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FDM ELECTROMAGNETIC ANALYSIS IN BUSHING REGIONS OF TRANSFORMER

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Abstract- According to the fact that Finite Difference Method (FDM) is more easily understood, easy-to-implement than Finite Element Method (FEM) and Analytical Method, it has been used to analyze magnetic field and stray losses at the bushing zones of transformer covers. Therefore, the FDM is used to solve Maxwell's equations and Ohm's law at the cross section area in axial symmetric page of a steel disk considering constant permeability, which is the novelty introduced in this paper. The solution algorithm was described in detail. The reliability of the proposed technique is confirmed by FEM. The comparison between the FDM results with those obtained from FEM declare the efficiency and capability of the applied numerical method.

Keywords: Finite Difference Method (FDM), Bushing Regions, Eddy Current Losses, Finite Element (FE) Simulation, Transformer Cover.

I. INTRODUCTION

High-current conductors of low voltage side of transformers are sources of power losses, generating undesirable thermal issues in their tanks. Heating hazard minimization in bushing plates becomes a significant role in the design steps. Longer useful lives of utilities and cost savings due to reduction of power losses can be significant. On the other hand, precise assessment of the temperature in the steel cover of the tank is important. Therefore, the application of advanced techniques for precise estimation of temperature distributions in steel plates due to eddy current losses is of great interest [1].

Almost in all the studies that recently published in the area of computation of the eddy current losses at transformer covers, the FE [2-7] and analytical methods [8-10] have been used. There have been published several studies in which different solution methods of magnetic analysis of transformer cover are studied based on Poynting's theorem [11-13] or Maxwell's equations [3, 8-10]. Numerical simulations have a great role in the design of transformers [2, 3, 14, 15]. They can model exactly the electromagnetic fields phenomena by considering complicated geometries and nonlinear behavior of materials and boundary conditions.

As a result, this study proposes FD formulas [16] to determine the magnetic field and losses in transformers cover, by taking constant permeability of the steel cover and boundary conditions into account. The used algorithm in this work can be considered as a base for the non-linear electromagnetic FDM analysis of transformers cover which taking the true nature of the used material into account. FDM is easily implementable [17] and the computation time of design procedures can be reduced from economical point of view of the transformer manufacturers. Hence, the proposed formulas may be applied to make a better design for transformers, as improve their efficiency. Therefore, it is very useful for practical design and analysis problems. Furthermore, it is not necessary to purchase special expensive software licenses and powerful computers.

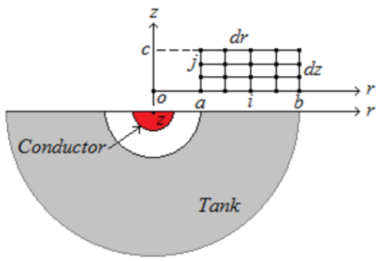


Figure 1. Idealized geometry, elements and solution area of the disk in the FDM approach.

II. PROPOSED FDM APPROACH

High-current conductors through the bushings of low voltage side of the tank cover, creates a magnetic field generating eddy currents which resulting losses in the bushing regions of the cover plate.

Since the large eddy current densities and therefore losses are concentrated near the circular bushing regions, the actual dimensional geometry and size of the tank cover will not matter much. Therefore, a reasonable tank cover losses estimation can be obtained by considering a transformer cover with circle cross section of large radius centered on the hole as shown in Figure 1. Besides, we assume an enough long circular cross section conductor perpendicularly crossing the center of tank cover hole [8].

In the mentioned geometry, the combining of Ohm's law with Maxwell's equations in the transformer cover and reduces to Equation (1) as

$$\nabla^2 H = \sigma \frac{\partial B}{\partial t} \tag{1}$$

where, B and H are flux density, intensity of the magnetic field respectively and σ is the electrical conductivity of steel plate. For a constant permeability μ

$$B = \mu H \tag{2}$$

with $\mu = \mu_r \mu_0$, where μ_0 equals to $4\pi \times 10^{-7}$ H/m is the permeability of free space in SI units and μ_r is the relative permeability of the magnetic material [14, 17].

Since the problem solution is axisymmetric, it is convenient to write the Equation (1) in cylindrical coordinate system and taking into account a rectangular solution domain (Figure 2) as Equation (3).

$$\frac{1}{r}\frac{\partial H_\varphi}{\partial r} + \frac{\partial^2 H_\varphi}{\partial r^2} - \frac{H_\varphi}{r^2} + \frac{\partial^2 H_\varphi}{\partial z^2} = \sigma \frac{\partial B_\varphi}{\partial t} \tag{3}$$

where, H_φ is azimuthal component of H [9].

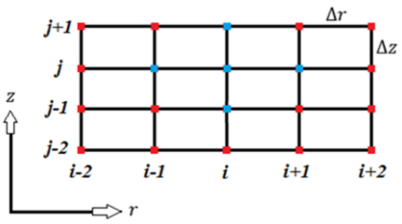


Figure 2. Typical mesh of domain solution and its 2D five-point stencil

For 2D FDM analysis, we need to divide the solution region into an equally spaced gird of nodes with different mesh in z and r-direction. The steel thickness will be replaced with a grid of nodes which is equal to 10 times of skin depth number of thickness [14]. Therefore, for the used configurations a grid of 60 points for the z-axis should be sufficient. Therefore, the magnitude of B and H of the field are determined on the nodes and denoted as $B(i,j)$ and $H(i,j)$, respectively, where $i = 1,\cdots,N_r$, $j = 1,\cdots,N_z$ for an $N_r \times N_z$ nodes. $B^{new}(i,j)$ and $H^{new}(i,j)$ are considered as magnitude of magnetic field properties at the next time step. It is not necessary to save all computations on grid nodes during work time of solution.

The FDM approximation of Equation (3) can be derived as follows:

$$\begin{aligned} &\frac{1}{r_i} \times \frac{H(i+1,j) - H(i-1,j)}{2\Delta r} + \\ &+ \frac{H(i+1,j) - 2H(i,j) + H(i-1,j)}{(\Delta r)^2} - \frac{H(i,j)}{r_i^2} + \\ &+ \frac{H(i,j+1) - 2H(i,j) + H(i,j-1)}{(\Delta z)^2} = \\ &= \sigma \frac{B^{new}(i,j) - B(i,j)}{\Delta t} \end{aligned} \tag{4}$$

where, $r_i = a + (i-1)\cdot \Delta r$ is the radial distance of mesh points, Δz and Δr are the distances between space points in z and r directions and Δt is the time interval between $B(i,j)$ and $B^{new}(i,j)$. The magnetic flux density at next time step 5 can be derived by modifying and simplifying the Equation (4) as follows:

$$\begin{aligned} B^{new}(i,j) = B(i,j) + \frac{\Delta t}{\sigma(\Delta r)^2} \begin{bmatrix} (1+\Delta r/2r_i) H(i+1,j) + \\ (1-\Delta r/2r_i) H(i-1,j) - \\ \left(2 + (\Delta r/r_i)^2\right) H(i,j) \end{bmatrix} + \\ + \frac{\Delta t}{\sigma(\Delta z)^2} \begin{bmatrix} H(i,j+1) - \\ 2H(i,j) + \\ H(i,j-1) \end{bmatrix} \end{aligned} \tag{5}$$

Here, it should be noted that Equation (5) under the following conditions have stable and converged solutions:

$$\frac{\Delta t_1}{\sigma(\Delta z)^2} \le \frac{\mu_{r,diff}\,\mu_0}{2}, \quad \frac{\Delta t_2}{\sigma(\Delta r)^2} \le \frac{\mu_{r,diff}\,\mu_0}{2} \tag{6}$$

where, $\mu_{r,diff}$ is the relative differential permeability. In addition, we can consider $\Delta z \le \Delta r$ condition as an acceptable assumption due to small thickness and small penetration depth of steel cover. By using all of these conditions together, time step size is determined as follows;

$$\Delta t \le \frac{\mu_{r,diff}\,\mu_0}{2} \sigma (\Delta z)^2 \tag{7}$$

The time step may vary during process [14, 18].

The surface boundary conditions at $j=1$, $j=N_z$ and all i are [12, 14]

$$H(i,j) = \frac{I_m}{2\pi r_i} \sin(\omega t) \tag{8}$$

$$H(1,j) = \frac{I_m}{2\pi a} \sin(\omega t), \text{ at } i=1 \text{ and all } j \tag{9}$$

$$H(N_r,j) = \frac{I_m}{2\pi b} \sin(\omega t), \text{ at } i=N_r \text{ and all } j \tag{10}$$

At starting time of the solution $t=0$, B and H values at all grid nodes are set to zero. At $t = \Delta t$, boundary values are set according to Equations (8)-(10) and all the non-boundary values of $H(i,j)$ are still zero. The new values of magnetic flux boundary, $B^{new}(i,j)$, can be calculated using Equation (5). $H^{new}(i,j)$ can be found at all non-boundary nodes by (2). The solution algorithm usually continues for about 6 periods of oscillation time of bushing current. Thus, at new time interval of $t+\Delta t$, magnetic field intensity at boundary condition $H(i,1)$, $H(i,N_z)$, $H(1,j)$ and $H(N_r,j)$ are derived according to Equations (8)-(10). The H values of other points changes from their initial values which is zero. Then, $B^{new}(i,j)$ are derived by (5) and the $H^{new}(i,j)$ for non-boundary points are determined using Equation (2).

According to the fact that the time average eddy current losses can be found at the end of each cycle, the algorithm flowchart for eddy current losses calculations (Figure 3) terminates when the losses error between two successive cycles is less than a given tolerance.

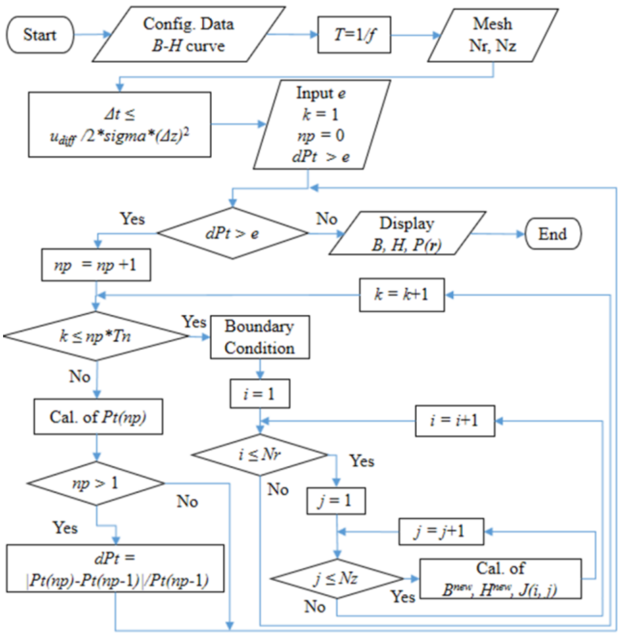


Figure 3. The flowchart of the electromagnetic FDM analysis

The eddy current density $J = \nabla \times H$ in the solution domain of the problem is given by;

$$J = -\frac{\partial H_\varphi}{\partial z}\hat{r} + \left(\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r}\right)\hat{k} \tag{11}$$

For non-boundary nodes, a central difference approach can be derived as follows;

$$\frac{\partial H_\varphi}{\partial r} = \frac{H(i+1,j) - H(i-1,j)}{2\Delta r} \tag{12}$$

$$\frac{\partial H_\varphi}{\partial z} = \frac{H(i,j+1) - H(i,j-1)}{2\Delta z} \tag{13}$$

For points on the $r = a$ surface where, $i = 1$ [14]

$$\frac{\partial H_\varphi}{\partial r} = \frac{-3H(1,j) + 4H(2,j) - H(3,j)}{2\Delta r} \tag{14}$$

At the other surface where, $i = N_r$ [14]

$$\frac{\partial H_\varphi}{\partial r} = \frac{3H(N_r,j) - 4H(N_r-1,j) + H(N_r-2,j)}{2\Delta r} \tag{15}$$

For points on the $z = 0$ surface where, $j = 1$

$$\frac{\partial H_\varphi}{\partial z} = \frac{-3H(i,1) + 4H(i,2) - H(i,3)}{2\Delta z} \tag{16}$$

At the other surface where, $j = N_z$

$$\frac{\partial H_\varphi}{\partial z} = \frac{3H(i,N_z) - 4H(i,N_z-1) + H(i,N_z-2)}{2\Delta z} \tag{17}$$

The Equations (14)-(17) are second order differential types to satisfy the required accuracy.

The time average eddy current losses of steel plate over a period can be determined from

$$Loss_{bush} = \frac{2\pi}{T\sigma}\sum_{k=1}^{k=T_n}\Delta t\sum_{j=1}^{j=N_z}\Delta z\sum_{i=1}^{i=N_r}\left|J(i,j,k)\right|^2 r\Delta r \tag{18}$$

where, T is the time period of bushing current and J is an instantaneous eddy current density at any points of grid.

Normally, the minimum value of differential permeability can be found by using the peak of magnetic field intensity $I_m/2\pi a$ over a period from Equation (9) [14]. Therefore, the time step can be determined based on Equation (7). For some margin, time step should be set lower.

III. SIMULATIONS AND RESULTS

To validate the proposed methodology we use a disk with a hole radius a = 30 mm, a thickness c = 6 mm, an external radius b = 50 cm, and a relative permeability $\mu_r = 900$ for linear case, a conductance $\sigma = 6.8\times10^6$ S/m. The lead cross section has a radius r = 24 mm, a length of 1 m, a relative permeability $\mu_r = 1$, a conductance $\sigma = 58\times10^6$ S/m. Through the conductor flows a 500 A, 1000 A and 50 Hz current [19].

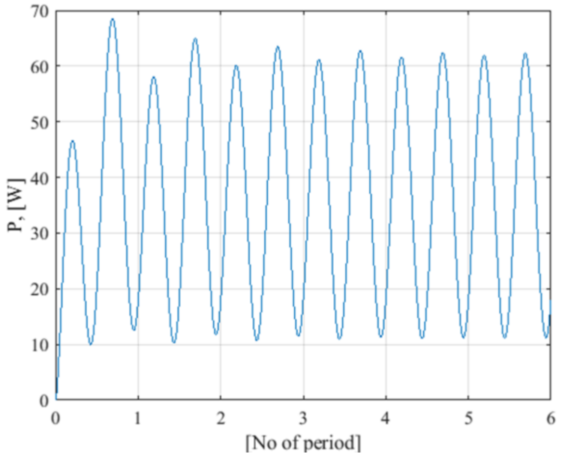
The transient solution of the eddy losses is based on the described algorithm in section II and the flowchart in Figure 3 by a developed program in a programming language like MATLAB [20] for the 2D FD model (Figure 1). According to Figure 4 the transient solution of the problem in the both case of 500 A and 1000 A current density has been reached to steady state condition after about 6 periods of time. The considered losses error between two successive cycles of oscillation is 0.001.

The given FDM results by developed program are then compared with the given FEM results by Maxwell software environment [21] possessing axisymmetric.

In the FE simulation model, only 1/720 of the disk has been considered due to axial and azimuthal symmetries. The used Flux boundary conditions in the faces of the model is shown in Figure 5. A total of 36700 FEs used in the represented model. For accurate calculation of the magnetic field and losses in the penetration depth of steel disk it is sufficient to consider the skin effect of the disk 0.91 mm by allowing four FE layers of 0.227 mm on disk surface at Maxwell's eddy current solution type for 50 Hz as given in Figure 6. FEM results of eddy current losses in steel disk have been given in Table 1.

Magnetic field and eddy current losses of transformer cover in the case of constant behavior of permeability have been computed by FDM with mesh of N_r = 100, N_z = 30 were compared with those of FEM as Losses Deviation (*LD*) in percent (%) in Table 1. There is an agreement about 2% error between the obtained losses by two methods. It is possible to decrease the error by applying a fine mesh or an adaptive mesh refinement technology. The *LD* has considerable decrement of 0.6% (from 2% to 0.6%) in the case of fine mesh of N_r = 300, N_z = 60 in Table 1.

The magnetic field distributions on and in the cover steel near the bushing zone illustrated in Figure 7 and Figure 9 have been solved by 3D FEM and 2D FDM respectively at the steady state condition. Besides, the 3D FEM and 2D FDM distributions of eddy current losses density on and in the cover steel have been shown in Figure 8 and Figure 10, respectively.



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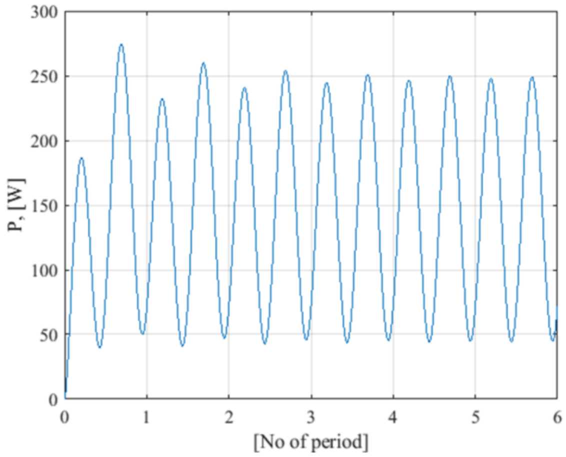


(b)

Figure 4. 2D FDM eddy current losses of disk in transient solution at fine mesh: (a) 500 A, (b) 1000 A current

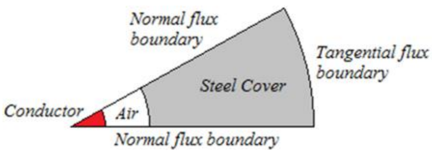


Figure 5. Flux boundaries of the cover at FEM model

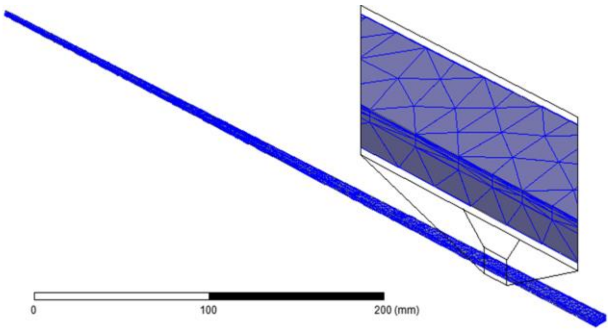
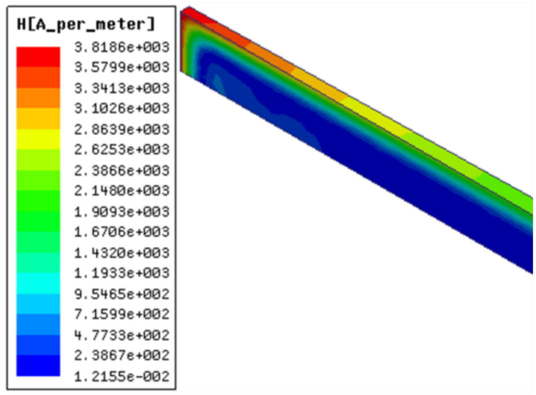


Figure 6. 3D FEM fine mesh in the penetration depth region of the plate

Table 1. Eddy current losses, [W] in cover plate

	Mesh	Linear, P_{total}		
Current, [A]	N_r×N_z	FDM	FEM	*LD* [%]
500	100×30	36	36.8	2.2
500	300×60	35.8	36.8	2.7
1000	100×30	143.9	147.25	2.3
1000	300×60	146.44	147.25	0.6



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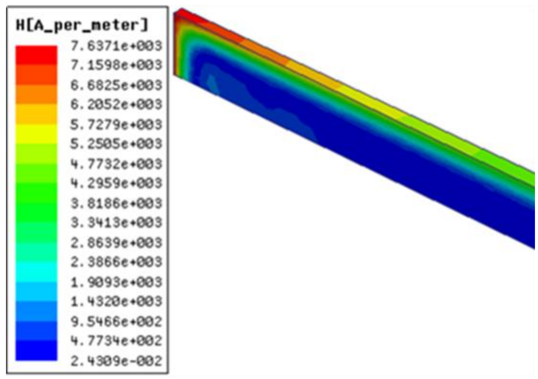

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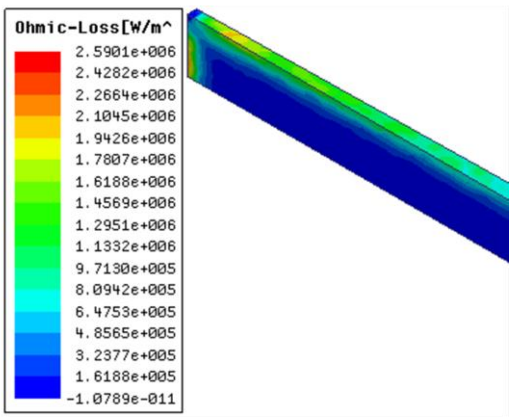
Figure 7. 3D FEM magnetic field intensity distribution on cover at the steady state condition: (a) 500 A, (b) 1000 A current



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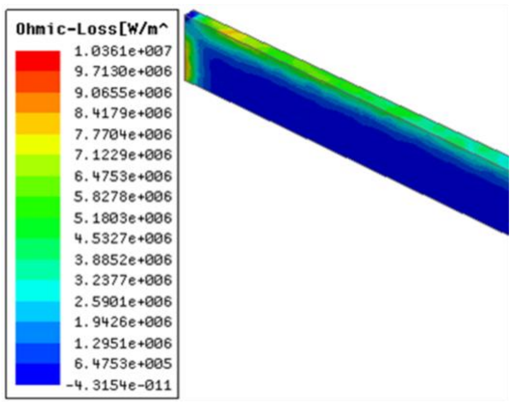
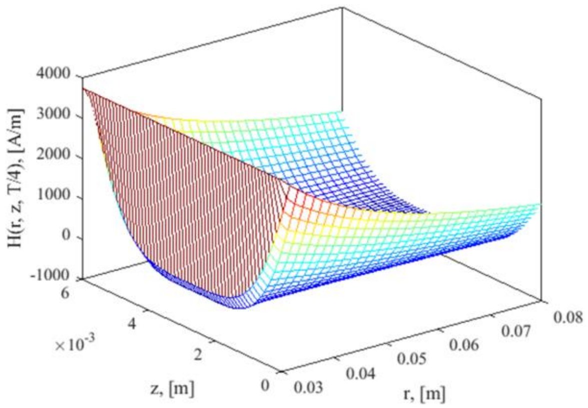


(b)

Figure 8. 3D FEM eddy current losses density on the cover at steady state condition: (a) 500 A, (b) 1000 A current

FD solution of magnetic field intensity $H_\varphi(r,z,t)$ for $t = T/4$ s at the steady state condition were shown in Figure 9 for all r, z and in for all r and z = 3 mm near the bushing region in Figure 11 the both case of 500 A and 1000 A current intensity.

According to the results at the steady state condition, the magnetic field intensity is maximum over the surfaces of plate and exponentially decreased as penetrating inside the plate. In addition, the simulation results declare that the power losses of transformer cover, due to the high current of low voltage side, concentrated near the bushing region. Therefore, in the case of non-linear behavior of cover plate, the study of this region has a great role.



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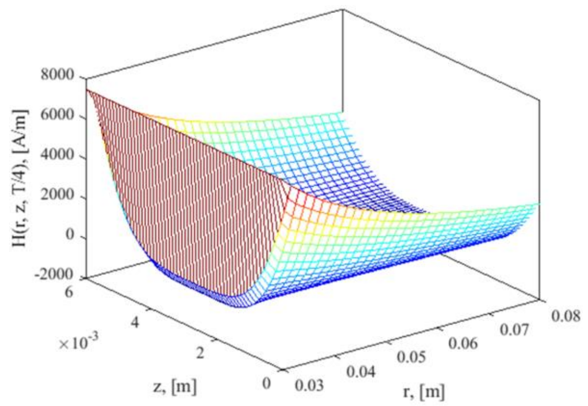
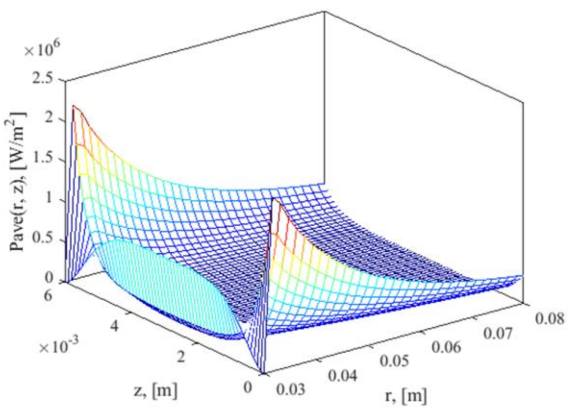


(b)

Figure 9. Magnetic field intensity distribution in the cover plate at steady state conditions. 2D FDM fine mesh: (a) 500 A, (b) 1000 A current



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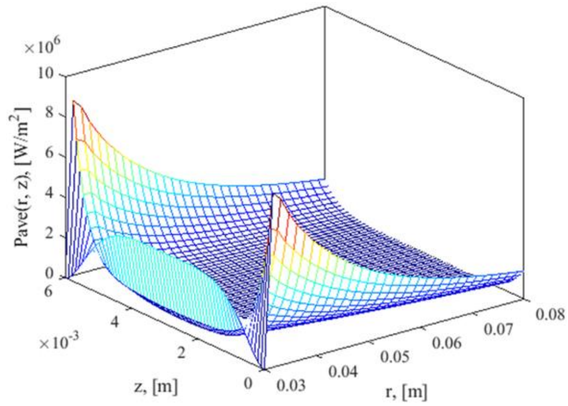
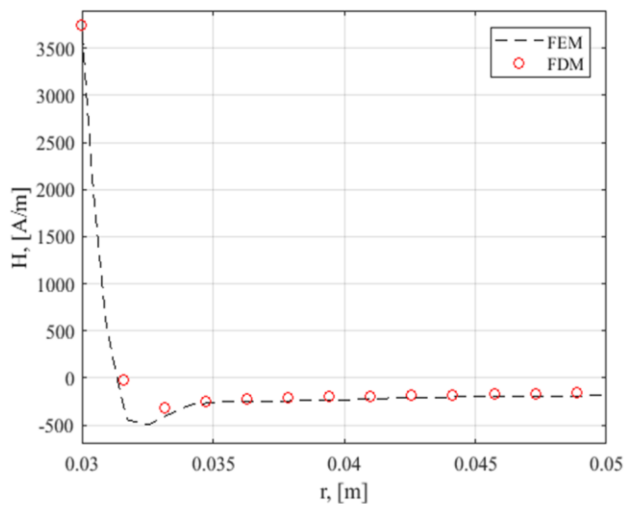


(b)

Figure 10. Eddy current losses density distribution in the cover plate at steady state conditions. 2D FDM fine mesh: (a) 500 A, (b) 1000 A current

The 2D FDM magnetic field intensity solution in the middle surface of the steel disk has been compared with those of 3D finite element simulations in Figure 11 which shows very close correspondence between them.



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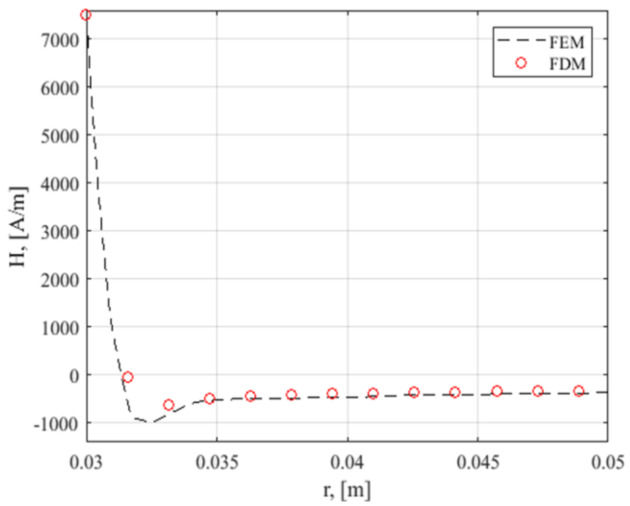

(b)

Figure 11. Magnetic field intensity distribution in middle surface of steel disk at steady state condition. FEM vs FDM: (a) 500 A, (b) 1000 A current

IV. CONCLUSIONS

FDM is easily implementable and has low computational time that reduces design procedures from economical point of view of the transformer manufacturers. So that, this work proposes a FDM algorithm that would be able to compute the magnetic field and eddy current losses distribution in the bushing zone of transformer covers. The assumptions and solution procedures have been described and justified in detail. The algorithm has been developed as a program in a programming tool like MATLAB. Therefore, it is not necessary to purchase special expensive software licenses and powerful computers.

The given results successfully validated with those of FE simulations results. The used algorithm in this work can be considered as the basis for the 2D FD non-linear analysis of cover steel field and losses which considering the true nature of the used material.

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BIOGRAPHIES

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