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WINGLETS INFLUENCE ON INDUCTIVE DRAG AND CIRCULATION OF FLOW VELOCITY AROUND THE WING

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Abstract- In this paper, the influence of winglets on inductive drag is studied by the method of $\Pi-$shaped vortex system. The author suggests an original idea that the $\Pi-$shaped vortex system which is realized on the plane passing through the ends of winglets parallel to the datum plane of the airplane. Using the Bio-Savar equation, the expression for inductive drag and angle of drift of the flow is written, and then the expression for inductive drag is determined. An integro-differential equation of a finite span wing with winglets for velocity circulation around the wing is derived. Using this equation, it is shown that velocity circulation is a monotonically decreasing function of the height of winglets. Reduction of inductive drag under the influence of winglets is also shown.

Keywords: Inductive Drag, Winglet, Bio-Savar Equation, $\Pi-$Shaped Vortex Circuit, Angle of Drift, Wing Integro-Differential Equation.

I. INTRODUCTION

Winglets are constructions mounted at wing ends for reducing inductive drag. The operation principle of a winglet is in reduction of frontal drag by eliminating the vortex intensity at the end part of the wing of the airplane. Despite the fact that winglets increase maximum lifting weight of FM (flying machine) by three tons, the aircraft rises faster and flies relatively silently, the engines operate economically. Initially the winglets were developed by NASA in 1960's years to reduce frontal drag. Starting from 1974 to 1976 R. Vitcombe conducted tests and persistently defended the idea of winglets. In 1977, L. Longhard first used winglets in civilian and military jet crafts.

In 1985 Boeing inserted winglets in 747-400. In 1990, Mc Donnel Douglas introduced the concept of winglets to their project MD-11. However, theoretical studies of influence of winglets on inductive drag and other aerodynamical characteristics were carried out in the works [4-8]. The methods of the theorem, of impulses of vortices, equations of steady horizontal flight, of theory of "carrier line", of continuity and Bernoulli equations were used. Mathematical model describing influence of winglets on aerodynamical field around the wing of an airplane were written by all these methods. Character of the change in the aerodynamic characteristics of the wing and airplane due to the influence of winglets, was studied. The obtained results confirmed the results of experimental studies.

II. PROBLEM STATEMENT

Using the model of ideal fluid, Zhukovsky suggested to look for the source of force impact of the flow on the body in the formation of circulation. According to Zhukovsky's theorem, the lifting force of an infinite span wing is proportional to velocity circulation along the contour that covers the wing. Therefore, the wing can be replaced by an infinitely long vortex cord with the same velocity distribution as of the wing. Such a vortex cord is called attached.

Closing a pair of free cortices that begin at the end of the span and at a right angle go away from the wing back, by a part of the attached vortex on the wing, $\Pi-$shaped vortex circuit equivalent to the finite span wing [1-3] is obtained. This vortex system induces additional velocities in the air flow and causes lbevel. In this paper we suggest an original idea that the $\Pi-$shaped vortex system is realized on the plane passing through the ends of winglets, parallel to the datum plane of the airplane.

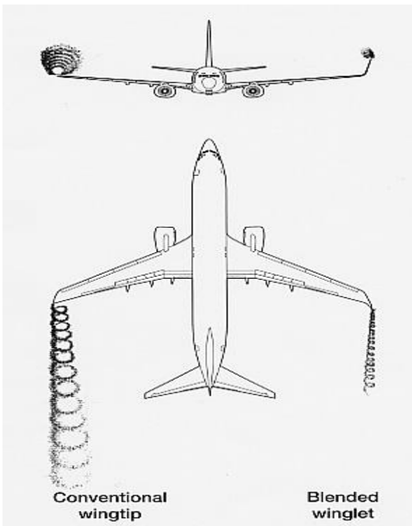

Figure 1. Reduction of intensity of vortex trace by the influence of winglet

For obtaining a simplest result on the winglet effect on aerodynamic characteristics of the wing, we assume that the rectangular winglets in the plan have the length r and cord b equal to the cord of the wing. By the Bio-Savar equation, the velocity induced by the vortex cord at the certain point is proportional to the distance of this point from the vortex axis. Therefore, removal of vortex bundle from the wing ends reduces the velocities induced by them under the wing. It was proved that just this phenomenon leads to reduction of induced drag of wing.

III. STUDY OF WINGLETS INFLUENCE ON INDUCTIVE DRAG BY CARRIER LINE METHOD

Let us consider a rectangular wing of span l and chord b streamlined by an incompressible fluid. Denote the height of the winglet by r. We assume that free vortices arise at the level of winglet tips and go away in the direction of the stream. We denote the distance between the vortices by $l_1 = l + 2e$, where e is some positive number. Depending on the shape of the wing the ratio e/l changes in the interval $0.01 \div 0.02$. We close these vortices with an attached vortex of length l_1, located above the wing at altitude r, parallel to the wing.

The origin of coordinates is located at the center of the left free vortex and direct the axis Oz along the span to the right, the axis Oy upwards and the axis Ox along unperturbed flow. Accepting that free vortices originate from the ends of winglets, then $\Pi-$shaped vortex system is built above the wing at the height of winglet.

We take any point A with the coordinate z on the attached vortex and draw a perpendicular from this point to the wing. The base of perpendicular on the wing will be the point A' with the coordinates $A'(0,-r,z)$. The distance of this point from the center of the left free vortex will be $OA' = \sqrt{z^2 + r^2}$. As the left free vortex is an infinite vortex semi-chord, then the velocity V_y induced by this vortex at the point A', according to the Bio-Savar equation [1-3], will

$$V_y = -\frac{\Gamma}{4\pi\sqrt{z^2 + r^2}} \tag{1}$$

The minus sign is set because the velocity V_y is directed down, and the axis y up.

From Equation (1) we see that the inductive velocity V_y decreases in the span of the wing almost by the hyperbolic law and has the greatest value $V_y = \Gamma/(4\pi r)$ at the end face $z = 0$ and the least value $V_y = \Gamma\Big/\left(4\pi\sqrt{l^2 + r^2}\right)$ on the another end face $z = l$ of the wing. The inductive velocity V_y has the same nature of change in the direction of flow as well. Indeed, at the point with the coordinates (x,z) behind the wing by Bio-Savar Equation, similar to Equation (1) it can be written

$$V_y = -\frac{\Gamma}{4\pi\sqrt{x^2 + z^2 + r^2}}.$$

In order to assess the influence of both vortices, we introduce the induced velocity w average in span. The velocity average in span from the left free vortex, obviously equals $\frac{1}{l}\int_e^{l+e} V_y dz$.

By the symmetry, the value of the mean velocity from the right vortex will by the same, and therefore the mean induced velocity

$$w = \frac{2}{l}\int_e^{l+e} V_y dz \tag{2}$$

Substituting in Equation (2) the expression of the velocity V_y from Equation (1), for the velocity we have the expression

$$w = \frac{-\Gamma}{2\pi l}\int_e^{l+e} \frac{dz}{\sqrt{z^2 + r^2}} \tag{3}$$

Using the value of the undetermined integral $\int \frac{dx}{\sqrt{x^2 + \lambda}} = \ln\left|x + \sqrt{x^2 + \lambda}\right| + C$, after calculating integral (3), we get

$$w = \frac{-\Gamma}{2\pi l}\ln\left|\frac{l + e + \sqrt{(l+e)^2 + r^2}}{e + \sqrt{e^2 + r^2}}\right| \tag{4}$$

So, we obtained the velocity w induced by free vortices moving away from the ends of winglets, i.e. in the case $r = 0$ the Equation (4) turns into a Equation for inductive velocity in the case of winglets without endings [1-3].

Prove that the modulus of the function w, determined by Equation (4) is a monotonically decreasing function r of the length of the winglet. To this end, we calculate the derivative of the function w with respect to r

$$\frac{dw}{dr} = \frac{-\Gamma}{2\pi l}\frac{1}{\left[l + e + \sqrt{(l+e)^2 + r^2}\right]\left(e + \sqrt{e^2 + r^2}\right)} \times$$

$$\times\left\{\frac{r}{\sqrt{(l+e)^2 + r^2}}\left(e + \sqrt{e^2 + r^2}\right) -\right.$$

$$\left. -\frac{r}{\sqrt{e^2 + r^2}}\left[l + e + \sqrt{(l+e)^2 + r^2}\right]\right\}$$

After calculations, we get

$$\frac{d|w|}{dr} < -\frac{\Gamma}{2\pi l}\frac{rl}{\sqrt{l^2 + r^2}} \times$$

$$\times\frac{1}{\left[l + e + \sqrt{(l+e)^2 + r^2}\right]\left(e + \sqrt{e^2 + r^2}\right)} < 0$$

Thus, the velocity w induced by free vortices with respect to modulus is a monotonically decreasing function of the height of winglets r and has the greatest value $|w| = \frac{\Gamma}{2\pi l}\ln\left|\frac{l+e}{e}\right|$ for $r = 0$, in the case of absence of winglets. Represent the function (4) in the following form

$$w = \frac{-\Gamma}{2\pi l}\ln\left|\frac{(l+e)[1+\sqrt{1+r^2(l+e)^{-2}}]}{e(1+\sqrt{1+r^2e^{-2}})}\right| = \\ = \frac{-\Gamma}{2\pi l}\left[\ln\frac{l+e}{e}+\ln\frac{1+\sqrt{1+r^2(l+e)^{-2}}}{1+\sqrt{1+r^2e^{-2}}}\right] \quad (5)$$

The right part of this Equation consists of the sum of the addends: the first of them corresponds to inductive velocity for a wing without winglets, the second one expresses influence of winglets. It is easy to see that the fraction under the logarithm of the second term is less than a unit, i.e. this term is a negative value and shows decrease of inductive velocity modulus. As was indicated above, depending on the shape of the wing, the ratio e/l changes in the interval $0.01 \div 0.02$. Therewith for the first logarithm we get the estimation

$$\ln\left|\frac{l+e}{e}\right| \approx 4(1+\delta)\frac{l+2e}{l} \quad (6)$$

Additionally, considering $r=e$, it is easy to get the estimation of the second logarithm

$$\ln\frac{1+\sqrt{1+r^2(l+e)^{-2}}}{1+\sqrt{1+r^2e^{-2}}} \approx \ln\frac{2}{1+\sqrt{2}} \quad (7)$$

Assuming $e/l = 0.01$ we easily find that the ratio of (7) and (6) are approximately about 4.5%, and provided $r=2e$, $e/l=0.01$ about 7.5%.

From the equality $\tan(\Delta\alpha) \approx \Delta\alpha = -\frac{w}{V_\infty}$ and Equation (5) we determine the angle of drift

$$\Delta\alpha = \frac{\Gamma}{2\pi l V_\infty}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right] \quad (8)$$

where in the second logarithm, the negative sign was brought forward, the fraction was written in the opposite ratio.

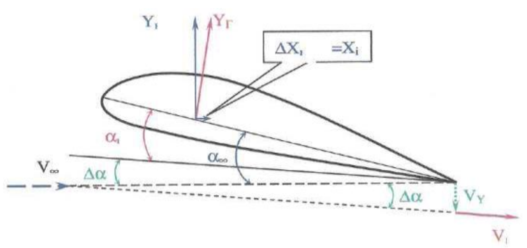

Figure 2. Angle of a drift $(\Delta\alpha)$, real attack angle (α_1) and inductive force $X_i = Y_1\tan(\Delta\alpha) \cong Y_1\Delta\alpha$

Let us determine circulation Γ from the condition of equality of the value of the lifting force, written by dynamic head and circulation

$$Y_a = C_y\frac{\rho V_\infty^2}{2}S = \rho V_\infty \Gamma l \quad (9)$$

Hence, we find $\Gamma = \frac{1}{2l}C_y V_\infty S$.

Substituting this expression in Equation (8), we get an expression for the angle of a drift

$$\Delta\alpha = \frac{C_y S}{4\pi l^2}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right]$$

Taking into account that the ratio l^2/S is the lengthening of the wing λ, we rewrite the equation in the form

$$\Delta\alpha = \frac{C_y}{4\pi\lambda}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right]$$

Then the inductive drag force will have the expression

$$X_i = Y_a\Delta\alpha = = Y_a\frac{C_y}{4\pi\lambda}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right]$$

Using the representations of the lifting force and inductive drag forces by the dynamic head

$$Y_a = C_y\frac{\rho V_\infty^2}{2}S, \quad X_i = C_{xi}\frac{\rho V_\infty^2}{2}S,$$

we find an expression for inductive drag coefficient C_{xi}

$$C_{xi} = \frac{C_y^2}{4\pi\lambda}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right]$$

Taking into account Equation (6), we can write

$$C_{xi} = \frac{C_y^2}{\pi\lambda}\left[(1+\delta) - \frac{1}{4}\ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^{-2}}}\right] \quad (10)$$

where the first addend in the square bracket corresponds to the inductive drag coefficient of a simple wing, the second one reflect the influence of winglets on inductive drag coefficient. As seen, the inductive drag coefficient found in the form of Equation (10) is less than the same one for the wing without winglets.

Obviously, the ratio of the second added to the first one in the square bracket in Equation (10) as above is assessed approximately about 4.5% for $r=e$, $e/l=0.01$ and about 7.5% for $r=2e$, $e/l=0.01$. We rewrite Equation (10) in the form

$$C_{xi} = \frac{C_y^2}{\pi\lambda}(1+\delta) - \frac{C_y^2}{4\pi\lambda}\ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2(l+e)^2}} \quad (11)$$

We call the modulus of the second addend in this Equation a function of winglets. This Equation admits to make the following conclusions:

- with the increase in the lifting force, the angle of a drift of flow, chord of the profile and height of winglets, the function of winglets increases, and consequently, the inductive drag decreases;
- Decrease of C_{xi} is proportional to C_y^2 and this dependence is monotonic;
- With the increase of the wingspan, the function of winglets decreases as the inductive drag itself.

Herewith, the fraction under logarithm in Equation (11) tends to constant value equal to $\frac{1+\sqrt{1+r^2e^{-2}}}{2}$.

IV. INFLUENCE OF WINGLETS ON VELOCITY CIRCULATION AROUND THE WING

In this section, using the $\Pi-$shaped vortex system, as the main integro-differential equation of a finite span wing is derived, and winglets influence on aerodynamical characteristics is studied.

We accept the following system of coordinates. We place the origin of coordinates in the middle of the attached vortex of the drawn at the end at the height of the winglet r from the wing and direct the axis Oz along the span to the right, the axis Oy upwards and the axis Ox along the unperturbed flow. The distance between the centers of vortexes will be denoted by $l_1 = l + 2e$, where e is some positive number, l is the span of the wing without winglets. Then the coordinate z will change on the segment $-\frac{l}{2} - e \le z \le \frac{l}{2} + e$. For the carrier line the circulation $\Gamma(z)$ is the same as for appropriate sections of the wing itself. Under such replacement of the wing by a carrier line, a flat vortex sheet begins directly on the carrier line and has linear intensity $d\Gamma(z)/dz$ along the span. The velocity induced by this vortex at the point with coordinate ξ, by the Bio-Savar law will be

$$dV_y = \frac{d\Gamma}{4\pi(\xi - z)}$$

The velocity induced by total vortex plane is found spanwise by the integral from this magnitude

$$V_y = -\frac{1}{4\pi}\int_{-l/2-e}^{l/2+e} \frac{d\Gamma(\xi)}{d\xi}\frac{d\xi}{\sqrt{(\xi - z)^2 + r^2}} \tag{12}$$

The corresponding total skew angle in this section will be expressed by the Equation (13).

$$\varepsilon = -\frac{1}{4\pi V_\infty}\int_{-l/2-e}^{l/2+e} \frac{d\Gamma(\xi)}{d\xi}\frac{d\xi}{\sqrt{(\xi - z)^2 + r^2}} \tag{13}$$

Nonsingular integral in these Equations is considered in the sense of its mean value. According to Equation (14), span wise mean skew angle of the flow equals

$$\varepsilon_{cp} = -\frac{1}{4V_\infty l}\int_{-l/2-e}^{l/2+e}\left[\int_{-l/2-e}^{l/2+e} \frac{d\Gamma(\xi)}{d\xi}\frac{d\xi}{\sqrt{(\xi - z)^2 + r^2}}\right]dz \tag{14}$$

The coefficient of the lift C_y of the wing may be determined by the known law of distribution of circulation along the span. With this definition, one can start from the hypothesis of plane sections according to which the element of the wing under consideration is streamlined as the corresponding profile belonging to the cylindrical wing of infinite span. Calculations based on this hypothesis give satisfactory accuracy for wings with small sweep and elongation $\lambda > 3 \div 4$. According to Zhukovsky's Equation, the lift force of the profile $dY_a = \rho_\infty V_\infty \Gamma(z)dz$, its total value for the wing

$$Y_a = \rho_\infty V_\infty \int_{-l/2-e}^{l/2+e} \Gamma(z)dz \tag{15}$$

and corresponding aerodynamical coefficient

$$C_y = \frac{Y}{q_\infty S_{kp}} = \frac{2}{V_\infty S_{kp}}\int_{-l/2-e}^{l/2+e} \Gamma(z)dz \tag{16}$$

To find the law of distribution of circulation, we consider the profile in the arbitrary section of the wing for which we can represent the lift in the form of $dY_a = C_y(z)q_\infty b(z)dz$, where $b(z)$ is the profile chord in the section under consideration. Taking account the Zhukovsky Equation $dY_a = p_\infty V_\infty \Gamma(z)dz$ we find the coupling equation determining the dependence of circulation velocity in the given section

$$\Gamma(z) = \frac{1}{2}C_{ya}(z)b(z)V_\infty \tag{17}$$

According to hypothesis of plane sections, the coefficient of the lift $C_{ya}(z)$ of the section under consideration is the same as for the corresponding cylindrical wing of in finite span. Its size may be determined with regard to skew angle by Equation (18).

$$C_{ya}(z) = C_{ya}^{\alpha}(z)(\alpha - z) \tag{18}$$

where, the derivative $C_{ya}^{\alpha}(z) = \partial C_{ya}(z)/\partial\alpha$ is determined for a wing of infinite span and the range of attack angles corresponding to the linear portion of the curve $C_{ya}(a)$. After substituting in (17) the values of (12) and (18), we obtain:

$$\Gamma(z) = \frac{1}{2}C_{ya}^{\alpha}(z)b(z)V_\infty \times \times\left(\alpha + \frac{1}{4\pi V_\infty}\int_{-l/2-e}^{l/2+e} \frac{d\Gamma(\xi)}{d\xi}\frac{d\xi}{\sqrt{(\xi - z)^2 + r^2}}\right) \tag{19}$$

In the absence of a winglet, i.e. for $r = 0$, this equation coincides with the main integro-differential equation of a finite span wing. We call this equation the main integro-differential equation of a finite span wing with winglets. This is called an integro-differential equation because the sought-for function $\Gamma(z)$ enters it under the sign of derivative and under the sign of integral.

Equation (19) enables to determine the circulation of the wing $\Gamma(z)$ by the known functions $b(z)$ and $\alpha(z)$ given by the construction of the wing. Establishment of the law of distribution of circulation along the span of the wing enables to determine all aerodynamical characteristics of the wing.

In the case (19) there are no general methods $r = 0$ for solving equation (and also similar equations). In this connection, the Clauret-Treftz approximate solution method and its modifications are mainly used. This method is based on the use of trigonometric series. There are other approximate methods, for example, the Multgopp, S.G. Nuzhin, B.N. Yuriev, V.V. Golubev [1], G.F. Burago, A.B. Risberg [2] methods and others. A part of these methods are based on the application of the methods of theory of functions of a complex variable, in particular, theory of residues. The Equation (19) obtained by us in even more complicated, therefore we will use this equation for obtaining a qualitative effect of winglets on aerodynamical characteristics.

First suppose that depending on the shape of the wing in the plan, the distribution of velocity distribution along the span is subject to the linear law $\Gamma(z)=\Gamma_0+\Gamma' z$. Then $\frac{d\Gamma}{dz}=\Gamma'$ and we can take this factor out of the sign of integral (19). This time the integral in Equation (19) takes the form

$$I=\int_{-l/2-e}^{l/2+e}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}}$$

Calculating this integral from the integration tables, we obtain

$$I=\ln\frac{a+\sqrt{a^2+r^2}}{-b+\sqrt{b^2+r^2}}$$

where $a=\frac{l}{2}+e-z$ and $b=\frac{l}{2}+e+z$.

This expression reflects the nature of influence of winglets on the value of circulation around the wing in the case under consideration. Show that this expression is a decreasing function of the quantity r. For that we calculate the derivative dI/dr

$$\frac{dI}{dr}=\frac{-b+\sqrt{b^2+r^2}}{a+\sqrt{a^2+r^2}}\times$$

$$\times\left(r\left(a^2+r^2\right)^{-\frac{1}{2}}\left(-b+\sqrt{b^2+r^2}\right)-\right.$$

$$\left.-r\left(a+\sqrt{a^2+r^2}\right)\left(b^2+r^2\right)^{-\frac{1}{2}}\right)/\left(-b+\sqrt{b^2+r^2}\right)^2$$

It is easy to see that the first fraction and the denominator of the second fraction are positive. Therefore, we investigate the sign of the numerator of the second fraction that may be written in the form

$$N=-\frac{br}{\sqrt{a^2+r^2}}+\frac{r\sqrt{b^2+r^2}}{\sqrt{a^2+r^2}}-\frac{ar}{\sqrt{a^2+r^2}}-\frac{r\sqrt{a^2+r^2}}{\sqrt{b^2+r^2}}$$

In the case $z=0$, we have $a=b$, and from the last equality we get

$$N=-\frac{2r\left(\frac{l}{2}+e\right)}{\sqrt{r^2+\left(\frac{l}{2}+e\right)^2}}$$

As can be seen, this expression is negative. Then the derivative dI/dr for $z=0$ is also negative. In the case $z=l/2+e$ we have $a=0, b=l+2e$ and we easily get the expression

$$N=-\frac{b^2}{\sqrt{b^2+r^2}}\left(\sqrt{1+\frac{r^2}{b^2}}-1\right)<0$$

The same result is obtained in the case $z=l/2-e$ as well. Herewith $a=l+2e, b=0$ and we easily get the expression

$$N=-\frac{a^2}{\sqrt{a^2+r^2}}\left(1+\sqrt{1+\frac{r^2}{a^2}}\right)<0$$

As can be seen, in all these cases the derivative dI/dr is negative. This shows that the velocity circulation around the wing is a decreasing function of quantity r.

Now suppose that distribution of velocity circulation along the span is subject to the parabolic law of the form

$$\Gamma(z)=\Gamma_0-\frac{\Gamma'}{2}\left(z^2-\frac{l^2}{4}\right)$$

and we carry out the same research as above. Since the velocity distribution along the span is symmetric with respect to the wing center, we consider the right semi-wing. Herewith

$$\frac{d\Gamma}{dz}=-\Gamma' z$$

and integral in Equation (8) for the first semi-wing takes the form

$$I=-\int_{0}^{l/2+e}\frac{\xi d\xi}{\sqrt{(\xi-z)^2+r^2}}$$

We transform this integral in the following form

$$I=-\int_{0}^{l/2+e}\frac{(\xi-z)d\xi}{\sqrt{(\xi-z)^2+r^2}}-z\int_{0}^{l/2+e}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}}$$

where the second integral was studied above, and as it was shown, it is a decreasing function from the height of winglets r. Let us investigate the character of dependence of the first integral on r

$$I_1=-\int_{0}^{l/2+e}\frac{(\xi-z)\,d\xi}{\sqrt{(\xi-z)^2+r^2}}=-\frac{1}{2}\int_{0}^{l/2+e}\frac{d\,(\xi-\xi)^2}{\sqrt{(\xi-z)^2+r^2}}=$$

$$=-\sqrt{(\xi-z)^2+r^2}\Big|_{0}^{l/2+e}=\sqrt{z^2+r^2}-\sqrt{a^2+r^2}$$

For the left semi-wing we have

$$\frac{d\Gamma}{dz}=-\Gamma' z$$

and the corresponding integral is written as following

$$\bar{I}=-\int_{-l/2-e}^{0}\frac{\xi d\xi}{\sqrt{(\xi-z)^2+r^2}}$$

Proceeding in the same way as above, we get

$$\bar{I}=-\int_{-l/2-e}^{0}\frac{(\xi-z)d\xi}{\sqrt{(\xi-z)^2+r^2}}-z\int_{-l/2}^{0}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}}$$

where the second integral in the sum with the corresponding second integral above, give a monotonically decreasing function of the height of winglets r. Let us study the nature of dependence of the first integral on r

$$\bar{I}_1=-\int_{-l/2-e}^{0}\frac{(\xi-z)\,d\xi}{\sqrt{(\xi-z)^2+r^2}}=-\frac{1}{2}\int_{-l/2-e}^{0}\frac{d\,(\xi-\xi)^2}{\sqrt{(\xi-z)^2+r^2}}=$$

$$=-\sqrt{(\xi-z)^2+r^2}\Big|_{l/2+e}^{0}=\sqrt{b^2+r^2}-\sqrt{z^2+r^2}$$

Thus,

$$I = I_1 + \bar{I}_1 = -\int_{-l/2+e}^{l/2+e} \frac{\xi d\xi}{\sqrt{(\xi - z)^2 + r^2}} = \sqrt{b^2 + r^2} - \sqrt{a^2 + r^2}$$

Calculate the derivative of this function with respect to r

$$\frac{dI}{dr} = \frac{r}{\sqrt{b^2 + r^2}} - \frac{r}{\sqrt{a^2 + r^2}}$$

This expression is negative as $b > 0$.

Thus, in the case of parabolic distribution of spanwise velocity distribution, it is a monotonically decreasing function of the height of winglets.

Using the above Equations for inductive velocity (1), the skew of the flow (2), the lift (4) and the lift factor (5) we see that these quantities are monotonically decreasing functions of the height of winglets. Reduction of the lift with increasing the height of winglets is a very important result. This result leads to conclusion that not the lifting force created only by a wing, but a lifting force created by the entire structure, by the wing together with winglets, balances the weight force of the aircraft in the horizontal flight.

V. CONCLUSION

Accepting that free vortices originate from the ends of winglets, the Π – shaped vortex circuit is built above the wing at the height of the winglet. The air is incompressible and using the Bio-Savar quation, we write an expression for inductive velocity created by free vortices at the level of the wing and find its mean value with respect to span. Determining the average angle of the drift of the flow, we find the inductive drag and its coefficient. Finally, decrease of inductive drag is shown.

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BIOGRAPHY

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