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NONLINEAR PARAMETRIC VIBRATIONS OF A LONGITUDINALLY STRENGTHENED ANISOTROPIC CYLINDRICAL SHELL WITH VISCOELASTIC FILLER

F.S. Latifov [1] R.A. Iskanderov [1] M.A. Mehdiyev [2]

1. Azerbaijan University of Architecture and Construction, Baku, Azerbaijan
flatifov@mail.ru, r.iskanderov@gmail.com
2. Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Baku, Azerbaijan
mahirmehdiyev@mail.ru

Abstract- In this paper by means of the variational principle in the geometric nonlinear statement a problem on longitudinally strengthened, orthotropic cylindrical shell contacting with external visco-elastic medium and located under the internal pressure, is solved. Amplitude-frequency dependences of parametric vibrations of a strengthened, viscoelastic medium-filled cylindrical shell were constructed.

Keywords: Variational Principle, Parametric Oscillations, Orthotropic Cylindrical Shell, Visco-Elastic Medium, Strengthened Shell.

I. INTRODUCTION

In recent times, investigation of stress-strain state of ridge anisotropic shells under the action of dynamical loads draws great attention of researchers. The monograph [1] was devoted to studies of deformation of cylindrical shells under the action of various dynamical loads.

A few of papers [2-5] have been devoted to nonlinear vibrations of ridge cylindrical shells. In the paper [2], the successive approximations method, in [4, 5] the finite elements method were used. In geometrically nonlinear statement, by using the variational principle, nonlinear parametric vibrations of an external viscoelastic medium-contacting, strengthened cylindrical shell situated under internal pressure were studied in [6-12]. Therewith the shells were strengthened with longitudinal, lateral and cross system of ribs. Investigations were conducted both without regard to lateral shift of shells [6-8] and with considering them [9-12].

Under parametric vibrations we understand vibrations that occur under the action of a force changed as a time parameters. Such loads are called parametric [14].

II. PROBLEM STATEMENT

We get differential equations of motion and natural boundary conditions for a longitudinally strengthened orthotropic, medium-contacting cylindrical shell on the basis of Ostrogradskii-Hamilton's variational principle.

For using Ostrogradskii-Hamilton's principle we a priori write total energy of the system. For orthotropic cylindrical shell, the potential energy is [2]:

$$\begin{aligned}
V_0 = \frac{hR}{2}\iint\Bigg\{ & B_{11}\left(\frac{\partial u}{\partial x}\right)^2 - 2\left(B_{11}+B_{12}\right)\frac{w}{R}\frac{\partial u}{\partial x} + B_{11}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2 - \\
& -\left(B_{11}+B_{12}\right)\frac{w}{R}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{B_{11}}{4}\left(\frac{\partial w}{\partial x}\right)^4 + \frac{w^2}{R^2}\left(B_{11}+2B_{12}+B_{22}\right) + \\
& +\frac{B_{22}}{R^2}\left(\frac{\partial \vartheta}{\partial \theta}\right)^2 - \left(B_{12}+B_{22}\right)\frac{w}{R^3}\left(\frac{\partial w}{\partial \theta}\right)^2 - 2\left(B_{12}+B_{22}\right)\frac{w}{R^2}\frac{\partial \vartheta}{\partial \theta} + \\
& +B_{22}\frac{1}{R^3}\frac{\partial \vartheta}{\partial \theta}\left(\frac{\partial w}{\partial \theta}\right)^2 + B_{22}\frac{1}{4R^4}\left(\frac{\partial w}{\partial \theta}\right)^4 + 2B_{12}\frac{1}{R^2}\frac{\partial u}{\partial x}\frac{\partial \vartheta}{\partial \theta} + \\
& +B_{12}\frac{1}{R^2}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial \theta}\right)^2 + B_{12}\frac{1}{R}\frac{\partial \vartheta}{\partial \theta}\left(\frac{\partial w}{\partial x}\right)^2 + \\
& +\frac{1}{2R^2}\left(B_{12}+2B_{66}\right)\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial \theta}\right)^2 + B_{66}\frac{1}{R^2}\left(\frac{\partial u}{\partial \theta}\right)^2 + \\
& +B_{66}\left(\frac{\partial \vartheta}{\partial x}\right)^2 + B_{66}\frac{1}{R}\frac{\partial \vartheta}{\partial x}\frac{\partial u}{\partial \theta} + \\
& +2B\frac{1}{R^2}\frac{\partial u}{\partial \theta}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta} + 2B_{66}\frac{1}{R^2}\frac{\partial \vartheta}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta}\Bigg\}dx\,d\theta
\end{aligned} \tag{1}$$

where,

$$B_{11} = b_{11}\cos^4\varphi + b_{22}\sin^4\varphi + \left(b_{66}+0.5b_{12}\right)\sin^2 2\varphi;$$

$$B_{22} = b_{11}\sin^4\varphi + b_{22}\cos^4\varphi + \left(b_{66}+0.5b_{12}\right)\sin^2 2\varphi;$$

$$B_{12} = \left(b_{11}+b_{22}-4b_{66}\right)\sin^2\varphi\cos^2\varphi + b_{12}\left(\sin^4\varphi+\cos^4\varphi\right);$$

$$B_{66} = -\left(b_{11}+b_{22}-2b_{2}\right)\sin^2\varphi\cos^2\varphi + b_{66}\cos^2 2\varphi;$$

$$\begin{aligned}
B_{26} = {} & 1/2\left(b_{22}\cos^2\varphi - b_{11}\sin^2\varphi\right)\sin 2\varphi - \\
& -1/6\left(b_{12}+2b_{66}\right)\sin 4\varphi;
\end{aligned}$$

$$\begin{aligned}
B_{16} = {} & 1/2\left(b_{22}\sin^2\varphi - b_{11}\cos^2\varphi\right)\sin 22\varphi - \\
& -1/6\left(b_{12}+2b_{66}\right)\sin 4\varphi.
\end{aligned}$$

$$B_{11}=\frac{E_1}{1-\nu_1\nu_2};\ B_{22}=\frac{E_2}{1-\nu_1\nu_2};\ B_{12}=\frac{\nu_2 E_1}{1-\nu_1\nu_2}=\frac{\nu_1 E_2}{1-\nu_1\nu_2};$$

$B_{66}=G_{12}=G$, R is the radius of the shell's median surface, h is the shell's thickness, u,v,w are the components of displacements of the median surface points of the shell. The expressions for potential energy of elastic deformation of the *i*th longitudinal rib are as follows [13]:

$$\prod\nolimits_i=\frac{1}{2}\int_{x_1}^{x_2}\left[\tilde{E}_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2+\tilde{E}_iJ_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+\right.$$
$$\left.+\tilde{E}_iJ_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2+\tilde{G}_iJ_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right]dx \quad (2)$$

In Equation (2) x_1,x_2 are the coordinates of edges of the shell; F_i, J_{zi}, J_{yi}, J_{kpi} are the area and inertia moments of the cross section of the *i*th longitudinal bar with respect to oz axis and the axis parallel to oy axis and passing through the gravity center, and also its inertia moment under torsion; E_i, G_i are the elasticity and shift module of the material of the *i*th longitudinal bar, u_i,ϑ_i,w_i are the components of displacements of the points of the axis of ribs $\varphi_{kpi}(x)=\varphi_2(x,y_i)=-\left.\left(\frac{\partial w}{\partial y}+\frac{\vartheta}{R}\right)\right|_{y=y_i}$.

The potential energy of the shell, under the action of all loads applied to the shell is determined as a work, and is represented in the form:

$$A_0=-\int_{x_1}^{x_2}\int_{y_2}^{y_2}\left(q_xu+q_y\vartheta+q_zw\right)dxdy-$$
$$-\int_{y_1}^{y_2}\left.\left(T_1u+S_1\vartheta+Q_1w+M_1\varphi_1\right)\right|_{x=x_1}^{x=x_2}dy- \quad (3)$$
$$-\int_{x_1}^{x_2}\left.\left(S_2u+T_2\vartheta+Q_2w+M_2\varphi_2\right)\right|_{y=y_1}^{y=y_2}dx$$

where, q_x,q_y,q_z are the components of external loads, moreover $q_z=q_{z1}+q_{z2}$ where q_{z1} is the load intensity acting on the shell from visco-elastic filler, q_{z2} is the intensity of external surface loads, T_1,S_1,Q_1,M_1 and T_2,S_2,Q_2,M_2 are internal forces and moments in shell.

The potential energy of the *i*th longitudinal bar, are similarly determined by the following expressions (it is accepted that only edge loads are applied to ribs):

$$A_i=-\left(T_iu_i+S_i\vartheta_i+Q_iw_i+\right.$$
$$+M_i\varphi_i+M_{1i}\varphi_{zi}+ \quad (4)$$
$$\left.\left.+M_{kpi}\varphi_{zi}+M_{kpi}\varphi_{kpi}\right)\right|_{x=x_1}^{x=x_2}$$

where, $T_i,S_i,Q_i,M_i,M_{1i},M_{kpi}$ are internal forces and moments in ribs.

The total potential energy of construction is equal to:

$$\prod=V_0+\sum_{i=1}^{k_1}\Pi_i+A_0+\sum_{i=1}^{k_1}A_i \quad (5)$$

Kinetic energies of the shell and ribs are written as follows:

$$K_0=\frac{\rho_0R^2h}{2}\int_0^{\xi_1}\int_0^{2\pi}\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]d\xi d\theta \quad (6)$$

$$K_i=\rho_iF_i\int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2+\frac{J_{kpi}}{F_i}\left(\frac{\partial \phi_{kpi}}{\partial t}\right)^2\right]dx \quad (7)$$

where, t is a time coordinate, ρ_0,ρ_i are densities of materials from which the shell and *i*th longitudinal bar were made.

The kinetic energy of the ribbed shell is determined as:

$$K=K_0+\sum_{i=1}^{k_1}K_i \quad (8)$$

From the condition for continuity of strain, we have the following relations between displacement and torsion angles:

$$u_i(x)=u(x,y_i)+h_i\varphi_1(x,y_i);$$
$$\vartheta_i(x)=\vartheta(x,y_i)+h_i\varphi_2(x,y_i); \quad (9)$$
$$w_i(x)=w(x,y_i);\ \varphi_i(x)=\varphi_1(x,y_i);\ \varphi_{kpi}(x)=\varphi_2(x,y_i)$$

where, $h_i=0.5h+H_i^1$, h is the shell thickness; H_i^1 is the distance from the axes of the *i*th longitudinal bar to the surface of the shell; (x,y_i) are the coordinates of the conjunction lines of ribs with the shell; φ_1 and φ_2 are turning angles of normal elements with respect to coordinate lines y and x, where $\varphi_1=-\frac{\partial w}{\partial x}$; $\varphi_2=-\left(\frac{\partial w}{\partial y}+\frac{\vartheta}{R}\right)$; φ_i, φ_{kpi} are angle deformations of longitudinal bars.

The equations of motion of a strengthened, orthotropic, medium-contacting shell are obtained based on Ostrogradskii-Hamilton's action stationarity principle:

$$\delta W=0 \quad (10)$$

where, $W=\int_{t''}^{t''}Ldt$ is Hamilton's action $L=K-\Pi$ is Lagrange's function, t' and t'' are given arbitrary times.

The load intensity acting on the shell from the elastic filler may be written in the form:

$$q_{z1}=k_cw-\int_{-\infty}^{t}\Omega(t-\tau)w(\tau)d\tau \quad (11)$$

where, the coefficient k_c is determined by the dependence $k_c=q_1+q_0\nabla^2$ (Pasternak's model), where ∇^2 is Laplace's two-dimensional operator on the contact surface, w is the shell flexure, q_0,q_1 are constants $\Omega(t-\tau)=Ae^{-\Psi(t-\tau)}$. Allowing for Equation (9) we express the displacement of bars by displacement of the shell. From the stationarity principle (10) we get a system of algebraic and differential equations with respect to sought-for unknowns.

III. PROBLEM SOLUTION

On an example we consider nonlinear parametric vibrations of a longitudinally strengthened annular orthotropic shell under the action of loads $q_{z2} = \tilde{q}_0 + \tilde{q}_0 \sin \omega_1 t$, where $\tilde{q}_0$ is the mean or main load, $\tilde{q}_1$ is rangeability of the load, ω_1 is the frequency of pressure change of the shell situated in visco-elastic medium. Assuming that the shell's edges are hingely supported, i.e. for $x = 0$ and $x = l$, $N_x = 0$, $M_x = 0$, $w = 0$, and $\vartheta = 0$.

Consider the problem solution in a first approximation and with respect to both coordinate and time functions. The boundary conditions are fulfilled exactly if we assume

$$u = u_0(t) \sin kx \cdot \sin m\theta \ , \ \vartheta = \vartheta_0(t) \sin kx \cdot \cos m\theta$$
$$w = w_0(t) \sin kx \cdot \sin m\theta \qquad (12)$$

where, m is the number of waves in peripheral direction; u_0, ϑ_0, w_0 are unknown amplitudes of sought-for quantities u, ϑ, w; $k = \frac{\pi n}{l}$, where n is the number of half-waves in longitudinal direction. Substitute approximation (12) in functional L and taking into account that $x_1 = 0$, $x_2 = l$, $y_1 = 0$, $y_2 = 2\pi$, $t' = 0$, $t'' = \frac{\pi}{\omega}$, integrate with respect to x, y and t. Then instead of the functional L we obtain a function from the sought-for quantities u_0, ϑ_0, w_0. The stationary value of obtained function is determined by the following ordinary differential nonlinear system:

$$\frac{\partial W}{\partial u_0} = 0; \ \frac{\partial W}{\partial \vartheta_0} = 0; \ \frac{\partial W}{\partial w_0} = 0 \qquad (13)$$

or

$$\left(\rho_0 h \cdot \frac{\pi L}{2} + L\sum_{i=1}^{k_1} \rho_i F_i \sin^2 m\theta_i\right)\frac{d^2 u_0}{dt^2} = \left(\frac{hRL\pi k^2 B_{11}}{2} + \frac{hm^2 \pi L B_{66}}{2R} + \frac{L}{2}\sum_{i=1}^{k_1} E_i F_i k^2 \sin^2 m\theta_i\right)u_0 + \frac{hRk^2 \pi L B_{11}}{4} w_0;$$

$$\left(\rho_0 h \cdot \frac{\pi L}{2} + L\sum_{i=1}^{k_1} \rho_i F_i \cos^2 m\theta_i + L\sum_{i=1}^{k_1} \frac{\rho_i J_{kpi} \cos^2 m\theta_i}{R^2}\right)\frac{d^2 \vartheta_0}{dt^2} + L\sum_{i=1}^{k_1} \frac{m \rho_i J_{kpi} \cos^2 m\theta_i}{R^2}\frac{d^2 w_0}{dt^2} = \frac{hR}{2}\frac{B_{22}\pi m^2}{R^2} L + \frac{hR}{2} B_{66} k^2 \pi L + \sum_{i=1}^{k_1} E_i J_{zi} k^4 \cos^2 m\theta_i + \frac{L}{2}\sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} k^2 \cos^2 m\theta_i \cdot \vartheta_0 + \frac{L}{2}\sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} k^2 m \cos^2 m\theta_i \cdot w_0; \qquad (14)$$

$$L\sum_{i=1}^{k_1} \rho_i \frac{J_{kpi} \cos^2 m\theta_i}{R^2} \cdot m \frac{d^2 \vartheta_0}{dt^2} + \left(\frac{\rho_0 h \pi L}{R^2} + L\sum_{i=1}^{k_1} \rho_i F_i \sin^2 m\theta_i + L\sum_{i=1}^{k_1} \rho_i \frac{J_{kpi} \cos^2 m\theta_i \ m^2}{R^2}\right) \cdot \frac{d^2 w_0}{dt^2} =$$
$$= \frac{hRk^2 \pi L B_{11}}{4} u_0 + \left(\frac{hm\pi L(B_{12} + B_{22})}{2R} + \frac{L}{2}\sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} mk^2 \cos^2 m\theta_i\right)\vartheta_0 + \left(\frac{h\pi L(B_{11} + 2B_{12} + B_{22})}{2R} + \frac{L}{2}\sum_{i=1}^{k_1} E_i \cdot I_{yi} \cdot k^4 \sin^2 m\theta_i + \frac{L}{2}\sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} m^2 k^2 \cos^2 m\theta_i + L\pi k_c + A_* \int_{\infty}^{0} e^{-\Psi\xi} w_0(t-\xi)\, d\xi + \pi \tilde{q}_0 \left(1 + \tilde{q}_1 / \tilde{q}_0 \sin \omega_1 t\right)\right) w_0 + \left[\frac{9k^4 \pi L B_{11}}{32} + \frac{9m^4 \pi L B_{22}}{32} + \frac{\pi m^2 k^4 L(B_{12} + 2B_{66})}{16R^2}\right] w_0^3$$

In the second stage of the solution we integrate the system of Equations (14). Give the solution in a first approximation and with respect to time function, assuming that the vibrations are harmonic. Represent the solutions of the system of Equations (14) in the form

$$u_0(t) = A\cos \omega t, \ \vartheta_0(t) B\cos \omega t, \ w_0(t) = C\cos \omega t \qquad (15)$$

where, A, B, C are dimensionaless amplitudes, ω are vibrations frequency. We conduct the further investigation with respect to time function by means of the Bubnov-Galerkin method. Then we obtain the system of nonlinear algebraic equations. This system was solved numerically for data given below:

$R = 0.16$ m; $h = 0.00045$ m; $l = 0.8$ m; $\rho_0 = \rho_i = 7800$ kg/m^3; $\tilde{E}_i = 66.7 \times 10^9$ Pa; $\nu_1 = 0.11$; $\nu_2 = 0.19$;

$h_i = 0.1375 \times 10^{-1} R$; $k_1 = 16$; $m = 8$; $\frac{F_i}{2\pi R h} = 0.1591 \times 10^{-1}$;

$\frac{J_{yi}}{2\pi R^3 h} = 0.8289 \times 10^{-6}$; $\frac{J_{zi}}{2\pi R^3 h} = 0.13 \times 10^{-6}$;

$\frac{J_{kpi}}{2\pi R^3 h} = 0.5305 \times 10^{-6}$; $\frac{\tilde{q}_0}{E_1} = 0.002$; $\frac{\tilde{q}_1}{\tilde{q}_0} = 0.3$; $\frac{k_c}{E_1} = 0.028$.

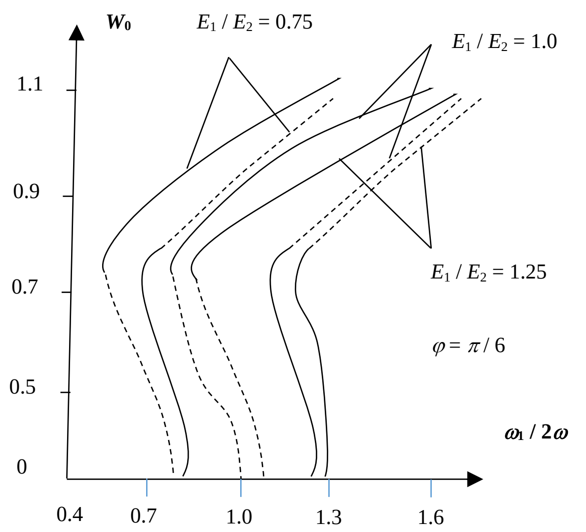

Figure 1. Amplitude-frequency dependences in the case of parametric vibrations of a strengthened, viscoelastic medium-filled cylindrical shell

The results of calculations are given in Figure 1. Amplitude-frequency curves in the case of considered vibrations of a strengthened orthotropic shell with a filler for different values of ratio E_1/E_2 were given in this figure. The solid lines point out stable branches of curves, the dotted lines point out unstable ones. It is seen that by decreasing the ratio E_1/E_2, the amplitude of considered vibrations of a strengthened orthotropic shell with a filler increases.

Consider the refined solution of the problem by approximating the curved surface of the shell by means of some parameters. Such a solution enables to follow the change of the form of the curved surface in time.

Represent the function $u(t), \vartheta(t) w(t)$ in the form of series satisfying the above given boundary conditions:

$$u = u_0(t)\sin kx \cdot \sin m\theta + u_1(t)\sin 2kx \cdot \sin m\theta$$
$$\vartheta = \vartheta_0(t)\sin kx \cdot \cos m\theta + \vartheta_1(t)\sin 2kx \cdot \cos m\theta \qquad (16)$$
$$w = w_0(t)\sin kx \cdot \sin m\theta + w_1(t)\sin 2kx \cdot \sin m\theta$$

Applying the Ostrogradskii-Hamilton principle, we get six equations of motion

$$\left(\rho_0 h \cdot \frac{\pi L}{2} + L\sum_{i=1}^{k_1} \rho_i F_i \sin^2 m\theta_i\right)\frac{d^2 u_0}{dt^2} =$$
$$= \left(\frac{hRL\pi k^2 B_{11}}{2} + \frac{hm^2\pi L k^2 B_{66}}{2R} + \frac{L}{2}\sum_{i=1}^{k_1} E_i F_i k^2 \sin^2 m\theta_i\right)u_0 +$$
$$+\frac{hRk^2\pi L B_{11}}{4} w_0 - \frac{h\pi}{3}\left(B_{11}+B_{12}\right)\left((-1)^n + 2\right)w_1 +$$
$$+x\frac{hB_{66}m\pi}{3}\left((-1)^n - 1\right)\vartheta_1;$$
$$\left(\rho_0 h \cdot \frac{h\pi L}{2} + L\sum_{i=1}^{k_1}\rho_i F_i \cos^2 m\theta_i + \sum_{i=1}^{k_1}\frac{\rho_i J_{kpi}\cos^2 m\theta_i}{R^2}\right)\frac{d^2\vartheta_0}{dt^2} +$$
$$+L\sum_{i=1}^{k_1}\frac{\rho_i J_{kpi}\cos^2 m\theta_i m}{R^2}\frac{d^2 w_0}{dt^2} = \left(\frac{hR}{2}\frac{B_{22}\pi m^2 L}{R^2} + \frac{hR}{2}B_{66}k^2\pi L +\right.$$
$$\left.+\frac{L}{2}\sum_{i=1}^{k_1}\frac{G_i J_{kpi}}{R^2}k^2\cos^2 m\theta_i\right)\vartheta_0 +$$
$$+\left(\frac{L}{2}\sum_{i=1}^{k_1}\frac{G_i J_{kpi}}{R^2}k^2 m\cos^2 m\theta_i + L\pi k_c\right)w_0 - \frac{4B_{66}}{3R}m\pi\left((-1)^n - 1\right)u_1; \qquad (17)$$
$$L\sum_{i=1}^{k_1}\rho_i\frac{J_{kpi}\cos^2 m\theta_i}{R^2}m\frac{d^2\vartheta_0}{dt^2} +$$
$$+\left(\frac{\rho_0 h\pi L}{R^2} + L\sum_{i=1}^{k_1}\rho_i F_i \sin^2 m\theta_i + L\sum_{i=1}^{k_1}\rho_i\frac{J_{kpi}\cos^2 m\theta_i m^2}{R^2}\right)\frac{d^2 w_0}{dt^2} =$$
$$= \frac{hRk^2\pi L B_{11}}{4}u_0 + \left(\frac{hm\pi L\left(B_{12}+B_{22}\right)}{2R} + \frac{L}{2}\sum_{i=1}^{k_1}\frac{G_i J_{kpi}}{R^2}mk^2\cos^2 m\theta_i\right)\vartheta_0 +$$
$$+\left(\frac{h\pi L\left(B_{11}+2B_{12}+B_{22}\right)}{2R} + \frac{L}{2}\sum_{i=1}^{k_1}E_i \cdot I_{yi}\cdot k^4\sin^2 m\theta_i +\right.$$
$$\left.+\frac{L}{2}\sum_{i=1}^{k_1}\frac{G_i J_{kpi}}{R^2}m^2 k^2\cos^2 m\theta_i + L\pi k_c +\right.$$
$$\left.+A_*\int_\infty^0 e^{-\Psi\xi}w_0(t-\xi)d\xi + \pi\tilde{q}_0\left(1+\tilde{q}_1/\tilde{q}_0\sin\omega_1 t\right)\right)w_0 +$$
$$+\left[\frac{9k^4\pi L B_{11}}{32} + \frac{9m^4\pi L B_{22}}{32} + \frac{\pi m^2 k^4 L\left(B_{12}+2B_{66}\right)}{16R^2}\right]w_0^3 +$$
$$+\frac{2h\pi\left(B_{11}+B_{12}\right)}{3}\left(\left(-1^n\right)+1\right)\cdot u_1 +$$
$$+\left[\frac{9B_{11}k^4\pi L}{4} + \frac{9B_{22}m^4\pi L}{4} + \frac{5\pi m^2 k^4 L\left(B_{12}+1B_{66}\right)}{16R^2}\right]w_0 w_1^2;$$
$$\left(\frac{\rho_0 h\pi L}{R^2} + L\sum_{i=1}^{k_1}\rho_i F_i\sin^2 m\theta_i\right)\frac{d^2 u_1}{dt^2} =$$
$$= \frac{2h\pi}{3}\left(B_{11}+B_{12}\right)\left(\left(-1^n\right)+1\right)w_0 - \frac{4B_{66}}{3R}m\pi\left((-1)^n - 1\right)\vartheta_0 +$$
$$+\left(2hRk^2 L\pi B_{11} + \frac{B_{66}m^2\pi L}{R^2} + 2\sum_{i=1}^{k_1}E_i F_i k^2 L\sin^2 m\theta_i\right)u_1 +$$
$$+hRB_{11}k^2\pi L w_1;$$
$$\left(\frac{\rho_0 h\pi L}{R^2} + L\sum_{i=1}^{k_1}\rho_i F_i\cos^2 m\theta_i + L\sum_{i=1}^{k_1}\rho_i\frac{J_{kpi}\cos^2 m\theta_i}{R^2}\right)\cdot\frac{d^2\vartheta_1}{dt^2} =$$
$$= 2L\sum_{i=1}^{k_1}m\rho_i\frac{J_{kpi}\cos^2 m\theta_i}{R^2}\frac{d^2 w_1}{st^2} + \frac{4B_{66}m\pi}{32}\left((-1)^n + 1\right)u_0 +$$
$$+\left(\frac{hB_{22}\pi m^2 L}{2R} + 4B_{66}k^2\pi L + 16\sum_{i=1}^{k_1}E_i J_{zi}k^4\cos^2 m\theta_i +\right.$$
$$\left.+2L\sum_{i=1}^{k_1}\rho_i\frac{G_i J_{kpi}}{R^2}k^2\cos^2 m\theta_i\right)\vartheta_1 + 2Lmk^2\sum_{i=1}^{k_1}\frac{G_i}{R^2}\cos^2 m\theta_i\cdot w_1;$$
$$2mL\sum_{i=1}^{k_1}\rho_i\frac{J_{kpi}\cos^2 m\theta_i}{R^2}\cdot\frac{d^2\vartheta_1}{dt^2} +$$
$$+\left(\frac{\rho_0 h\pi L}{R^2} + L\sum_{i=1}^{k_1}\rho_i F_i\sin^2 m\theta_i + 4m^2 L\sum_{i=1}^{k_1}\rho_i\frac{J_{kpi}\cos^2 m\theta_i}{R^2}\right)\frac{d^2 w_1}{dt^2} =$$
$$= -\frac{h\pi}{3}\left(Bl11\right) + B_{12}\left(\left(-1^n\right)+2\right)u_0 + hRB_{11}k^2\pi L u_1 +$$
$$+\left[\frac{h\pi L}{2R}\left(B_{11}+2B_{12}+B_{22}\right) + A_*\int_\infty^0 e^{-\Psi\xi}w_1(t-\xi)d\xi +\right.$$
$$+8Lk^4\sum_{i=1}^{k_1}E_i J_{yi}\sin^2 m\theta_i +$$
$$\left.+2L\frac{k^2 m^2}{R^2}\sum_{i=1}^{k_1}G_i J_{kpi}\cos^2 m\theta_i + L\pi k_c\right]w_1 +$$
$$+\left[\frac{hm\pi L}{2R}\left(B_{12}+B_{22}\right) + \frac{2mLk^2}{R^2}\sum_{i=1}^{k_1}G_i J_{kpi}\cos^2 m\theta_i\right]\vartheta_1 +$$

$$+\left[\frac{9hRk^4\pi LB_{11}}{8}+\frac{9hRm^4\pi LB_{22}}{8}+\frac{5h\pi m^2k^2L\left(B_{12}+2B_{66}\right)}{32R}\right]w_0^2w_1+$$
$$+\left[\frac{3hRk^4\pi B_{11}L}{4}+\frac{hR\pi m^2k^2L\left(B_{12}+2B_{66}\right)}{8}+\frac{3hm^4\pi LB_{22}}{4R^3}\right]w_1^3$$

The system of Equations (17) was integrated by the Runge-Kutta method. The initial deviation was given in the form of sinusoids in both directions, i.e. for $t=0$, $u_0=A$, $\frac{du_0}{dt}=0$, $u_1=0$, $\frac{du_1}{dt}=0$, $\vartheta_0=B$, $\frac{d\vartheta_0}{dt}=0$, $\vartheta_1=0$, $\frac{d\vartheta_1}{dt}=0$, $w_0=C$, $\frac{dw_0}{dt}=0$, $w_1=0$, $\frac{dw_1}{dt}=0$. Then, integrating the system (17), we find the values of u_1, ϑ_1, w_1. The time step is chosen so that the stability of the solution of the system of Equations (17) is provided.

Then, the calculations are repeated and $u_1(2\Delta t)$, $\vartheta_1(2\Delta t)$, $w_1(2\Delta t)$ are found by integrating the system, and so on. The values $u(t), \vartheta(t)$, $w(t)$ at arbitrary point of the shell are calculated for the given time by Equation (16).

Dependence of the flexure in central section of the shell on dimensionless variable $\tau=t/T_0$, where T_0 is the period of the main tone of small vibrations of the strengthened cylindrical shell with a filler, at different initial deviations is given in Figure 2. For $A=1$, the shell's vibrations are harmonic, while the form of the curved surface remains initial.

The obtained solution practically coincides with the solution in a first approximation. If the initial deviation exceeds the shell thickness, then the vibrations sharply differ from pure harmonic ones. Therewith the form of vibrations also at some times differ from the semiwave of the sinusoid, obtained for amplitude $A=3$. It should be noted that growth of vibrations period in comparison with a first approximation is observed.

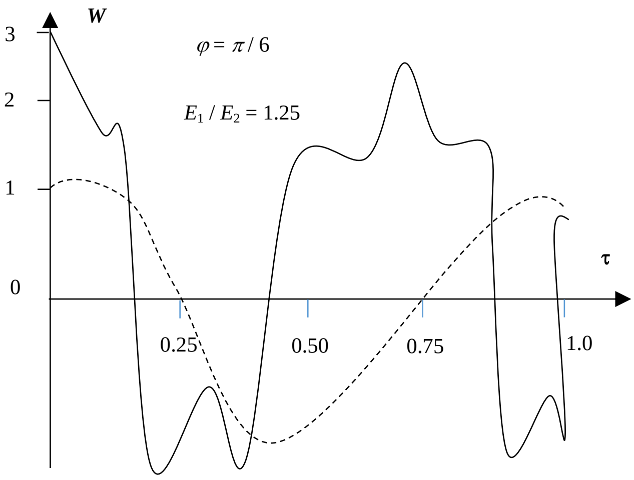

Figure 2. Time variation of flexure in median section of a visco-elastic medium-filled shell under different initial deviations

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BIOGRAPHIES

Fuad Seyfeddin Latifov was born in Ismayilly, Azerbaijan, in 1955. He graduated from Faculty of Mechanics-Mathematics, Azerbaijan State University, Baku, Azerbaijan in 1977. In 1983, he received his M.Sc. degree in Physics-Mathematics from Saint Petersburg State University, Saint Petersburg, Russia. In 2003, he got the Ph.D. degree in Physics-Mathematics. He is a Professor and the Chief of Department of Higher Mathematics in Azerbaijan University of Architecture and Construction Baku, Azerbaijan. He has written more than 80 scientific articles and 11 monographs. He is the coauthor of an encyclopedia on mathematics.

Ramiz Aziz Iskanderov was born in Krasnoselo, Armenia on July 7, 1955. He received the M.Sc. degree in Mathematics-Mechanics from Azerbaijan (Baku) State University, Baku, Azerbaijan, in 1977. He also received the Ph.D. degree in Mathematics-Mechanics from Azerbaijan University of Architecture and Construction, Baku, Azerbaijan, in 1983. He is a Professor in the field of Mathematics-Mechanics in Department of Theoretical Mechanics, Azerbaijan University of Architecture and Construction since 1983. He has published 70 papers and one book. His scientific interests are theoretical mechanics, solid state mechanics and mechanics of composite materials.

Mahir Aligulu Mehdiyev was born in Gazakh, Azerbaijan in 1976. He has studied at Mechanics Mathematics Faculty, Dnipropetrovsk State University Ukraine in 1981. He was a probationary researcher at Institute of Mechanics, Ukrainian National Academy of Sciences of Ukraine in 1983-1985, a post graduate student at the same institute in 1985-1988. He is a candidate of technical sciences since 1988. Now he is a Leading Research Associate of Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Baku, Azerbaijan. Since 2005 to present he is an Associate Professor of the chair of "Mathematics" of Azerbaijan State Economics University, Baku, Azerbaijan. He has published 50 papers and two books.