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WAVES PROPAGATION IN THE FLUID FLOWING IN AN ELASTIC TUBE, CONSIDERING VISCOELASTIC FRICTION OF SURROUNDING MEDIUM

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Abstract- The process of wave propagation in deformable tubes is containing a liquid taking into account interaction with the environment differs significantly from the properties of the hydroelastic system when the tube is not fixed. An explanation of the phenomena which is extremely important, will be the presence of external surface effects. In the present paper, a periodic pulsating flow of an ideal incompressible fluid including a periodic pulsating flow of an ideal incompressible fluid in a thin-walled elastic tube is described, taking into account viscoelastic external friction within the framework of a one-dimensional linear theory.

Keywords: Wave, Ideal Fluid, Viscoelasticity, Friction, Wave Velocity, Attenuation.

I. INTRODUCTION

We give the main theoretical propositions of a one-dimensional theory for a linearly elastic isotropic tube with an ideal fluid flowing in its cavity. Let to consider that a semi-infinite cylindrical tube is given which denotes by the dimensions of R and h, as its radius and thickness, respectively. The liquid is assumed to be homogeneous and incompressible, with the density of ρ_f. The viscosity can be neglected based on data of the velocity profiles, i.e. in large arteries [1]. The cylindrical tube can be approximately considered flat, i.e. the influence of viscosity is limited by thin boundary layers. The realization of the long-wave approximation, when the wavelengths are much larger than the diameter of the tube. In a one-dimensional model it is assumed that the flow rate $u=u(x,t)$, pressure $p=p(x,t)$, radial displacement $w=w(x,t)$. Then the continuity equation has the form of

$$\frac{\partial u}{\partial x}+\frac{2}{R}\frac{\partial w}{\partial t}=0 \tag{1}$$

and the equation of motion is

$$\frac{\partial u}{\partial t}+\frac{1}{\rho_f}\frac{\partial p}{\partial x}=0 \tag{2}$$

where, $x\in[0,\infty)$ is the longitudinal coordinate, and t is time. Taking $w/R<<1$, we write the equation of motion of the tube

$$\rho_* h=\sigma-\frac{Eh}{R^2\left(1-\nu^2\right)}w \tag{3}$$

where ρ_* is the density of the wall material, E is the modulus of elasticity, and ν is the Poisson ratio. Further, we assume that the quantity σ consists of two types of stresses: hydrodynamic p, acting on the liquid side in the tube, and the voltage arising under the assumption that the surrounding external medium introduces additional viscoelastic stiffness $G^\nu\frac{\partial w}{\partial t}$. Hence, by analogy with the hereditary theory of elasticity [2], for the operator G^ν we write

$$G^\nu=G\left\{\frac{\partial w}{\partial t}-\int_{-\infty}^{t}G_0(t-\tau)\frac{\partial w(x,\tau)}{\partial \tau}d\tau\right\} \tag{4}$$

Now taking into account equality (4), we rewrite Equation (3) in the form of

$$p=\rho_* h\frac{\partial^2 w}{\partial t^2}+G\left\{\frac{\partial w}{\partial t}-\int_{-\infty}^{t}G_0(t-\tau)\frac{\partial w(x,\tau)}{\partial \tau}d\tau\right\}+\frac{Eh}{R^2(1-\nu^2)}w \tag{5}$$

Thus, the closed system of hydroelasticity is described by Equations (1), (2) and (5).

II. INITIAL EQUATION OF PROBLEM AND ITS SOLUTION

Having Equations (1), (2) and (5) we reduce them to the solution of the integral-differential equation. With combining Equations (1) and (2), we find

$$\frac{1}{\rho_f}\frac{\partial^2 p}{\partial x^2}-\frac{2}{R}\frac{\partial^2 w}{\partial t^2}=0 \tag{6}$$

Calculating by Equation (5) and substituting the result obtained in (6), we have

$$\frac{\rho_*}{\rho_f} h \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{G}{\rho_f} \frac{\partial^3 w}{\partial x^2 \partial t} - \\ -\frac{G}{\rho_f} \int_{-\infty}^{t} G_0(t-\tau) \frac{\partial^3 w}{\partial w^2 \partial \tau} d\tau + \\ + \frac{E}{\rho_f} \frac{h}{R^2(1-\nu^2)} \frac{\partial^2 w}{\partial x^2} - \frac{2}{R} \frac{\partial^2 w}{\partial t^2} = 0 \tag{7}$$

We introduce the following notation

$$\frac{E}{2\rho_f} = c_0^2, \quad \frac{h}{R(1-\nu^2)} = \eta, \quad \frac{\rho_*}{\rho_f} = \rho$$

after a series of elementary transformations, we obtain the following integral-differential equation with respect to the deflection function w

$$\rho \frac{Rh}{2} \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{RG}{E} c_0^2 \frac{\partial^3 w}{\partial x^2 \partial t} - \\ -\frac{RG}{E} c_0^2 \int_{-\infty}^{t} G_0(t-\tau) \frac{\partial^3 w}{\partial x^2 \partial t} d\tau + \\ + c_0^2 \eta \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial t^2} = 0 \tag{8}$$

The model used here generalizes the previously proposed ones and, in a number of cases, reduces to known ones. So taking in (8) $G_0(t-\tau) = 0$ we come to the case when the friction of the external medium is elastic. Taking $G = 0$ we reduce the influence of the environment. Then, acquiring dynamic effects, we obtain a formula for the propagation velocity of the Moens-Korteweg wave as

$$\overset{\approx}{c} = c_0 \sqrt{\eta}$$

III. RESOLVING EQUATION

We shall find the value of the deflection at which steady oscillations are possible. In this case, applying Fura's method of separation of variables, we find the particular solution of Equation (8) in the form of [3]

$$w(x,t) = y(x) \exp(i\omega t) \tag{9}$$

where, ω is a given real value of the angular frequency, and y is in general, a complex position coordinate function.

We first turn to the calculation of the integral term in (8). Due to Equation (9), having adopted $t - \tau = \theta$ and introducing the notation

$$\alpha = \int_0^{\infty} G_0(\theta) \exp(-i\omega\theta) d\theta \tag{10}$$

We get

$$-\frac{RG}{E} c_0^2 \int_{-\infty}^{t} G_0(t-\tau) \frac{\partial^3 w}{\partial x^2 \partial \tau} d\tau = \\ = -i \frac{RG}{E} c_0^2 \omega \alpha \exp(i\omega t) y'' \tag{11}$$

where the primes denote the ordinary derivative with respect to the coordinate x.

Finally, taking into account equalities (11) and (9) in Equation (8) and introducing into the considered dimensionless elastic rigidity parameter

$$g = \omega \frac{RG}{E}$$

After a reduction to the common time multiplier, we finally obtain

$$y'' + \frac{\omega^2}{-\rho \frac{Rh}{2} \omega^2 + i c_0^2 g (1-\alpha) + c_0^2 \eta} y = 0 \tag{12}$$

Analysis of the experimental data of the elastic module, Poisson's coefficients and densities for materials such as rubber, celluloid and water densities allows us to conclude that $\rho \approx 1$, for the long-wave approximation, the first term in the denominator (12) is negligibly small

$$\rho \frac{hR}{2} \omega^2 << c_0^2 \eta$$

These considerations allow us to conclude that the gravitational effects in the region of the cross section of the tube are small and Equation (5) is rewritten in the form

$$p = G \left\{ \frac{\partial w}{\partial t} - \int_{-\infty}^{t} G_0(t-\tau) \frac{\partial^3 w}{\partial x^2 \partial \tau} d\tau \right\} + \\ + \frac{Eh}{R^2(1-\nu^2)} w \tag{13}$$

Thus, Equation (12), with sufficient accuracy, can be approximately replaced by the following

$$y'' + \frac{\omega^2}{i c_0^2 g (1-\alpha) + c_0^2 \eta} y = 0 \tag{14}$$

Writing the dispersion relation by means of equality

$$\delta^2 = \frac{\omega^2}{c_0^2} \frac{1}{\eta + g(1-\alpha)} \tag{15}$$

we reduce Equation (13) to the form

$$y'' + \delta^2 y = 0 \tag{16}$$

IV. SOLUTION OF DISPERSION EQUATION

Representing α with $\alpha = \alpha_0 + i\alpha_1$ we express expression (15) as

$$\delta^2 = \frac{\omega^2}{c_0^2} \frac{1}{(\eta + g\alpha_1) + ig(1-\alpha_0)}$$

By the rule of extracting the square root of a complex number, we have

$$\delta = \delta_0 - i\delta_1$$

where,

$$\delta_0 = \sqrt{\frac{m+a}{2}}$$

$$\delta_1 = \sqrt{\frac{m-a}{2}}$$

$$m = \sqrt{a^2 + b^2}$$

$$\operatorname{Im} \alpha < 0$$

Here, in turn, for brevity we introduce the notation

$$a = \frac{\omega^2}{c_0^2}\frac{\eta + g\alpha_1}{(\eta + g\alpha_1)^2 + g^2(1-\alpha_0)^2} \quad (17)$$

$$b = \frac{\omega^2}{c_0^2}\frac{g(1-\alpha)}{(\eta + g\alpha_1)^2 + g^2(1-\alpha_0)^2} \quad (18)$$

Equation (16) must be supplemented with boundary conditions

$$y(0) = y_0, y \to 0 \text{ at } x \to \infty \quad (19)$$

We note that the value of the quantity y_0 will be determined in the future, assuming that the pressure p varies with the law $x = 0$

$$p(0,t) = p_0 \exp(i\omega t) \quad (20)$$

Taking into account conditions (19) and (17), the solution of Equation (16) is written in the form

$$w = y_0 \exp\left[i(\omega t - \delta x)\right] \quad (21)$$

Now from Equation (13) we can directly obtain an expression for the pressure

$$p = y_0\left\{i(1-\alpha)g\frac{E}{R} + \frac{2\rho_f}{R}c_0^2\eta\right\}\exp\left[i(\omega t - \delta x)\right] \quad (22)$$

By comparing relations (20) and (22), we obtain an expression for y_0 in the following form of

$$y_0 = \frac{p_0}{i(1-\alpha)g\frac{E}{R} + \frac{2\rho_f}{R}c_0^2\eta}$$

The equality obtained allows us to write down

$$p = p_0 \exp\left[i(\omega t - \delta x)\right] \quad (23)$$

$$w = \frac{p_0}{i(1-\alpha)g\frac{E}{R} + \frac{2\rho_f}{R}c_0^2\eta}\exp\left[i(\omega t - \delta x)\right] \quad (24)$$

Using equality (2), it is easy to compute the function u

$$u = i\frac{\delta}{\omega}p_0\frac{\frac{2}{R}(1-\alpha)gc_0^2 - i\frac{2}{R}c_0^2\eta}{i(1-\alpha)g\frac{E}{R} + \frac{2\rho_f}{R}c_0^2\eta}\exp\left[i(\omega t - \delta x)\right] \quad (25)$$

We note that, in view of the linearity of the problem, we are only interested in the real parts of the quantities (23) to (25).

V. NUMERICAL IMPLEMENTATION

First of all, you need to specify a difference core $G_0(t-\tau)$. For qualitative analysis, we set this function as follows

$$G_0(\theta) = g_0 = \text{const}$$

This equality makes it possible to determine α by Equation (10) as a function of g_0 and ω with

$$\alpha = -ik$$

where, $k = g_0\omega^{-1}$.

In this case $\alpha_0 = 0$, $\alpha_1 = -k$ and in place of (18), we have

$$a = \frac{\omega^2}{c_0^2}\frac{\eta - gk}{(\eta - gk)^2 + g^2}$$

$$b = \frac{\omega^2}{c_0^2}\frac{g}{(\eta - gk)^2 + g^2}$$

when,

$$g = k = 0$$

then,

$$\sqrt{\eta} = \frac{c}{c_0} \approx 0.259, \text{ and } \frac{c_0}{\omega}\delta_1 = 0.$$

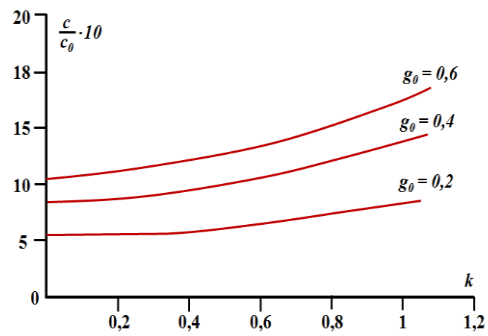

Figure 1. The dependence of the velocity from the value of k

Figure 1 shows dependence of the dimensionless velocity $\frac{c}{c_0} \approx \frac{\omega}{\delta_0 c_0}$ of the wave on the value of k, for different values of g_0, with the initial data of the problem: $\frac{h}{R} = 5\times10^{-2}$ and $\nu = 0.5$ which corresponds to the value $\eta \approx 0.067$.

VI. CONCLUSION

The performed calculations allow us to formulate the following conclusions:

1. With increasing k, the dimensionless wave velocity increases and for fixed k its value increases with increasing g_0;
2. With increasing k the damping increases; However, it decreases with increasing value of g_0.

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BIOGRAPHY

Reyhan Sayyad Akbarli was born in Neftchala, Azerbaijan on July 18, 1989. She works in Azerbaijan University of Architecture and Construction, Baku, Azerbaijan since 2012. She has published 10 scientific papers. Her scientific interests are theoretical mechanics, solid state mechanics and mechanics of composite materials and fluid mechanics.