International Journal on "Technical and Physical Problems of Engineering" (IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

December 2018	Issue 37	Volume 10	Number 4	Pages 31-35

PROBABILITY OF ERROR ESTIMATION FOR AN OPTICAL LINK USING PHASE DISTRIBUTION OF GAUSSIAN VORTEX BEAM

H.T. Eyyuboglu [1]   M. Ari [1]   M.C. Taplamacioglu [2]   H. Gozde [3]

1. Electrical and Electronics Engineering Department, Cankiri Karatekin University, Cankiri, Turkey, eyyubogluhalil@gmail.com, mari@karatekin.edu.tr
2. Electrical and Electronics Engineering Department, Gazi University, Ankara, Turkey, taplam@gazi.edu.tr
3. Elect. and Comm. Eng. Dept., Military Academy, National Defense University, Ankara, Turkey, hgozde@kho.edu.tr

Abstract- In this paper, the probability of error is evaluated for an optical link using the phase distribution correlation of Gaussian vortex beam. This evaluation is implemented in a computer environment, where the turbulent atmosphere is modelled as a series of random phase screens. The transmitted message signal consists of eight *M*ary levels and is encoded into the topological charge of the source vortex beam. The detection technique employed is based on correlating the aperture confined and topological charge related phase distribution of the received beam with those of the free space equivalents. It is seen that this technique performs well in weak turbulence, degrading as moderate turbulence levels are approached. It is anticipated that the obtained results will be beneficial to optical links incorporating the use of Gaussian vortex beams.

Keywords: Probability of Error, Vortex Beam, Atmospheric Turbulence.

I. INTRODUCTION

Due to their information carrying capability in the topological charge (TC), vortex beams have become quite popular recently. In this scheme, the symbols of the electrical message signal are encoded into the different TC or correspondingly optical angular momentum (OAM) values at the transmitter side of an optical communication link. Then the receiver will make an attempt to decode the messaged embedded in the TC, a process which is impaired by the presence of atmospheric turbulence. The recovery of the TC value is enabled by examining the phase distribution of the vortex beam. There are several works the literature on this topic, some of them being cited below.

The effects of perturbing and misalignment of the Laguerre Gaussian vortex beam carrying information encoded OAM states are analysed at details in [1]. In another study [2], vortex beam propagation through atmospheric turbulence and TC conservation is examined and it is found that increasing the azimuthal order of the beam, and increasing the receiver aperture size will contribute positively to this act of conservation.

A successful pre and post turbulence compensation is implemented in [3] by utilizing a Gaussian beam to achieve a crosstalk reduction of 12 dB in an optical link employing OAM multiplexing. In [4], for a communication link using OAM multiplexing, wavefront correction is applied to mitigate the turbulence effects, simultaneously channel coding is also employed to further enhance the performance. Using a probe Gaussian beam and Gerchberg-Saxton algorithm, pre-turbulence compensation is implemented for multiplexed OAM beams. Recently formulations and assessments were made on the performance bounds of an optical communication system using irradiance profile modulation based on the information carrying capability of the TC of the various vortex beams [5].

In the present study, for the detection of the TC carrying the transmitted symbol, a detection technique based on correlating the aperture limited and the TC related phase distribution with those of free space equivalents is proposed and partially evaluated.

II. PHASE DISTRIBUTIONS ON SOURCE AND RECEIVER PLANES IN FREE SPACE

The source field of a Gaussian vortex beam possessing a TC of m_s can be expressed as

$$\begin{aligned} u_s(s,\phi_s) &= \left(\frac{s}{\alpha_s}\right)^{|m_s|} \exp\left(-\frac{s^2}{\alpha_s^2}\right) \exp\left(-jm_s\phi_s\right) \\ u_s(s_x,s_y) &= \left[\frac{s_x + j\,\mathrm{sign}(m_s)\,s_y}{\alpha_s}\right]^{|m_s|} \exp\left(-\frac{s_x^2+s_y^2}{\alpha_s^2}\right) \end{aligned} \qquad (1)$$

where, the first line is written in the radial coordinate representation of (s,ϕ_s), whereas the second line is in the Cartesian coordinate representation of (s_x,s_y). In Equation (1), $|m_s|$ means the absolute value of m_s with m_s referring to the TC value, $\mathrm{sign}(m_s)=\pm1$ depending on m_s being a positive or a negative integer, α_s is the Gaussian source size.

After propagating an axial distance of z in free space (i.e. in the absence of turbulence), the receiver field expression of the Gaussian vortex beam of Equation (1) is found with the help of Huygens Fresnel integral [6] and will be given by

$$u_r(r,\phi_r) = (-1)^{|m_s|}(-jk)^{|m_s|+1} r^{|m_s|}\alpha_s^{|m_s|+2}\left(\frac{1}{2z - jk\alpha_s^2}\right)^{|m_s|+1} \times \\ \times \exp(jkz)\exp(-jm_s\phi_r)\exp\left(-\frac{jkr^2}{jk\alpha_s^2 - 2z}\right) \quad (2)$$

where, (r,ϕ_r) are the radial receiver plane coordinates and $k = 2\pi/\lambda$ is the wave number, with λ being the wavelength of the optical source.

The phase distributions of the source and the received fields will respectively be

$$\Phi_s(s,\phi_s,m_s) = \tan^{-1}\left\{\frac{\mathrm{Im}[u_s(s,\phi_s)]}{\mathrm{Re}[u_s(s,\phi_s)]}\right\} = -m_s\phi_s$$

$$\Phi_r(r,\phi_r,m_s) = \tan^{-1}\left\{\frac{\mathrm{Im}[u_r(r,\phi_r)]}{\mathrm{Re}[u_r(r,\phi_r)]}\right\} = \\ = -(|m_s|+1)\frac{\pi}{2} + (|m_s|+1)\tan^{-1}\left(\frac{k\alpha_s^2}{2z}\right) + \\ + \frac{2kzr^2}{4z^2 + k^2\alpha_s^2} + kz - m_s\phi_r \quad (3)$$

Bearing in mind that, the transmitted message signal will be embedded into the TC, it is then instructive to examine the phase distributions on the source and receiver planes against the various values of the TC, m_s. It is assumed that in the alphabet of message signal, there are eight *M*ary symbols. This way, it is appropriate to adopt the symbol alphabet for m_s as done in spread spectrum systems and continuous phase modulation [7, 8], thus

$$m_s = \{m_{-7},\ m_{-5},\ m_{-3},\ m_{-1},\ m_1,\ m_3,\ m_5,\ m_7\} = \\ = \{-7,\ -5,\ -3,\ -1,\ 1,\ 3,\ 5,\ 7\} \quad (4)$$

By taking $\alpha_s = 1$ cm, $\lambda = 1.55$ μm, $z = 3$ km, Figure 1 displays the phase distributions on source and receiver planes at $m_s = m_{-5} = -5$ and $m_i = m_5 = 5$. As seen from the subplots on the left hand side of Figure 1 and Equation (3), the phase distribution on the source plane has no radial coordinate, i.e., no s dependence. On the other hand, this is modified as we move to the receiver plane. Additionally it is seen from Figure 1 that there will be a change of rotation in the phase distribution when the sign of m_s is inverted.

III. DETECTION AND DECISION TECHNIQUE

An optical link operating in turbulent atmosphere is envisaged. In this setup, the source beam of Equation (1) is transmitted after being modulated with one of the m_s values selected from the alphabet of eight *M*ary symbols given in Equation (4).

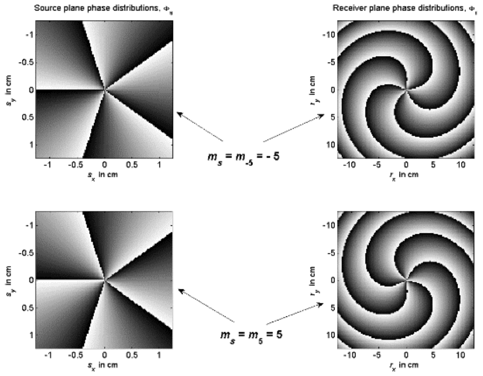

Figure 1. Source and receiver plane phase distributions with the topological charges of $m_s = m_{-5} = -5$ and $m_s = m_5 = 5$

This modulated beam passes through the turbulent atmosphere and arrives at the receiver. The primary task on the receiver side is to estimate as correctly as possible which m_s was sent from the transmitter. For this detection and decision process, the technique of correlating the received phase distribution against the free space phase distribution is used.

The setup that will perform this task is shown in Figure 2, which also illustrates the complete optical link as well as the simulation concept.

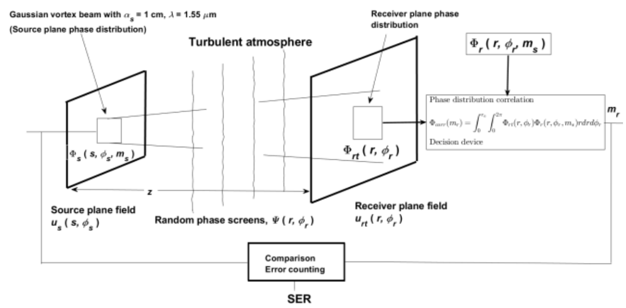

Figure 2. Block diagram illustration of the complete optical link and simulation concept

To this end, if the received field of the turbulent atmosphere is denoted as $u_{rt}(r,\phi_r)$, then in Rytov representation [6], this can be written in terms of the free space expression of Equation (2) in the following way.

$$u_{rt}(r,\phi_r) = u_r(r,\phi_r)\exp[\psi(r,\phi_r)] \quad (5)$$

where, $\psi(r,\phi_r)$ is the random complex phase arising from the atmospheric turbulence of the propagation environment. From Equation (5), the corresponding phase distribution of the receiver field can be extracted as

$$\Phi_{rt}(r,\phi_r) = \tan^{-1}\left\{\frac{\mathrm{Im}[u_{rt}(r,\phi_r)]}{\mathrm{Re}[u_{rt}(r,\phi_r)]}\right\} \quad (6)$$

Now by carrying out the following correlation

$$\Phi_{corr}(m_s) = \int_0^{r_a}\int_0^{2\pi} \Phi_{rt}(r,\phi_r)\,\Phi_r(r,\phi_r,m_s)\, r dr d\phi_r \quad (7)$$

and spanning over all possible values of m_s listed in Equation (4), it is possible to arrive at a decision by selecting the $\Phi_{corr}\left(m_s\right)$ which has the maximum value. Such a decision can be formulated as

$$\begin{aligned} m_r &\equiv \max\left[\Phi_{corr}\left(m_s\right)\right]\equiv \\ &\equiv \max\left[\Phi_{corr}\left(m_s=m_{-7}\right),\ \Phi_{corr}\left(m_s=m_{-5}\right),\right. \\ &\qquad \Phi_{corr}\left(m_s=m_{-3}\right),\ \Phi_{corr}\left(m_s=m_{-1}\right), \\ &\qquad \Phi_{corr}\left(m_s=m_{1}\right),\ \Phi_{corr}\left(m_s=m_{3}\right), \\ &\qquad \left.\Phi_{corr}\left(m_s=m_{5}\right),\ \Phi_{corr}\left(m_s=m_{7}\right)\right] \end{aligned} \tag{8}$$

which means that m_r is to be equated to m_s of $\max\left[\Phi_{corr}\left(m_s\right)\right]$. As seen from Equation (7), the correlation is performed over a limited aperture opening of radius r_a rather than the full one. The detection and decision technique defined in Equations (7) and (8) is similar to the correlation metrics evaluations described in [8, 9]. A pictorial representation of the correlation operation implemented in Equation (7) is given in Figure 3. As also seen from this figure, for computational convenience, the numeric equivalence of the integration of Equation (7) is over a square aperture instead of a circular one.

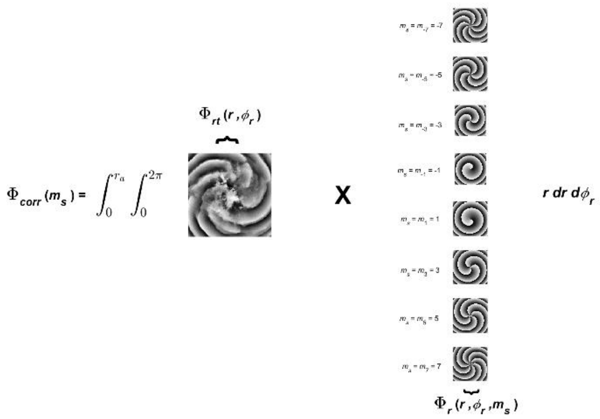

Figure 3. Pictorial representation of the correlation operation in Equation (7)

IV. PROBABILITY OF ERROR RESULTS AND DISCUSSIONS

As seen from the left hand side of Figure 3, after passing through the turbulent atmosphere, the phase distribution on the receiver plane, $\Phi_{rt}\left(r,\phi_r\right)$ becomes distorted. Hence the task of Equation (7) is to find to which free space equivalent, this distorted phase distribution correlates best.

This way it is possible to perform an estimation on the TC and correspondingly the message signal sent from the transmitter. It is shown in [10, 11], that it is possible to deal with the random events by analytic formulation as well as simulation runs. Here the random complex phase term, $\psi\left(r,\phi_r\right)$ of Equation (5), i.e. the effects of atmospheric turbulence has been modeled by random phase screens. The construction details of such modelling are given elsewhere [12]. Thus implemented simulation runs will deliver the desired symbol error rate (SER) results.

Upon setting $\alpha_s = 1$ cm, $\lambda = 1.55$ μm, $z = 3$ km, Figure 4 displays the SER results for a range of, $C_n^2 = 10^{-15}$ m$^{-2/3}$ to 10^{-14} m$^{-2/3}$, where C_n^2 is the structure constant indicating the strength of turbulence. From this figure, it is observed that as turbulence strength is increased, SER will also increase, as expected. As well known, reliable communication occurs below SER values of 10^{-3}. From the data cursor values of Figure 4, it is seen that the detection and decision technique proposed in Equations (7) and (8) is favorable only towards the low end of weak turbulence region. Unfortunately, it takes quite an incredible amount of computational time to evaluate the numeric values of SER below 10^{-3}.

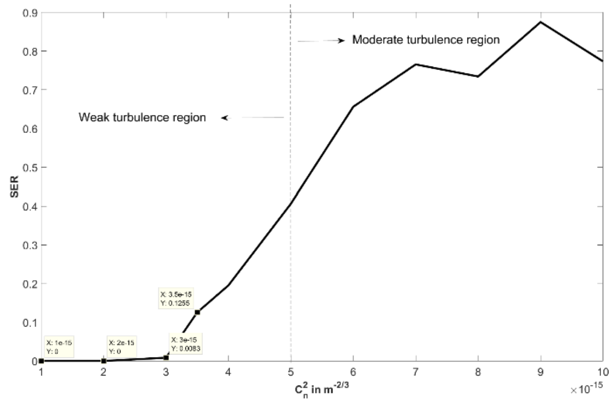

Figure 4. SER variation against C_n^2

V. CONCLUSIONS

For an optical link operating in atmospheric turbulence and using the topological charge of the Gaussian vortex beam as the information carrier, the performance of a detection and decision technique based on correlating the phase distribution in the receiver beam with those of the free space ones is analyzed. The initial findings show that such a technique yields favorable results at the low end of weak turbulence region.

Extending this operating region to moderate and even strong levels by applying correction to the distorted phase distributions will be the subject of future work.

NOMENCLATURES

A. Acronyms

OAM	Optical Angular Momentum
SER	Symbol Error Rate
TC	Topological Charge

B. Parameters

α_s : Gaussian source size
k : Wave number
λ : Wavelength of source beam
m_s : Topological charge of source beam
m_r : (Detected) Topological charge of received beam
s,ϕ_s : Radial source plane coordinates

s_x, s_y : Cartesian source plane coordinates
r, ϕ_r : Radial receiver plane coordinates
$u_s(s, \phi_s)$: Source field expression of Gaussian vortex beam
$u_r(r, \phi_r)$: Receiver field expression of Gaussian vortex beam
z : The axial propagation distance between the source and receiver planes
C_n^2 : Structure constant (of atmospheric turbulence)
$\Phi_s(s, \phi_s, m_s)$: Phase distribution on source plane
$\Phi_r(r, \phi_r, m_s)$: Phase distribution on receiver plane in free space conditions
$\Phi_{rt}(r, \phi_r)$: Phase distribution on receiver plane in atmospheric turbulence
$\Phi_{corr}(m_s)$: Result of correlation integral

REFERENCES

[1] G. Gibson, J. Courtial, M.J. Padgett, "Free-Space Information Transfer Using Light Beams Carrying Orbital Angular Momentum", Optics Express, Vol. 12, No. 22, pp. 5448-5456, Nov. 2004.
[2] G. Gbur, R.K. Tyson, "Vortex Beam Propagation Through Atmospheric Turbulence and topological Charge Conservation", Journal of Optical Society of America A, Vol. 25, Issue 1, pp. 225-230, Jan. 2008.
[3] Y. Ren, G. Xie, H. Huang, N. Ahmed, Y. Yan, L. Li, C. Bao, M.P.J. Lavery, M. Tur, M.A. Neifield, R.W. Boyd, J.H. Shapiro, A.E. Willner, "Adaptive-Optics-Based Simultaneous Pre- and Post-Turbulence Compensation of Multiple Orbital-Angular-Momentum Beams in a Bidirectional Free-Space Optical Link", Optica, Vol. 1, Issue 6, pp. 376-382, Dec. 2014.
[4] S. Zhao, L. Wang, L. Zou, L. Gong, W. Cheng, B. Zheng, H. Chen, "Both Channel Coding and Wavefront Correction on the Turbulence Mitigation of Optical Communications Using Orbital Angular Momentum Multiplexing", Optics Communications, Vol. 376, pp. 92-98, 2016.
[5] H.T. Eyyuboglu, "The Performance Bounds of an Optical Communication System Using Irradiance Profile Modulation", Journal of Modern Optics, Vol. 64, Issue 20, pp. 2110-2116, 2017.
[6] L.C. Andrews, R.L. Phillip, "Laser Beam Propagation through Random Media, 2nd ed." SPIE Press, Bellingham, Washington, USA, 2005.
[7] F. Xiong, "Digital Modulation Techniques", Artech House, 2002.
[8] J.G. Proakis, M. Salehi, "Communication Systems Engineering", 2nd Ed., Prentice Hall, Upper Saddle River, New Jersey, USA, 2002.
[9] http://ece376.cankaya.edu.tr/uploads/files/ECE376_%20Dimensionality%20of%20Signals_ASK_PSK_QAM_FSK_Jan%202013_HTE.pdf.
[10] M. Ari, H.H. Goktas, M.C. Taplamacioglu, H. Gozde, "A Model for Optimization of Soliton Based Communication Systems in the View of Renewable Energy Research", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 28, Vol. 8, No. 3, pp. 36-41, September 2016.
[11] B. Baydar, H. Gozde, M. Ari, M.C. Taplamacioglu, "A Research on Evolutionary Computation Techniques in Optimal Power Flow Solution", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 33, Vol. 9, No. 4, pp. 26-33, December 2017.
[12] H.T. Eyyuboglu, "Bit Rate Analysis of Gaussian, Annular Gaussian, Cos Gaussian and Cosh Gaussian Beams with the Help of Random Phase Screen", Applied Optics, Vol. 53, Issue 17, pp. 3758-3763, 2014.

BIOGRAPHIES

Halil Tanyer Eyyuboglu was born in Elazig, Turkey, 1953. With the government educational grant, he received his B.Sc. degree in Electronics Engineering from Birmingham University, UK in 1976 and M.Sc. degree in Digital Communications and Ph.D. degree in Optical Fibres from Loughborough University, UK in 1977 and 1982, respectively. Returning to Turkey in 1982, he took up several positions in Turkish PTT, eventually serving as the R&D Director between 1990 and 1995. He was the Ankara Regional Director in Telsim, then the second largest GSM operator of Turkey during 1999 to 2001. Between 2002 and 2018, he was an academician in Cankaya University, Ankara, Turkey. Currently, he a Professor in Cankiri Karatekin University, Cankiri, Turkey. He has over 130 SCI publications mainly in the area of optical propagation and communication in turbulent atmosphere.

Murat Ari received the B.Sc. degree from Department of Electrical and Electronics Engineering, Engineering Faculty, Middle East Technical University, Ankara, Turkey. He received the M.Sc. and Ph.D. degrees from Department of Electrical and Electronics Engineering, Gazi University, Ankara, Turkey. He is a Professor of the Electrical and Electronics Engineering since 2017 and Dean of Engineering Faculty and Vice Rector of Cankiri Karatekin University, Cankiri, Turkey. His research interests include optical fiber commutations and its applications, electromagnetic fields and using and evaluation of video conference platform.

M. Cengiz Taplamacioglu graduated from Department of Electrical and Electronics Engineering, Gazi University, Ankara, Turkey. He received the M.Sc. degrees in Industrial Engineering from Gazi University and also in Electrical and Electronics Engineering from Middle

East Technical University, Ankara, Turkey. He received his Ph.D. degree in Electrical, Electronics and System Engineering from University of Wales, Cardiff, UK. He is a Professor of the Electrical and Electronics Engineering since 2000. His research interests and subjects are high voltage engineering, corona discharge and modelling, electrical field computation, measurement and modelling techniques, optical HV measurement techniques, power systems control and protection, lighting techniques, renewable energy systems and smart grid applications.

Haluk Gozde received the B.Sc. degree in Electrical and Electronics Engineering from Karadeniz Technical University, Trabzon, Turkey in 1997. He received the M.Sc. and the Ph.D. degrees in Electrical and Electronics Engineering from Gazi University, Ankara, Turkey in 2004 and 2010, respectively. He is an Assoc. Prof. of the Electrical and Electronics Engineering since 2016. His main research area consists of power system dynamics and control, renewable energy systems, smart grid technologies, artificial intelligence-based control methods, and swarm intelligence-based optimization algorithms.