International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

September 2015 | Issue 24 | Volume 7 | Number 3 | Pages 63-67

FREE OSCILLATIONS OF FLOWING LIQUID-FILLED ANISOTROPIC CYLINDRICAL SHELL STRENGTHENED WITH CROSSED SYSTEMS OF RIBS

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Abstract- In this paper, we represent the results of finding the free oscillations frequency of a flowing liquid-filled cylindrical anisotropic shell strengthened with the crossed systems of ribs under Navier's boundary conditions. It is assumed that the axis of elastic symmetry in the shell with peripheral direction forms an angle. Using the Ostrogradsky-Hamilton variational principle, we construct a frequency equation and realize it numerically. The results of calculations of eigen frequencies of oscillations are represented in the form of dependences on the winding angle for the shell and on the flowing liquid velocity at different values of wave formation parameters and different relations between the parameters characterizing geometrical sizes of the shell.

Keywords: Shell, Oscillation, Variational Principle, Modulus of Elasticity, Deformation, Liquid.

I. INTRODUCTION

In engineering practice the use of polymeric materials, in particular glass reinforced plastics makes necessary to take into account anisotropy of elastic properties when studying low-frequency oscillations of shells. For rigidity the thin walled part of the shell is strengthened with ribs that essentially increases its strength at negligible increase of the mass of construction even if the ribs have small height.

Dynamical analysis is one of the important problems on designing stage of thin walled shelled constructions widely used in aviation, rocket-cosmic engineering and various fields of industry. The necessary element for studying dynamics of shells is definition of eigen frequencies and forms of small oscillations, where the frequencies from lower sector present great interest for applications. The results of finding of eigen frequencies of axially symmetric oscillations of orthotropic, liquid-filled, unstrengthen cylindrical shells in infinite elastic medium were represented in the papers [1, 2].

The monograph [3] was devoted to investigation of stability and oscillations of strengthened isotropic shells without medium. Eigen oscillations of a flowing liquid-filled isotropic cylindrical shell strengthened with longitudinal and crossed system of ribs were considered in the papers [4, 5].

In this paper, we present the results of finding free oscillations frequencies of a cylindrical, structural, flowing liquid-filled anisotropic shell made of a glass reinforced plastic and strengthened with a cross system of ribs at Navier's boundary conditions. It is assumed that all the ribs are strengthened on the external surface of the casing, are arranged at equal distances and have the identical geometrical and mechanical characteristic.

The results of calculations of eigen frequencies of oscillations are represented in the form of dependences on the winding angle of the glass fibre for a shell made of tissue glass-reinforced plastic and on the velocity of flowing liquid at various values of wave formation parameters and different relations between the parameters characterizing geometrical sizes of shell. The solution of problem is based on Ostrogradsky-Hamilton's principle.

II. PROBLEM STATEMENT

The frequency equations of free oscillations of a flowing liquid-filled anisotropic shell strengthened with cross systems of ribs were obtained on the principle of Ostrogradsky-Hamilton's action stationarity principle

$$\delta W = 0 \qquad (1)$$

where $W = \int_{t'}^{t''} L dt$ is Hamilton's action, $\tilde{L}$ is the Lagrange function, t' and t'' are the given arbitrary moments.

For applying the Ostrogradsky-Hamilton Equation (1), we preliminarily write the total energy of the system consisting of a shell, longitudinal and lateral bars and liquid. It is accepted that the longitudinal ribs are arranged along the generator, the lateral ones along the perimeter of the shell's cross section and are rigidly connected with it. The system of coordinates was chosen so that the coordinate lines coincide with the principles curves of the median surface of the shell; x along the generator, φ along the arch of the shell's cross section, z along the normal of the shell's median surface.

The total energy of the elastic deformation of flowing liquid-filled anisotropic cylindrical shell strengthened with cross systems of ribs is of the form [3, 10, 11]:

$$\begin{aligned}
J &= \frac{1}{2}R^2\int_0^L\int_0^{2\pi}\{N_{11}\varepsilon_{11}+N_{22}\varepsilon_{22}+N_{12}\varepsilon_{12}-M_{11}\chi_{11}-\\
&-M_{22}\chi_{22}-M_{12}\chi_{12}\}dxd\theta+\\
&+\frac{1}{2}\sum_{i=1}^{k_1}\int_0^L\left[\tilde{E}_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2+\tilde{E}_iJ_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+\right.\\
&\left.+\tilde{E}_iJ_{zi}\left(\frac{\partial^2 v_i}{\partial x^2}\right)^2+\tilde{G}_iJ_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right]dx+\\
&+\frac{R}{2}\int_0^{2\pi}\left[\breve{E}_jF_j\left(\frac{\partial \vartheta_j}{R\partial\theta}-\frac{w_j}{R}\right)^2+\breve{E}_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right. \qquad (2)\\
&\left.+\breve{E}_jJ_{zj}\left(\frac{\partial^2 u_j}{R^2\partial\theta^2}-\frac{\varphi_{kpj}}{R}\right)^2+\breve{C}_jJ_{kpj}\left(\frac{\partial\varphi_{kpi}}{R\partial\theta}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]d\theta+\\
&+\rho_0Rh\int_0^L\int_0^{2\pi}\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dxd\theta+\breve{\rho}_jF_jR\times\\
&\times\int_0^{2\pi}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial \vartheta_j}{\partial t}\right)^2+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{\kappa pi}}{F_j}\left(\frac{\partial \varphi_{\kappa pj}}{\partial t}\right)^2\right]d\theta+\\
&+\sum_{i=1}^{k_1}\tilde{\rho}_iF_i\int_0^L\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2+\right.\\
&\left.+\frac{J_{\kappa pi}}{F_i}\left(\frac{\partial \varphi_{\kappa pi}}{\partial t}\right)^2\right]dx-R\int_0^L\int_0^{2\pi}q_z w\,dxd\theta
\end{aligned}$$

The expressions for inner forces and moments are represented in the following way [10]:

$$N_{ij}=\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)dz;\ M_{ij}=\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)zdz \qquad (3)$$

where,

$$w_{11}=B_{11}\chi_{11}+B_{12}\chi_{22}+B_{16}\chi_{12},$$
$$w_{22}=B_{12}\chi_{11}+B_{22}\chi_{22}+B_{26}\chi_{12},$$
$$w_{21}=w_{12}=B_{16}\chi_{11}+B_{22}\chi_{22}+B_{66}\chi_{12}.$$

The stresses σ_{ij} and strains ε_{ij} in the median surface in Equation (3) are determined as follows [10]:

$$\begin{aligned}
\sigma_{11}&=B_{11}\varepsilon_{11}+B_{12}\varepsilon_{22}+B_{16}\varepsilon_{12}\\
\sigma_{22}&=B_{12}\varepsilon_{11}+B_{22}\varepsilon_{22}+B_{26}\varepsilon_{12} \qquad (4)\\
\sigma_{12}&=B_{16}\varepsilon_{11}+B_{26}\varepsilon_{22}+B_{66}\varepsilon_{12}
\end{aligned}$$

$$\begin{aligned}
&\varepsilon_{11}=\frac{\partial u}{\partial x};\ \varepsilon_{22}=\frac{\partial v}{\partial y}+w;\ \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\
&\chi_{11}=\frac{\partial^2 w}{\partial x^2};\ \chi_{22}=\frac{\partial^2 w}{\partial y^2};\ \chi_{12}=-2\frac{\partial^2 w}{\partial x\partial y}
\end{aligned} \qquad (5)$$

The elasticity constants depend on the winding angle φ of the fiberglass and are determined as [10]:

$$B_{11}=b_{11}\cos^4\varphi+b_{22}\sin^4\varphi+\left(b_{66}+0.5b_{12}\right)\sin^2 2\varphi$$
$$B_{22}=b_{11}\sin^4\varphi+b_{22}\cos^4\varphi+\left(b_{66}+0.5b_{12}\right)\sin^2 2\varphi$$
$$B_{12}=\left(b_{11}+b_{22}-4b_{66}\right)\sin^2\varphi\cos^2\varphi+b_{12}\left(\sin^4\varphi+\cos^4\varphi\right)$$
$$B_{66}=-\left(b_{11}+b_{12}-2b_{12}\right)\sin^2\varphi\cos^2\varphi+b_{66}\cos^2 2\varphi$$
$$B_{26}=1/2\left(b_{22}\cos^2\varphi-b_{11}\sin^2\varphi\right)\sin^2\varphi-$$
$$-1/6\left(b_{12}+2b_{66}\right)\sin 4\varphi$$
$$B_{16}=1/2\left(b_{22}\sin^2\varphi-b_{11}\cos^2\varphi\right)\sin^2\varphi-$$
$$-1/6\left(b_{12}+2b_{66}\right)\sin 4\varphi$$

where, φ is the angle formed by the direction of glass fiber with peripheral direction, R is the radius of the shell's median surface, h is the shell thickness, u,ϑ,w are the components of displacements of the points of the shell's median surface, F_i, J_{zi}, J_{yi}, J_{kpi} are the square and inertia moments of the cross section of the ith longitudinal bar with respect to the axis Oz and the axis parallel to the axis Oy and crossing through the gravity center of the cross section, and also its inertia moment at torsion; F_i is the area of the cross section of the longitudinal rib; $\tilde{E}_i,\tilde{G}_i$ are modulus of elasticity and shift of the material of the ith longitudinal bar, ρ_0, $\tilde{\rho}_i$, $\breve{\rho}_j$ are densities of materials from which the shell was made, i is the longitudinal bar, j is the lateral bar, respectively, L is the shell length, q_z is the pressure of liquid on the shell, $F_j, J_{zj}, J_{yj}, J_{kpj}$ area and inertia moments of the cross section of the jth lateral bar with respect to the axis Oz, and the axis parallel to the axis Oy and passing through the gravity center of the cross section, and also its inertia moment at torsion; $\tilde{E}_i,\tilde{G}_i$ are the module of elasticity and shift of the material of the φ jth lateral bar, respectively, t is the time coordinate, $\omega_0=\sqrt{\frac{b_{11}}{\left(1-\nu^2\right)\rho_0R^2}}$, $b_{11},b_{22},b_{12},b_{66}$ are the basic module of elasticity of the orthotropic material, that are expressed by the module of elasticity E_1,E_2 and the Poisson ratio ν_1,ν_2 in coordinate directions by relations $b_{11}=\frac{E_1}{1-\nu_1\nu_2}$, $b_{22}\frac{E_2}{1-\nu_1\nu_2}$ and $b_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2}=\frac{\nu_1E_2}{1-\nu_1\nu_2}$.

Supposing that the main velocity of the flow equals U and deviations from this velocity are small, we use the wave equation for the potential of perturbed velocities φ according to [10]:

$$\Delta\varphi-\frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2}+2U\frac{\partial^2\varphi}{R\partial\xi\partial t}+U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right)=0 \qquad (6)$$

On the contact surface a shell-liquid we observe the continuity of radial velocities and pressures. The condition of impermeability or fluency at the shell wall is of the form [10, 11]:

$$\vartheta_r\big|_{r=R} = \frac{\partial \varphi}{\partial r}\bigg|_{r=R} = -\left(\omega_0 \frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial \xi}\right) \tag{7}$$

Equality of radial pressures of liquid on the shell:

$$q_z = -p\big|_{r=R} \tag{8}$$

Complementing by contact conditions (7) and (8) the expression for the total energy of the shell (1), equation of motion of liquid (6), we arrive at the problem of eigen oscillations of anisotropic flowing liquid-filled cylindrical shell strengthened with crossed systems of ribs.

III. PROBLEM SOLUTION

We will look for displacements of the shell in the form of:

$$\begin{aligned} u &= u_0 \sin\chi\xi \cos n\theta \sin\omega_1 t_1 \\ \vartheta &= \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega_1 t_1 \\ w &= w_0 \cos\chi\xi \cos n\theta \sin\omega_1 t_1 \end{aligned} \tag{9}$$

where u_0, ϑ_0, w_0 are unknown constants; χ, n are wave numbers in longitudinal and peripheral directions, respectively, $\xi = x/R$, $\chi = kR$, $t_1 = \omega_0 t$, $\omega_1 = \omega/\omega_0$ ω is the sought-for frequency.

We look for the potential of perturbed velocities φ in the form:

$$\varphi(\xi, r, \theta, t) = f(r)\cos n\theta \sin\chi\xi \sin\omega_1 t_1 \tag{10}$$

Using (10), from conditions (7) and (6) we have:

$$\begin{aligned} \varphi &= -\Phi_{\alpha n}\left(\omega_0 \frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right) \\ p &= \Phi_{\alpha n}\rho_m\left(\omega_0^2 \frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0 \frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \end{aligned} \tag{11}$$

where

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta r), M_1 < 1 \\ J_n(\beta_1 r)/J_n'(\beta_1 r), M_1 > 1 \\ \dfrac{R^n}{nR^{n-1}}, \ M_1 = 1 \end{cases} \tag{12}$$

where $M_1 = \dfrac{U + \omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, I_n is the n order modified Bessel function of first kind, J_n is n order Bessel function of first kind.

In the sequel, in (8) and place of q_z we should take the value $q_z = -p$, where p is the pressure according to (11). Allowing for (9) we can represent the pressure p as follows:

$$p = \frac{\rho_m \Phi_{\alpha n}}{\rho_0 \omega_0^2 h}\left(\omega_0^2\omega_1^2 + 2\omega_0\omega_1\chi U + \chi^2 U^2\right)w \tag{13}$$

After substituting (13), (9) in (5), the problem is reduced to the homogeneous systems of linear algebraic equations of third order

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0 \qquad (i = 1,2,3) \tag{14}$$

The elements $a_{i1}, a_{i2}, a_{i3}\ (i = 1,2,3)$ have a bulky form and we don't cite them. The nontrivial solution of the system of linear algebraic equations in (14) of third order is possible only in the case when ω_1 is the root of its determinant. The determination of ω_1 is reduced to transcendental equation as ω_1 enters into the argument of the Bessel function J_n:

$$\begin{vmatrix} 2\left(\breve{\varphi}_{11} - \psi_{11}\omega_1^2\right) & \breve{\varphi}_{44} & \breve{\varphi}_{55} \\ \breve{\varphi}_{44} & 2\left(\breve{\varphi}_{22} - \psi_{22}\omega_1^2\right) & \breve{\varphi}_{66} \\ \breve{\varphi}_{55} & \breve{\varphi}_{66} & 2\left(\breve{\varphi}_{33} - \psi_{33}\omega_1^2 + q_z^{(0)}\psi_2\right) \end{vmatrix} = 0 \tag{15}$$

Note that for $U = 0\ (\varphi_2 = 0)$ in Equation (15) goes into the frequency equation of free oscillations of an anisotropic cylindrical shell filled with liquid in rest and strengthened with lateral systems of ribs. Consider some results of calculations executed proceeding from the above mentioned dependences by means of ICM.

For geometrical and physical parameters characterizing the shell's materials we adopted:

$\rho_0/\rho_m = 0.105\ \rho_0 = \breve{\rho}_j = \tilde{\rho}_i = 1850$ KJ/m^3;

$L = 10000$ mm, $F_i = 3.4$ mm^2, $h_j = 1.39$ mm;

$R = 160$ mm; $h = 0.45$ mm; $F_j = 5.75$ mm^2;

$I_{xj} = 19.9$ mm^4; $I_{kp.j} = 0.48$ mm^4; $J_{yi} = 5.1$ mm^4;

$h_i = 1.39$ mm; $\tilde{E}_i = \breve{E}_j = 6.67\times10^9$ H/m^2

In Figure 1 the dependence of the frequency parameter ω_1 on relative velocity of the flow $U^* = U/c$, $c = \omega_0 R$ at various values of χ and n is given. It is seen that increase in velocity reduces to decrease in frequency of the system's oscillations. It is very important to notice the value of U^* at which the frequency of oscillations approaches to zero. Here the loss of shell's stability should happen.

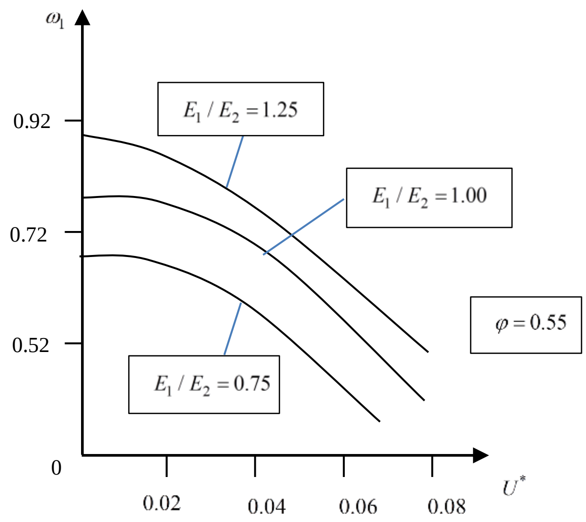

Figure 1. Dependence of frequency parameter on velocity of flow for a moving liquid-filled cylindrical shell strengthened with crossed system of ribs

In Figure 2 the dependence of frequency on the winding angle for various ratios of h/R was given. It follows from the figure that influence of change of the ratio of h/R on the value of eigen frequency is of very complicated character. The character of dependence is complicated when decreasing the ratio of h/R.

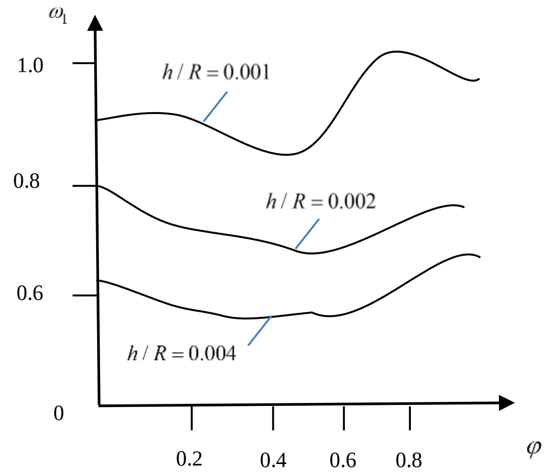

Figure 2. Dependence of the parameter of frequencies of free oscillations on the winding angle φ

At last the dependence of the frequency parameter of oscillations of the system under investigation on the number of longitudinal ribs at various values of lateral ribs is depicted in Figure 3, where $k_1=16$ corresponds to solid lines, $k_1=12$ to dotted lines. It is seen from the figure that with increasing the number of longitudinal and lateral ribs, at first the oscillations frequency of the construction under consideration increases, and then begins to decrease. This is explained by the fact that with increasing the number of ribs their mass increases and this effects on increase of inertial actions on the oscillations process of the system.

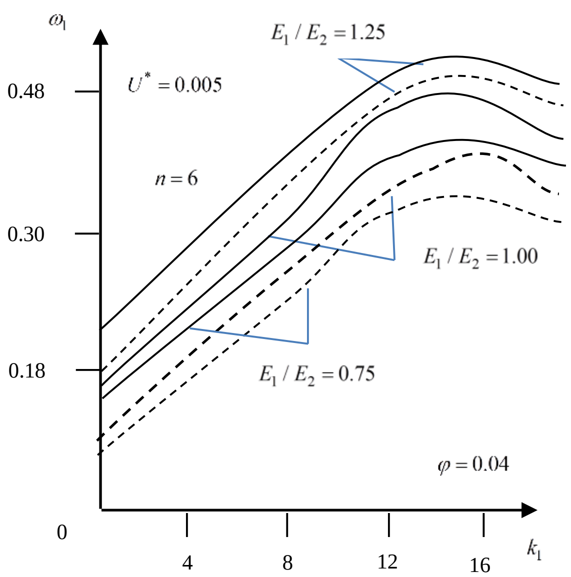

Figure 3. Dependence of the oscillation frequencies parameter on the number of longitudinal ribs

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