International Journal on "Technical and Physical Problems of Engineering" (IJTPE)
Published by International Organization of IOTPE
ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com
December 2019 Issue 41 Volume 11 Number 4 Pages 77-80

BEHAVIOUR OF LASER BEAM IN NONLINEAR MEDIA

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Abstract- In this paper the laser beam and its parameters are studied. Also the optical Kerr effect is discussed. Refractive index of the material has dependence on the intensity of the electric field. The laser beam described as Gaussian beam has higher intensity at the center comparing to edge of the beam so the refractive index is increased at the beam center. However, the laser field intensity is gradually decreased through the beam edge, resulting decrement of the refractive index at the beam edge. This phenomenon causes to self-focusing of the beam in the nonlinear medium. Gaussian beam parameters and the derivation of the optical Kerr effect are studied through the paper.

Keywords: Gaussian Laser Beam, Nonlinear Optics, Kerr Effect, High Energy Laser Field, Refractive Index, Nonlinear Refractive Index.

I. INTRODUCTION

In laser physics, laser beam is an electromagnetic radiation, and the electric field amplitude of the radiation is described by the Gaussian function. The laser beams are usually described in the form of the Gaussian beam. The divergence of the Gaussian beam is small, and the paraxial approximation can be applied for its solution. This approximation leads that the Gaussian beam remains Gaussian in free space. Gaussian beams have high beam quality, and they have special properties. For example, they remain Gaussian when they pass through optical elements, and there is no beam distortion. Also, the beam parameters do not change after passing through an optical element [1]. The response of the Gaussian beam interacting with materials has been studied for several researchers [2-5].

The square of the electric field amplitude (E^2) corresponds to intensity (I) of the laser beam, which induces the refractive index. This phenomenon is known as the Kerr effect. It has an important role in nonlinear optics. It causes self-focusing in medium, and it has application area for generating of short laser pulses. The optical Kerr effect can find applications in broad areas including spectroscopy of liquids [6], design of the waveguides having different refractive index [7]. It can also find applications in electro optical devices [8-10].

The intensity of the Gaussian beam is higher at the beam center, and it gradually decreases toward to beam edge. The laser beam passing through a nonlinear medium focuses more at the beam center than the beam edge since the intensity of the optical beam is higher at the center of the beam, and it causes the variation of the nonlinear refractive index.

In this paper, the Gaussian beam and the nonlinear optical Kerr effect are discussed. The laser beam propagation is described in terms of the laser beam parameters, which give information about the beam waist, spot size, radius of the curvature and Gouy phase. Moreover, nonlinear optical Kerr effect is analyzed, and change of the refractive index with the variation of the laser intensity is studied. The formulations lead to general understanding of the Gaussian beam and the nonlinear optics.

II. DESCRIPTION OF LASER BEAM PARAMETERS

The Maxwell equations in free space is described in the form of an Equation called the Helmholtz equation [11]

$$(\nabla^2 + k^2)E = 0 \tag{1}$$

where, E is the electric field amplitude, and $k = n\omega / c$. The field amplitude can be described as propagation of the field along the z direction, and it can be written as [11]

$$E = \psi e^{-ikz} \tag{2}$$

where, ψ is the variation of the field in propagation. If we put Equation (2) in Equation (3), it takes the form as

$$\nabla E = (\nabla \psi)e^{-ikz} - ik\psi e^{-ikz}\hat{z} \tag{3}$$

$$\nabla^2 E = (\nabla^2 \psi)e^{-ikz} - 2ik(\nabla \psi).e^{-ikz}\hat{z} - k^2\psi e^{-ikz} \tag{4}$$

Then, one can have

$$(\nabla^2 + k^2)E = (\nabla^2\psi - 2ik.\partial\psi / \partial z)e^{-ikz} = 0 \tag{5}$$

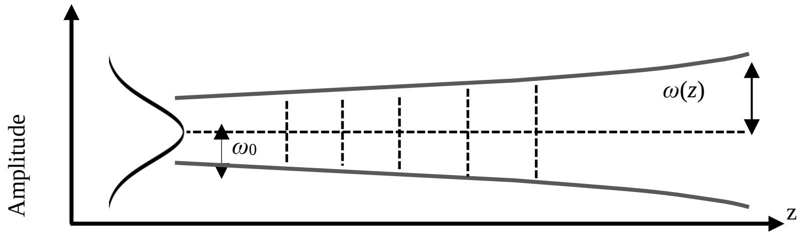

Figure 1. Propagation of the beam along the *z* direction. Field amplitude slowly changes along the *z* direction [11]

The amplitude ψ changes slowly with the propagation direction, and the wave front is normal to the propagation direction as presented in Figure 1. Then, the Helmholtz equation takes the form of

$$(\nabla_T^2 - 2ik\frac{\partial}{\partial z})\psi = 0 \tag{6}$$

The Equation (6) is called paraxial wave equation, and here it is described as $\nabla^2{}_T = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ [11]

The paraxial wave equation can be solved in cylindrical coordinates, and the fundamental mode of the Gaussian beam can be written as [11]

$$\frac{E}{E_0} = \frac{\omega_0}{\omega(z)} e^{(\frac{-r^2}{\omega^2(z)})} e^{(\frac{-ikr^2}{2R(z)})} e^{(-i\{kz-\phi(z)\})} \tag{7}$$

where, ω_0 is the beam waist, and it is given as $\omega_0 = \sqrt{\frac{2z_R}{k}} = \sqrt{\frac{\lambda_0 z_R}{n\pi}}$, $\omega(z)$ is called as spot size, and it is given as $\omega(z) = \omega_0\sqrt{1+(\frac{z}{z_R})^2}$ and $R(z)$ given as, $R(z) = z(1+(\frac{z_R}{z})^2)$ is called radius of the curvature. Lastly, the ϕ is called Gouy phase shift and is written as $\phi(z) = \tan^{-1}(\frac{z}{z_R})$. Theoretical demonstration of the Gaussian beam shape is shown in Figure 2.

Above equations are used to determine the properties of the Gaussian beam. Intensity of the optical field is $I(r) = |E(r)^2|$, and the intensity is a function of the axial and radial distance z and $r^2 = x^2 + y^2$. Then, the intensity of the Gaussian beam is given as [11]

$$I = I_0\left(\frac{\omega_0}{\omega(z)}\right)^2 e^{\left(-2r^2/\omega^2(z)\right)} \tag{8}$$

For the value of z, intensity is a Gaussian function. The Gaussian function has maximum value at $r=0$, and it drops monotonically with increasing r, as Figure 2.

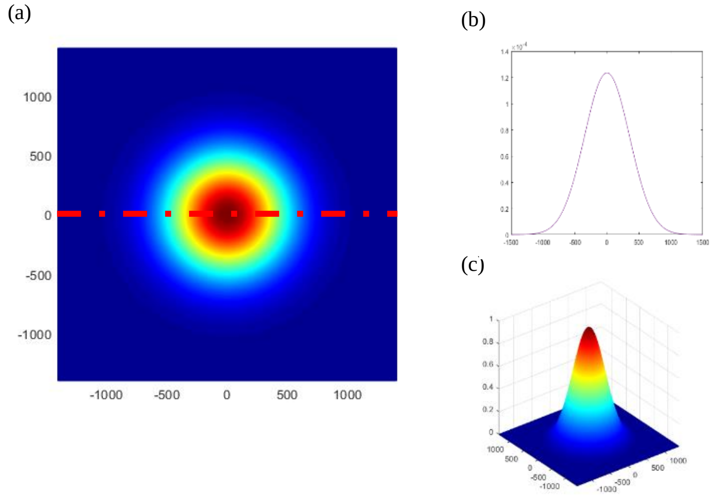

Figure 2. (a) Theoretical demonstration of the Gaussian beam profile, (b) Intensity distribution along the red dashed line in (a), (c) Gaussian distribution of the beam (a)

III. OPTICAL FIELD INTENSITY DEPENDENT NONLINEAR REFRACTIVE INDEX

Nonlinear optics is the study of the behavior of light-matter interactions when the response of the material is a nonlinear function of the electromagnetic field.

The interaction of an optical field with the nonlinear medium can be analyzed in terms of the nonlinear polarization. Electric field polarization, P can be written in terms of the electric field amplitude, E [12, 13].

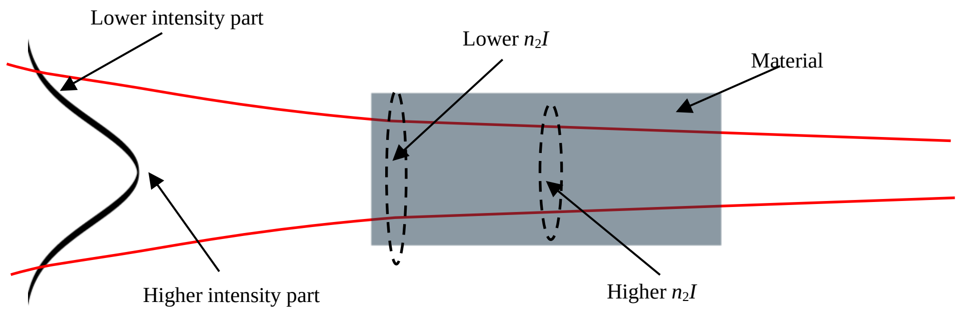

Figure 3. Refractive index modification due to high beam intensity [14]

$$P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + ... \quad (9)$$

where, ε_0 is the vacuum permittivity and χ^n is the nth order electric susceptibility of the medium. If the electric field is accepted in the form of $E(t) = E(\omega) e^{-i\omega t}$, the medium's total polarization is given as [12, 13]

$$P(\omega) = \varepsilon_0 \chi^{(1)} E(\omega) + 2\varepsilon_0 \chi^{(2)} E(\omega)^2 + +3\varepsilon_0 \chi^{(3)} |E(\omega)|^3 + ... \quad (10)$$

Complete explanation of the above equation can be found in [14]. First two terms in the above equation is known as linear electro optic Pockels effect. This effect results the electric field change in the refractive index [12]. The third term of the above equation gives the study of the optical Kerr effect. So Equation (10) can be simplified to

$$P(\omega) \sim \varepsilon_0 \chi^{(1)} E(\omega) + 3\varepsilon_0 \chi^{(3)} |E(\omega)|^2 E(\omega) \equiv \varepsilon_0 \chi_{eff} E(\omega) \quad (11)$$

where, the effective susceptibility can be described as $\chi_{eff} = \chi^{(1)} + 3\varepsilon_0 \chi^{(3)} |E(\omega)|^2$.

Refractive indexes of many materials change with the varying beam intensity. The optical Kerr effect occurs when intense optical field passes through a medium.

The physics of this effect is a nonlinear polarization generated in the medium so the propagation of the light is modified. The total refractive index at high beam intensities is given as [13]

$$n = n_0 + n_2 I \quad (12)$$

where, n_0 is weak field refractive index, and n_2 is the second order nonlinear refractive index, and the I is the optical field intensity. It increases with increasing optical intensity. The value of the nonlinear refractive index has smaller value, which is on the order of ~ 10^{-14} W/cm^2 and the intensity of the optical field is ~10^{14} W/cm^2. The high optical field intensity compensates the second order nonlinear refractive index so the total refractive index is modified. The relation between the nonlinear susceptibility (χ^3) and the nonlinear refractive index (n_2) is that $n^2 = 1 + \chi_{eff}$. So the linear and the nonlinear refractive indexes can be written as a function of nonlinear susceptibilities [12, 13] $n_0 = \sqrt{(1 + \chi^{(1)})}$ and $n_2 = \dfrac{3\chi^{(3)}}{4n_0}$, respectively.

IV. DISCUSSION

The intensity of the laser beam is usually on the order of 10^{14} W/cm^2. The optical Kerr effect is affected by the beam intensity. The response of the nonlinear medium plays an important role. The nonlinear refractive index increases with the increasing laser electric field [15]. At the lower beam intensities, the total refractive index does not depend on the nonlinear refractive index, which is ignored at low beam intensities. However, it increases as the field intensity increases, in which the nonlinear refractive index added to total refractive index term. That leads to self-focusing of the beam while passing through the medium. In addition, the intensity of the beam is higher in the inert part of the beam, and the intensity at the beam edge decreases gradually (Figure 2).

Thus, the contribution of the refractive index of the material is higher at the beam center due to optical Kerr effect. The modified refractive index behaves like a lens, resulting self-focusing of the beam. Figure 3 shows that the beam passing through a nonlinear material is self-focused due to refractive index variation along the material. The refractive index is independent of the light intensity at the low optical intensity, however the increase of the optical field intensity results that the nonlinear refractive index starts to depend on the electric field intensity. The intensity of the Gaussian beam is higher at the beam center, and thus change of refractive index leads to self-focusing of beam.

V. CONCLUSION

Laser-matter interaction results to different type of electro-optical effects if the medium response is a nonlinear function of the applied electric field. Gaussian wave equation is derived, and the laser parameters are discussed. The nonlinear optical Kerr effect is analyzed. Response of the medium at high field intensities leads to self-focusing of the beam. It has important properties to explain nonlinear effects. The Kerr effect is a nonlinear distortion of the electron in the material due to high optical power. Self-focusing is an important effect for laser engineering since the modification of the laser beam needs to be considered. In addition, the higher intensities damage the material, so the threshold intensities of the material must be checked. The refractive index of the beam is higher than the outside of the beam, so the beam is self-focused. The Gaussian beam and the optical Kerr effect have variety of applications areas, and they are carried out in many research fields, especially in photonics devices.

REFERENCES

[1] H. Kogelnik, T. Li, "Laser Beams and Resonators", Appl. Opt., No. 5, pp. 1550-1567, 1966.
[2] P.A. Belanger, "Beam Propagation and the ABCD Ray Matrices", Opt. Letters, No. 16, pp. 196-198, 1991.
[3] J.R. Pierce, "Modes in Sequences of Lenses", PNAS USA, No. 47, pp. 1808-1813, 1961.
[4] H. Kogelnik, "Imaging of Optical Mode-Resonators with Internal Lenses", Bell Sys. Tech. J., No. 44, p. 455, 1965.
[5] H. Kogelnik, "On the Propagation of Gaussian Beams of Light through Lenslike Media Including those with a Loss or Gain Variation", Appl. Opt., No. 4, pp. 1562-1569, 1965.
[6] Q. Zhong, J.T. Fourkas, "Optical Kerr Effect Spectroscopy of Simple Liquids", J. Phys. Chem. B, No. 112, pp. 15529-15539, 2008.
[7] R.H. Stolen, A. Ashkin, "Optical Kerr Effect in Glass Waveguide", App. Phys. Letters, No. 22, pp. 294-296, 1973.
[8] M. Qasymeh, M. Cada, S.A. Ponomarenko, "Quadratic Electro-Optic Kerr Effect: Applications to Photonic Devices", IEEE J. Quantum Elect., No. 44, pp. 740-746, 2008.
[9] F.J. Diaz, G. Li, C.M. de Sterke, B.T. Kuhlmey, S. Palomba, "Kerr Effect in Hybrid Plasmonic Waveguides", JOSA B, No. 33, pp. 957-962, 2016.
[10] T.Z. Shen, S.H. Hong, J.K. Song, "Electro-Optical Switching of Graphene Oxide Liquid Crystals with an Extremely Large Kerr Coefficient", Nat. Mater., No. 13, p. 394, 2014.
[11] B.E.A. Saleh, M.C. Teich, "Fundamentals of Photonics", Wiley-Interscience, 2007.
[12] K. Rottwitt, "Nonlinear Optics: Principles and Applications", 1st Ed., Boca Raton: CRC Press, 2015.
[13] R. Boyd, "Nonlinear Optics", 3rd Ed., San Diego, CA: Academic Press, 2003.
[14] G. New, "Introduction to Nonlinear Optics", Cambridge University Press, Cambridge, 2011.
[15] Z. Chang, "Fundamentals of Attosecond Optics", Boca Raton, FL, CRC Press, 2011.

BIOGRAPHIES

Muhammed Sayrac received his B.Sc. degree from the Department of Physics, Cumhuriyet University, Sivas, Turkey in 2009. In 2009, he was awarded a scholarship by the Turkish Ministry of National Education to pursue the Master and the Ph.D. degrees in the USA. He enrolled the Department of Physics and Astronomy in Texas A&M University, USA, where he earned his M.Sc. and Ph.D. degrees in 2013 and 2017, respectively. He joined the Physics Department, Cankiri Karatekin University, Cankiri, Turkey in 2017. His research interests include ultrafast lasers and spectroscopy, nonlinear optics, high harmonic generation, X-ray spectroscopy, ultrafast optics, attosecond science, and femtosecond lasers.

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