tag{4}$$

and V_{pq} is the rms of each three inserted voltages.

Equation (3) shows the effect of LLC on active power flow. It must be noted that, if an open circuit occurs in one of the three injecting transformers or inverters of LLC, unlike SSSC the LLC can still operate correctly [7, 8].

III. INTRODUCING TO V-LLC, TV-LLC AND CV-LLC

A. V-LLC

As discussed above, LLC needs a delta connection to insert controlled additive voltages between lines and adjust the line to line voltages, which results in line to neutral voltages adjustment to achieve the desired active power flow in the line. Because of the ability of delta connection to operate in open delta or V configuration, LLC also can be represented in a V-connection. Therefore, one of the converters and injecting transformers can be eliminated. Figure 3 shows the configuration of V-LLC.

V_{pqa} and V_{pqb} are two injected voltages by V-LLC. As presented in phasor diagram of Figure 4 the phase voltages of line after V-LLC, are changed as a result of these injected voltages. In this figure the turn ratios of two transformers in V connection are assumed unit to simplify the relations. This assumption does not affect the generality of the argument.

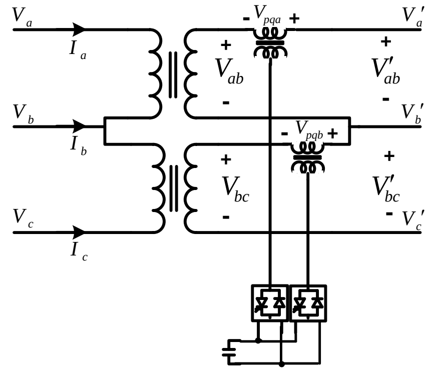

Figure 3. Structure of V-LLC

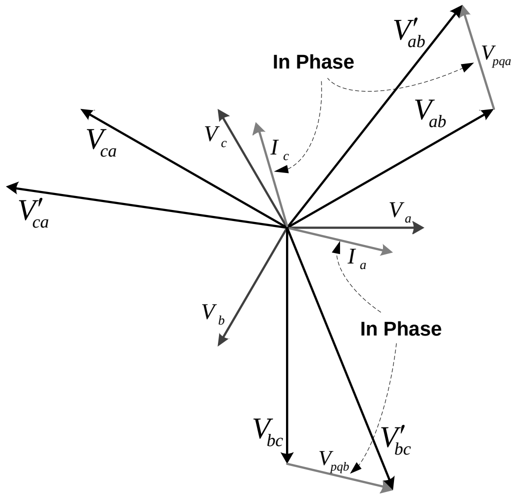

Figure 4. Phasor diagram of V-LLC with current frame control

In Figure 4 V'_{ab}, V'_{bc} and V'_{ca} are line to line voltages after V-LLC, and can be calculated from (5).

$$\begin{aligned} V'_{ab} &= V_{ab} + V_{pqa} \\ V'_{bc} &= V_{bc} + V_{pqb} \\ V'_{ca} &= -(V'_{ab} + V'_{bc}) \end{aligned} \tag{5}$$

where V_{pqa} and V_{pqb} are two inserted voltages in Figure 3. If V_{pqa} and V_{pqb} are assumed in phase with I_c and I_a respectively (as shown in Figure 4), then the exchanged active power between V-LLC and the network becomes zero.

$$\begin{aligned} P_{comp} &= V_{pqa} \times I_a \times \cos\theta - V_{pqb} \times I_c \times \cos\theta = \\ &= \left|V_{pq} \times I \times \cos\theta\right| \left[1\angle(\angle V_{pqa} + \angle I_a) - 1\angle(\angle V_{pqb} + \angle I_c)\right] \\ &= 0 \qquad (\angle V_{pqa} = \angle I_c \quad and \quad \angle V_{pqb} = \angle I_a) \end{aligned} \tag{6}$$

In the Equation (6) P_{comp} is the average active power of V-LLC and $\theta = \left|\angle V_{pqa} - \angle I_a\right| = \left|\angle V_{pqb} - \angle I_c\right|$.

It keeps the DC link capacitor voltage constant. The internal loss of V-LLC including inverters and injection transformers, decreases the DC link voltage. The output voltage is adjusted to compensate this loss [10]. With such an adjustment the required active power can be absorbed from network and the DC link voltage remains constant despite losses in elements of V-LLC.

The method used in Figure 4 is based on measurement of current phase angle and computation of voltage phase according to it. This method is, therefore, known as current frame control method [11]. The current frame is a conventional method in literature and experiment. If the current sensors are substituted with voltage sensors, which can be installed and operated easily, more reliably and with low expenses, the control will be in voltage frame [11]. The small phase difference between voltage and current in a transmission line is the base of voltage frame idea. In voltage frame control method, the phasor of injecting voltage is set to be perpendicular to voltage of PCC instead of its current.

Due to the discussed advantages of voltage frame method as well as less number of inverters and injecting transformers, an extra beneficial feature is obtained for V-LLC. These changes cause a remarkable improvement in operation and economy of introduced FACTS. Figure 5 shows the phasor diagram of V-LLC when it is controlled in voltage frame. Like Figures 1-4 the turn ratio of two V-connection transformers are assumed to be unit.

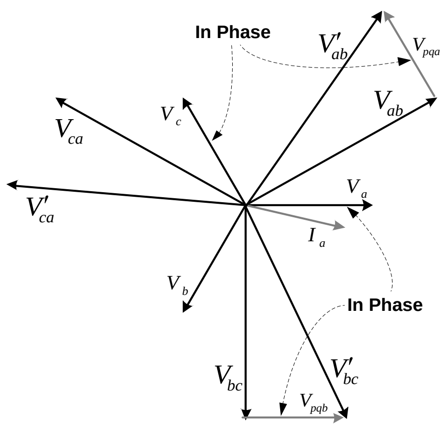

Figure 5. Phasor diagram of V-LLC with voltage frame control

B. TV-LLC

To achieve a voltage frame control in V-LLC, two inserted voltages V_{pqa} and V_{pqb} should be in phase with V_{cn} and V_{an} respectively. V_{an} and V_{cn} are the phase to ground voltages of two phases of the network line.

The structure of Transformer V-LLC (TV-LLC) comes from the above fact. Instead of two inverters to produce V_{pqa} and V_{pqb} in phase with V_{cn} and V_{an} , two transformers can be used. So the inverters will be omitted and the injecting transformers will adjust the magnitude of additive V_{pqa} and V_{pqb} voltages to achieve the desired active power flow. Figure 6 shows the represented FACTS with a very simple structure.

If the two mentioned injecting transformers have various taps, the predefined percent of variation in active power flow can be made in line. With two tap changer transformers these variations in power flow will have discontinuous values. But using power electronic transformers, matrix converters or AC-AC choppers instead of magnetic transformers, a continuous control can be achieved.

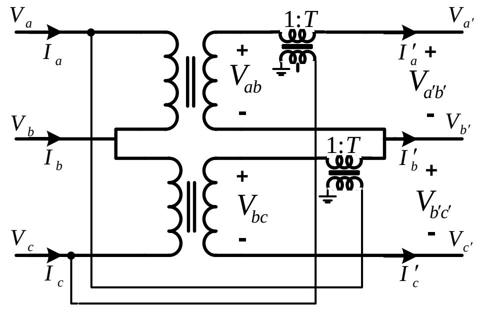

Figure 6. Structure of TV-LLC

A TV-LLC with two magnetic transformers (or their electronically identical) with turn ratio of T is named as TV-LLC with T factor. According to Figure 5 a T-factor TV-LLC can affect the sending (or receiving) end voltage and therefore the power flow. If the TV-LLC is installed in sending end the consequences can be discussed in the following three steps:

Step 1: Line to line voltages, after TV-LLC are shown as (Figure 6):

$$V_{a'b'} = V_{ab} + T\,V_{cn}$$
$$V_{b'c'} = V_{bc} + T\,V_{an} \tag{7}$$

That leads to a change in line to neutral voltages: (The proof is in Appendix 1)

$$V_{a'n} = V_{an}(1+\frac{T}{\sqrt{3}}\angle 90) \tag{8}$$

Step 2: The phase difference between sending and receiving ends varies from δ to $\delta+\Delta\delta$ in that: (The proof is in Appendix 2)

$$\Delta\delta = \tan^{-1}(\frac{T}{\sqrt{3}}) \tag{9}$$

Step 3: Because of this variation in phase difference the relative change in active power flow can be computed: (The proof is in Appendix 3)

$$\frac{\Delta P}{P} = \frac{P'-P}{P} = \frac{T}{\sqrt{3}}\cot\delta \tag{10}$$

In which P' is the new active power and P is the old one. T and δ are the factor of installed TV-LLC and phase difference between sending and receiving ends before installing TV-LLC respectively.

As it will be shown by an example, the index of TV-LLC is usually small, so the variation of voltage magnitude is negligible and a change in phase difference will affect the active power flow. Considering Equation (10) in a transmission line with e.g. $\delta = 5^\circ$, to reach a 10% change in active power flow ($\Delta P/P = 10\%$) the factor of TV-LLC must be $T = 0.015$.

The main concern may be in the effect of TV-LLC on current unbalance in three phases. After TV-LLC the line currents are named as I'_a , I'_b and I'_c. If the three line to line voltages $V_{a'b'}$, $V_{b'c'}$ and $V_{c'a'}$ are symmetric and the network on the right side of TV-LLC in Figure 6 is symmetric too, I'_a , I'_b and I'_c will be symmetric. To check this condition the following equations can be written: (The proof is in Appendix 4)

$$V_{c'a'} = -V_{bc} - T\,V_{an} - V_{ab} - T\,V_{cn} = V_{a'b'}(1\angle 120^\circ) \qquad (11)$$
$$V_{b'c'} = -(V_{a'b'} + V_{c'a'}) = V_{a'b'}(1\angle -120^\circ)$$

It shows the symmetry of voltages after TV-LLC is not affected. So the line currents after TV-LLC are symmetric. But assuming the turn ratio of two transformers in V-V (open delta) combination unit, the following equations can be written for the line currents before TV-LLC:

$$I_a = I'_a - T\,I'_c = I'_a(1 - T\angle 120) \qquad (12)$$
$$I_c = I'_c - T\,I'_a = I'_c(1 + T\angle -120)$$

So the amount of inequality caused by TV-LLC in line currents magnitude is presented as: (The proof is in Appendix 5)

$$\frac{|I_a| - |I_b|}{|I_b|} = \frac{T}{2} \qquad (13)$$

As an example and based on the above equation, in a line with $\delta = 5^\circ$ to reach a 5% change in active power flow ($\Delta P/P = 5\%$) the inequality of currents magnitude will be $(|I_a| - |I_b|)/|I_b| = 0.37\%$.

C. CV-LLC

If a power electronic AC-AC chopper [12, 13] is used instead of each injecting transformers in TV-LLC configuration, a new structure is obtained which is named as Chopper V-LLC (CV-LLC). Figure 7 shows an AC-AC chopper that can control the ratio between output voltage rms and input voltage rms like a transformer but by cheaper and lighter equipment.

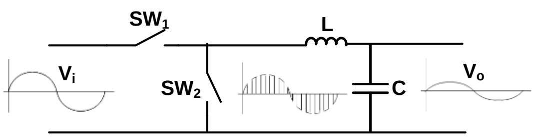

Figure 7. A simple AC chopper

The gain of the above chopper can be calculated in terms of duty cycle of SW_1.

$$\frac{V_o}{V_i} = \sqrt{D} \qquad (14)$$

where,
V_o is the rms of output voltage,
V_i is the rms of input voltage,
and D is the duty cycle of SW_1.

In practice if the ratio V_o/V_i is too small, the value of duty cycle is obtained about zero from equation (14). For example to reach the gain factor 0.02, the duty cycle of SW_1 must be 0.0004. To overcome this problem a combination of an AC chopper and a transformer can be used. Figure 8 shows the structure of Chopper V-LLC (CV-LLC).

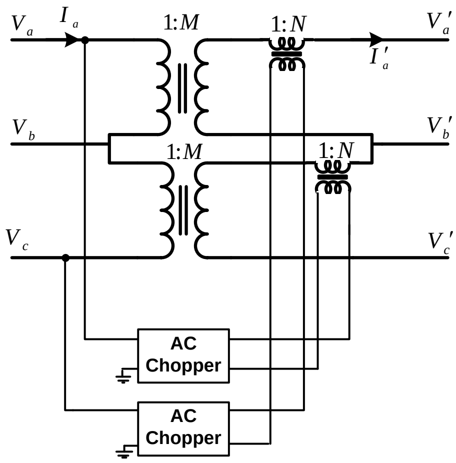

Figure 8. Structure of CV-LLC

All TV-LLC equations are true in the case of CV-LLC in Figure 8 by calculating T from:

$$T = \frac{\sqrt{D} \times N}{M} \qquad (15)$$

For example in the above situation to reach $T = 0.02$, if $M = 1$ then the duty cycle of chopper switches can be $D = 0.25$, and the turn ratio of injecting transformers should be $T = 0.04$.

If the chopper of Figure 7 is used in Figure 9 the current and voltage rating of inductors and capacitors and the turn ratio of injecting transformers must be studied. Equation (10) can be rewritten for CV-LLC as:

$$\frac{\Delta P}{P} = \frac{N\sqrt{D}\cot\delta}{\sqrt{3}M} \qquad (16)$$

The inductor's current and capacitor's voltage is considered as:

$$I_L = K_2 \times I_l \quad , \quad V_c = K_1 \times V_{l-l} \qquad (17)$$

where, I_L is line current and V_{l-l} is line-line voltage. So:

$$K_1 = \frac{\sqrt{D}}{M\sqrt{3}} \quad , \quad K_2 = N \qquad (18)$$

And then:

$$K_1 \times K_2 = \frac{\Delta P}{P}\tan\delta \qquad (19)$$

If 0.1 and 0.95 are assumed to be the minimum and maximum duty cycle limits of SW_1 and SW_2, then:

$$\mathrm{Min}(N) = \frac{M}{0.56}\frac{\Delta P}{P}\tan\delta \qquad (20)$$
$$\mathrm{Max}(N) = \frac{M}{0.18}\frac{\Delta P}{P}\tan\delta$$

For example for $\Delta P/P = 10\%$ in a line with $\delta = 5^\circ$ and $M = 230^{\mathrm{kv}}/20^{\mathrm{kv}}$:

$$\mathrm{Min}(N) = 0.18 \quad , \quad \mathrm{Max}(N) = 0.56 \qquad (21)$$

With a proof presented in Appendix 6, the minimum value of L is calculated from:

$$L_{\min} = \frac{0.31 V_{l-l}}{M \Delta i_L f_s} \qquad (22)$$

where,

V_{l-l} is line to line voltage of transmission line, Δi_L is the acceptable ripple of the current of the inductor and f_s is the switching frequency of the chopper.

According to the above parameters and Appendix 7 the minimum value of C comes from:

$$C_{\min} = \frac{0.16\, V_{l-l}}{M\, L\, f_s^2\, \Delta v_C} \tag{23}$$

where Δv_C is the acceptable ripple of the capacitor voltage.

Obviously using CV-LLC instead of TV-LLC changes *THD* of transmission line voltage from zero (TV-LLC) to a new value comes from (24) (The proof is in Appendix 8).

$$THD = \frac{0.11}{M\, LC\, f_s^2} \tag{24}$$

Table 1 summarizes the mentioned properties of proposed topologies, to compare with a conventional series FACTS, SSSC. Also a complete and precise comparison between line to line compensation and series compensation is presented in [8]. That shows the good features of LLC even in size and weight of injecting transformers [8].

Table 1. Characteristics of proposed circuits and SSSC

	SSSC	V-LLC	TV-LLC	CV-LLC
Simple design	✗	✗	✓	✗
Operation without DC link	✗	✗	✓	✓
Fast response	✓	✓	✗	✓
Active power flow control	✓	✓	✓	✓
Operation without power electronic switches	✗	✗	✓	✗
Control each phase separately	✗	✓	✓	✓
Changing the voltage level of network	✗	✓	✓	✓

IV. CV-LLC IN DISTRIBUTION SYSTEM

CV-LLC or other LLC-based configurations can be used in distribution system. This section presents an application of CV-LLC to decrease the unbalance of current in presence of unbalanced load. This device is chosen because of its abilities mentioned in Table 1.

Current unbalance may lead to increase in power loss in distribution lines and transformers. To improve current balance, each chopper in Figure 8 can inject different voltages according to situations. Injecting two additive voltages in Figure 8 changes the line to line voltages (V_{ab} and V_{bc}). So if these injecting voltages are different, then three voltages V_{an} , V_{bn} and V_{cn} can be controlled to compensate the effect of unbalanced load on currents. For example if I_a is less than I_b and I_c , then injecting a voltage to be added with V_{ab} increases V_a and decreases the difference between I_a , I_b and I_c .

To explain the strategy of control and the effect of voltage injection on each line currents, consider a three phase system in which phase voltages (V_a , V_b and V_c) can be written in terms of line to line voltages (V_{ab} and V_{bc}) as follows:

$$\begin{aligned} 3V_a &= 2V_{ab} + V_{bc} \\ 3V_b &= -V_{ab} + V_{bc} \\ 3V_c &= -V_{ab} - 2V_{bc} \end{aligned} \tag{25}$$

Two injected voltages may affect V_{ab} and V_{bc} , and change them to $V_{ab} + k_1 V_{cn}$ and $V_{bc} + k_2 V_{an}$ respectively. To simplify the control strategy, it is considered that in every situation one of the k_1 or k_2 is zero and the other one can increase or decrease the line to line voltage ($k > 0$ or $k < 0$ respectively). For example positive k_1 leads to an increase in V_{ab} , and V_{bc} remains constant, then V_a increases, V_b and V_c decrease. That is schematically shown as: $\{V_a \uparrow, V_b \downarrow, V_c \downarrow\}$.

Table 2 shows the other modes of k_1 and k_2 , and the effect of them on V_a , V_b and V_c . Six different unbalance types, shown in Table 3, may occur in an unbalanced line. According to each case a scenario from Table 2 is chosen to improve the balance. In some cases two scenarios can be used. So that scenario which needs fewer changes in configuration is chosen. Figure 9 shows the topology of CV-LLC to use in systems with unbalanced loads and Figure 10 shows the control block diagram of that. In Figure 9, SW_{1-8} are used to make k_1 and k_2 positive, negative or zero according to Table 4.

Table 2. Modes of operation

Scenario	Control Parameters	V_a	V_b	V_c
1	$k_1 > 0$, $k_2 = 0$	↑	↓	↓
2	$k_1 = 0$, $k_2 > 0$	↑	↑	↓
3	$k_1 < 0$, $k_2 = 0$	↓	↑	↑
4	$k_1 = 0$, $k_2 < 0$	↓	↓	↑

Table 3. Unbalance types and scenarios

Unbalance type	Scenario
$I_a < I_b < I_c$	1 or 2
$I_a < I_c < I_b$	1
$I_b < I_a < I_c$	2
$I_b < I_c < I_a$	3
$I_c < I_a < I_b$	4
$I_c < I_b < I_a$	3 or 4

Table 4. Status of each switch in Figure 9

k_i	Switches
$k_1 > 0$	SW_5,SW_6:ON SW_7,SW_8:OFF
$k_1 < 0$	SW_5,SW_6:OFF SW_7,SW_8:ON
$k_1 = 0$	SW_2,SW_5,SW_6:ON SW_1:OFF
$k_2 > 0$	SW_9,SW_{10}:ON SW_{11},SW_{12}:OFF
$k_2 < 0$	SW_9,SW_{10}:OFF SW_{11},SW_{12}:ON
$k_2 = 0$	SW_4,SW_9,SW_{10}:ON SW_3:OFF

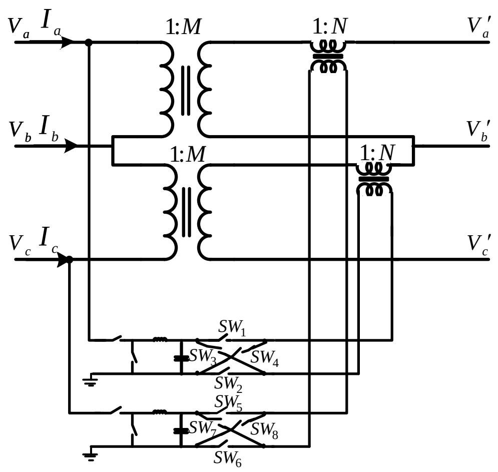

Figure 9. CV-LLC for unbalance reduction

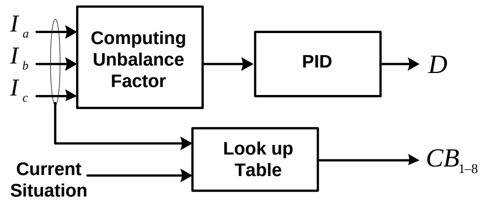

Figure 10. Control block diagram of CV-LLC for unbalance reduction in distribution system

V. SIMULATION RESULTS

In this section some simulations are carried out to show the abilities of the presented structures.

A. Transmission

MATLAB/SIMULINK software is used to simulate a two machine network shown in Figure 11. The location of the installed device (V-LLC, TV-LLC or CV-LLC) is shown in the sending side of the transmission line. The turn ratio of the main transformer in sending end is $20\,\text{kV}/230\,\text{kV}$ instead of unit turn ratio of transformer in previous sections. The type and connection of the second transformer in receiving end of line in Figure 11, do not affect the calculations, presented equations and methods. But, in Figure 11 common connection (Y/Δ) is used. The parameters of simulated network are shown in Table 5.

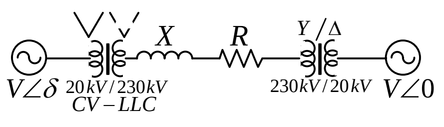

Figure 11. One line diagram of simulated network

Table 5. Parameters of simulation

Parameter	Value
Sending end voltage	20 kV rms L-L
Receiving end voltage	20 kV rms L-L
Phase difference between sending and receiving ends	5°
Transmission line voltage	230 kV rms L-L
Transmission line resistance	50 Ω
Transmission line impedance	600 Ω
Frequency	50 Hz

Simulations are performed to study the steady state operation of each topology. Power flow control with V-LLC, TV-LLC and CV-LLC is studied in this section. The desired active power is $2\,\text{MW/phase}$ for all of these cases. Figures 12-14 show the active power flow per phase, line voltages, line currents and injected voltage (V_{pqa}), when V-LLC, TV-LLC and CV-LLC are used to control the active power flow respectively.

The turn ratio of two injecting transformers in these simulations are $20\,\text{kV}/3\,\text{kV}$ for V-LLC and TV-LLC and $20\,kV/12\,kV$ for CV-LLC.

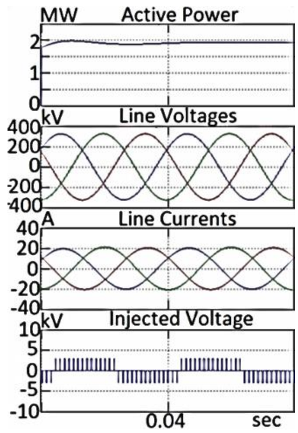

Figure 12. Simulation results for V-LLC

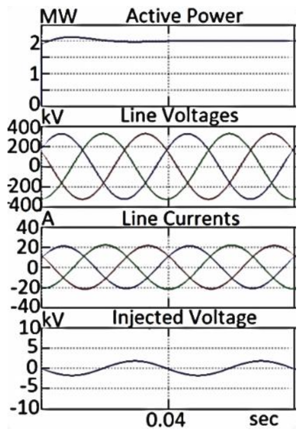

Figure 13. Simulation results for TV-LLC

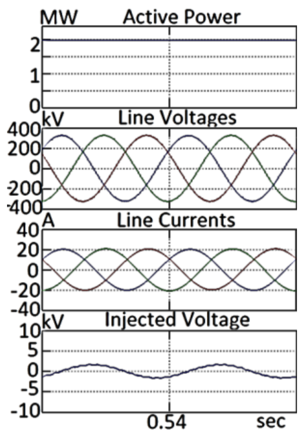

Figure 14. Simulation results for CV-LLC

Figures 12-14 show the ability of the presented structures to control the line active power flow. In these figures, the line voltages and line currents remain balance despite of using two injected voltage instead of three. In Figures 12-14, also the magnitude and rms of injected voltages are noticeable. These voltages are small in comparison with line voltages. That is a good feature, because it leads to small variation in line voltage magnitude and adjusts the power flow by controlling the line currents.

B. Distribution

To verify the efficiency of CV-LLC in presence of unbalance load some simulations are carried out. During these simulations three different loads are connected to a 380 V distribution network and the line currents are observed with and without CV-LLC. Simulated network is shown in Figure 15.

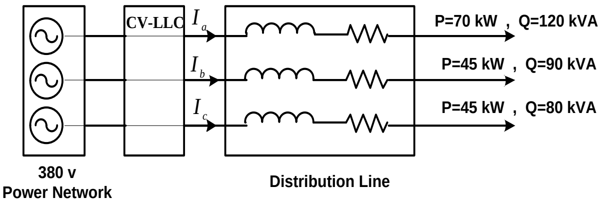

Figure 15. Simulated distribution unbalance network

Figure 16 shows the line currents with and without CV-LLC. The currents in presence of CV-LLC (16-below) are obviously more balanced than the system without CV-LLC (16-above). The difference between maximum and minimum current becomes 4 times smaller by using CV-LLC.

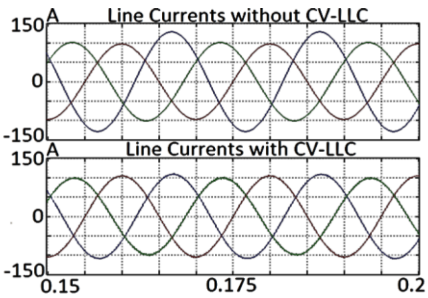

Figure 16. Line currents with unbalance load

Decreasing the unbalances of currents, leads to loss reduction in the distribution lines. In simulated network lines, power loss decreases 9.6% by using CV-LLC.

VI. CONCLUSIONS

In this paper, three new structures for FACTS devices are proposed. These structures are based on a recently introduced compensation method named as Line to Line Compensation. In this method an additive voltage is inserted between two phases, and then several parameters of the system can be controlled. The main idea of this paper comes from eliminating a converter from three conventional converters of a three phase system. So V-LLC, TV-LLC and CV-LLC structures are obtained. TV-LLC and CV-LLC are more noticeable, because of the absence of the common inverters in their topology. So they can be embedded in power transformers to lead to a new generation of transformers. Therefore they can expose more flexibility in addition to conventional uses in voltage level adjustment.

The main features in transmission and distribution systems are the ability of power flow control and decreasing the effect of the unbalance loads respectively. The proposed structures have the both mentioned capabilities. The principles of operation and related equations are presented and the effectiveness of proposed structures in transmission and distribution networks is verified through simulations in MATLAB software.

APPENDICES

Appendix 1

$$V_{a'n} = (\frac{1}{\sqrt{3}}\angle -30)V_{a'b'} =$$
$$=(\frac{1}{\sqrt{3}}\angle -30)\left[V_{ab} + T\times(1\angle 120)V_{an}\right]=$$
$$=(\frac{1}{\sqrt{3}}\angle -30)\left[(\sqrt{3}\angle 30)V_{an} + T\times(1\angle 120)V_{an}\right]=$$
$$=V_{an}(1+\frac{T}{\sqrt{3}}\angle 90)$$

Appendix 2

$$1+\frac{T}{\sqrt{3}}\angle 90 = \sqrt{1+\frac{T^2}{3}}\angle \tan^{-1}(\frac{T}{\sqrt{3}})$$
$$\Rightarrow \Delta\delta = \angle V_{a'n} - \angle V_{an} = \tan^{-1}(\frac{T}{\sqrt{3}})$$

Appendix 3

$$\frac{\hat{P}-P}{P} = \frac{\frac{V^2}{X}\left[\sin(\delta+\Delta\delta)-\sin\delta\right]}{\frac{V^2}{X}\sin\delta} =$$
$$=\frac{\sin\delta\cos\Delta\delta+\cos\delta\sin\Delta\delta-\sin\delta}{\sin\delta}$$

$\Delta\delta$ is small $\Rightarrow \cos\Delta\delta \cong 1$, $\sin\Delta\delta \cong \tan\Delta\delta = \frac{T}{\sqrt{3}}$

$$\Rightarrow \frac{\hat{P}-P}{P} = \frac{\sin\delta+\cos\delta\times(T/\sqrt{3})-\sin\delta}{\sin\delta} = \frac{T}{\sqrt{3}}\cot\delta$$

Appendix 4

$$V_{ab} = V_{bc}(1\angle 120^\circ) = V_{ca}(1\angle -120^\circ)$$
$$V_{c'a'} = -V_{bc} - T\,V_{an} - V_{ab} - T\,V_{cn} =$$
$$= -T(V_{an}+V_{cn}) - (V_{bc}+V_{ab})$$
$$V_{an}+V_{cn} = -V_{bn}\ ,\ V_{bc}+V_{ab} = -V_{ca} \Rightarrow V_{c'a'} = V_{ca} + T\,V_{bn}$$
$$V_{a'b'} = V_{ab} + T\,V_{cn} \Rightarrow V_{c'a'} = (1\angle 120^\circ)(V_{ab}+T\,V_{cn}) =$$
$$= (1\angle 120^\circ)V_{a'b'}$$

Appendix 5

$$\left.\begin{aligned} |I_a| &= |I'_a(1-T\angle 120)| = |I'_a|\left|1+0.5T - j\frac{\sqrt{3}}{2}T\right| \\ &= |I'_a|\sqrt{(1+0.5T)^2+(\frac{\sqrt{3}}{2}T)^2} = |I'_a|\sqrt{1+T+T^2} \\ |I_c| &= |I'_a(1\angle 120 - T)| = |I'_a|\left|-0.5+j\frac{\sqrt{3}}{2}-T\right| \\ &= |I'_a|\sqrt{(0.5+T)^2+(\frac{\sqrt{3}}{2})^2} = |I'_a|\sqrt{1+T+T^2} \end{aligned}\right\} \Rightarrow |I_a| = |I_c|$$

$$|I_b| = |I'_b| = |I'_a| \Rightarrow \frac{|I_a|-|I_b|}{|I_b|} = \frac{\sqrt{1+T+T^2}-1}{1} \cong \frac{T}{2}$$

Appendix 6

$$v_{in} = V_m \sin\omega t \times d(t)\,, v_{out} = \sqrt{D}V_m \sin\omega t \Rightarrow$$

$$v_L = v_{in} - v_{out} = V_m \sin\omega t\left[d(t) - \sqrt{D}\right]$$

$$v_L = L\frac{\Delta i_L}{\Delta t} \Rightarrow L = \frac{v_L \Delta t}{\Delta i_L}$$

$$t_{on} \Rightarrow L = \frac{V_m(1-\sqrt{D})DT_s}{\Delta i_L}\,, t_{off} \Rightarrow L = \frac{V_m\sqrt{D}(1-D)T_s}{\Delta i_L}$$

$$\text{Max}(D - D\sqrt{D}) = 0.148\,, \text{Max}(\sqrt{D} - D\sqrt{D}) = 0.385$$

$$V_m = \frac{\sqrt{2}V_{l-l}}{\sqrt{3}M} \Rightarrow L_{\min} = \frac{0.31\,V_{l-l}}{M\,\Delta i_L\, f_s}$$

Appendix 7

$$i_C = C\frac{dv_c}{dt} \Rightarrow C = \frac{\Delta i_L \frac{T_s}{2}}{\Delta v_C}$$

$$\Delta i_L = \frac{0.31\,V_{l-l}}{M\,L\,f_s} \Rightarrow C = \frac{0.16\,V_{l-l}}{M\,L\,f_s^2\,\Delta v_C}$$

Appendix 8

$$THD = \frac{\Delta v_C}{\sqrt{2}\,V_{l-l}} \Rightarrow THD = \frac{0.11}{M\,LC\,f_s^2}$$

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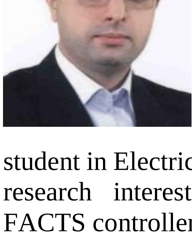
BIOGRAPHIES



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