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FREE VIBRATIONS OF LONGITUDINALLY STIFFENED SOLID CONICAL SHELL WITH OPTING ASSOCIATED MASS IN VISCOUS ELASTIC MEDIUM

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Abstract- Conical shells are most used in aircrafts and also engineering. We consider a circular closed, solid, viscous-elastic medium-filled conical shell of invariable layer, usually stiffened with longitudinally ribs, supporting shell associated mass in opposite as diametrically points as meaning of two springs of the same rigidity as was shown in Figure 1. The main problem of determination of natural frequency of vibrations of such a shells problems are solved in linear statement by the energetic ways. The discrete arrangement of stiffening longitudinal ribs are taken into account. To construct the frequency equation, the second-order Lagrange equation is used. The frequency equation is solved numerically and characteristic curves of dependence are constructed.

Keywords: Vibration, Conical Shell, Viscous-Elastic Medium, Longitudinal Ribs, Associated Mass.

I. INTRODUCTION

One of the papers on investigation of stability of conical shells was Kh.M. Mushtairs paper [1]. In the papers [2, 3], the equation of motion was obtained for conical shells stiffened with rigidity ribs under linear-elastic deformation with regard to lateral shear. The mathematical model of deformation of stiffened orthotropic shells of general form based on the function of total energy of deformation was represented in [4].

The paper [5] was devoted to the construction of the mathematical model of deformation of conical shell-type constructions based on the functional of total energy of deformation with regard to orthotropy of the material, geometrical non-linearity and also lateral shear. In the paper [6] around closed, truncated medium-contacting conical shell of invariable layer usually starch with cross ribs supporting shell associated mass in diametrically opposite points by means of two springs of the same rigidity, is considered the problem of determination of natural vibrations of frequencies of such a shell was solved in linear statement by the energetic method.

A similar method was solved in [7] for longitudinally stiffened truncated conical shells. In this paper [8], the inhomogeneity was taken into account by accepting the Young modulus and the density of the material as a function of coordinate changing in thickness. In [9], the free oscillation of a longitudinally strengthened, orthotropic, moving fluid-contacting cylindrical shell inhomogeneous in thickness, is studied. Using the Hamilton-Ostrogradsky variational principle, the systems of equations of motion of a longitudinally strengthened, orthotropic moving-fluid-contacting cylindrical shell inhomogeneous in thickness, is constructed.

II. PROBLEM STATEMENT

Such a system of coordinates was accepted; inconstant radius r and also dihedral angle φ between diametrical planes, the potential energy of the shell is calculated by means of the following expression [10]:

$$\Pi_1=\frac{Eh}{12(1-\nu^2)}\int_0^{r_1}\int_0^{2\pi}\left(\varepsilon_1^2+\varepsilon_2^2+2\nu\varepsilon_1\varepsilon_2+\frac{1-\nu}{2}\psi^2\right)\times$$
$$\times\frac{rdrd\phi}{\sin\gamma}+\frac{D}{2}\int_0^{r_1}\int_0^{2\pi}\left(\chi_1^2+\chi_2^2+2\nu\chi_1\chi_2+2(1-\nu)\tau^2\right)\frac{rdrd\phi}{\sin\gamma}$$

where in accordance with [14] it is accepted:

$$\varepsilon_1=\frac{\partial u}{\partial r}\sin\gamma,\ \varepsilon_2=\frac{u}{r}\sin\gamma+\frac{\partial\upsilon}{r\partial\phi}+\frac{w}{r}\cos\gamma,$$

$$\psi=\frac{\partial u}{r\partial\psi}+\frac{\partial\upsilon}{\partial r}\sin\gamma-\frac{\upsilon}{r}\sin\gamma,$$

$$\chi_1=-\frac{\partial^2 w}{\partial r^2}\sin^2\gamma;\quad \chi_2=-\frac{u}{r^2}\sin\gamma\cos\gamma-\frac{w}{r^2}\cos^2\gamma-$$
$$-\frac{\partial^2 w}{r^2\partial\phi^2}-\frac{\partial w}{r\partial r}\sin^2\gamma;\quad \tau=-\frac{\cos\gamma}{r}\left(\frac{\partial u}{r\partial\phi}-\frac{\partial\upsilon}{\partial r}\sin\gamma+\frac{\upsilon}{r}\sin\gamma\right)-$$
$$-\frac{2\sin\gamma}{r}\left(\frac{\partial^2 w}{\partial r\partial\phi}-\frac{\partial w}{r\partial\phi}\right)$$

where, E is elasticity modulus; h is shell layer; ν is a Poisson ratio; r_1, r_2 are the greatest radius and also smaller bases of the shell; γ is an angle as we know between the generatrix also axis of the husk; u, υ, w are the components displacement vector of the points of the median surface of the shell along the generatrix, in tangential direction and also along the normal main formula to the median surface; $D=Eh^3/12\left(1-\nu^2\right)$.

The Π_2 the potential energy of deformation of longitudinal ribs equals [11]:

$$\Pi_2 = \frac{1}{2}\sum_{i=1}^{k_1}\int_0^{r_1}\left[EF_1\left(\frac{\partial u}{\partial r}\sin\gamma\right)^2 + EI_1\left(\frac{\partial^2 w}{\partial r^2}\sin^2\gamma\right)^2 + \right.$$
$$\left. + E\tilde{I}_1\left(\frac{\partial^2 \vartheta}{r^2\partial\phi^2}\right)^2 + GI_{1kp}\left(\frac{\partial^2 w}{r\partial r\partial\phi}\sin\gamma\right)^2\right]_{\phi=\phi_i}\frac{dr}{\sin\gamma}$$

where F_1, I_1, $\tilde{I}_1$, I_{1kp} are area and also moments of the tia of the cross section of the longitudinal bar with respect to tangential and also radial axes respectively and also torsion inertia moment of; G is a shear modulus, k_1 is the number of longitudinal bars; ϕ_i are coordinates of their arraignment.

The kinetic energy, T_1 of longitudinal ribs are calculated by means of the expression:

$$T_1 = \frac{\gamma_1 h}{2g}\int_0^{r_1}\int_0^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2\right]\frac{rdrd\phi}{\sin\gamma} +$$
$$+ \frac{\gamma_1 F_1}{2g}\sum_{i=1}^{k_1}\int_0^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2\right]_{\phi=\phi_i}\frac{dr}{\sin\gamma}$$

in which γ_1 is shells particular gravity of the material and also ribs; g is gravity acceleration.

The influence of medium on the shell is determined of eternal surface loads applied to the shell and also is calculated as a work performed by these loads when taking their system from the deformed state to the initial nondeformed are and also is represented in the form:

$$A = \int_0^{r_1}\int_0^{2\pi} q_r rdrd\phi \qquad (1)$$

We will take into account the motion of mass only in the level of cross-section along the axis z (Figure 1). The motion of the mass from this plane and also deformation of springs caused by displacement of points of their fastening to the husk, in the command of displacement vector u is not taken into account. Since the problem is solved by our method like in linear method statement, i.e., under the assumption that displacement of points of the deformed system is small, such assumptions are valid.

The potential energy Π_3 of the springs and also kinetic energy T_2 of the mass are equal to $T_2 = 0.5M\dot{z}_2$, $\Pi_3 = (z - w_0\cos\gamma)^2 c$ respectively, where w_0 is the displacement of spring attachment points arranged in diametrical coordinate plane $\phi = 0$.

III. PROBLEM SOLUTION

Represent the shell displacement in this form:

$$w = (r^2/r_1^2)\sin(m\pi r/r_1)A_n(t)\cos n\phi$$
$$\vartheta = (r^2/r_1^2)\sin(m\pi r/r_1)B_n(t)\sin n\phi \qquad (2)$$
$$u = (r^2/r_1^2)(m\pi r/r_1)D_n(t)\cos n\phi$$

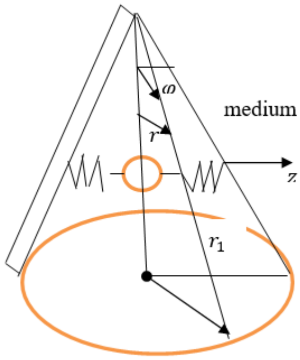

Figure 1. A longitudinally reinforced conical shell with a mass and also in contact with the medium

where, n is the number of waves in peripheral direction; m is the number of half-waves along the generatrix.

These expressions satisfy such conditions of flexible figure in of the husk at the edges for where $w = \vartheta = 0$. Expressions (2) are convenient in the sense that's saving to the multiplier r^2, the integrals encountered in calculating energy, are taken in squares which greatly facilitates the solution of the problem. Besides, expression (2) qualitatively reflects the fact that crests of half waves of the form of vibrations along the generatrix of the conical shell were slightly displaced towards its greater base.

It's necessary to note that for even n the springs are deformed, the mass doesn't more; for add n the mass performs vibrations along the axis z. In future, we will consider this case.

We suppose that the influence of the medium on the shell is subjected to the model [12]:

$$q_r = \left(q + q_0\frac{d^2}{dx^2}\right)w - \int_{-\infty}^{t}\Omega(t-\tau)w(\tau)d\tau \qquad (3)$$

where, $\Omega(t-\tau) = A^* e^{-\Psi(t-\tau)}$ and A^*, Ψ, q, q_0 are constant, and w is a shell deflection.

Taking into account (2) and also (3) in (1) we get:

$$A = \left\{\frac{\tilde{q}\pi}{r_1^4}S_p + \frac{2\tilde{q}_0\pi}{r_1^4}l^3\sin^4\gamma\left(\frac{1}{6} - \frac{1}{4m^2\pi^2}\right) + \right.$$
$$\left. + l^4\sin^4\gamma\left(-\frac{1}{2m\pi} + \frac{3}{4m^3\pi^3}\right) - \left(\frac{m\pi}{l}\right)^2 S_p\right\}\times$$
$$\times\left(A_1^2(t) + A_3^2(t) + \ldots\right) + \qquad (4)$$
$$+ \frac{\pi A_*}{r_1^4\sin\gamma}\left(\frac{1}{12}R_6 - \frac{5}{2\alpha^2}R_4 + \frac{30}{\alpha^4}R_2\right)\times$$
$$\times\left(\int_0^{2\pi} e^{-\Psi(t-\tau)}\sum_{n=1}^{\infty}A_1(\tau)\cos n\phi\right)d\tau d\phi A_n(t)\cos n\phi$$
$$S_p = l^5\left(\frac{1}{10} - \frac{2}{m^2\pi^2} - \frac{3}{4m^4\pi^4}\right).$$

If in the case of natural vibrations (4). If $A_n = A_n^*\sin\omega t$, we get:

$$A=\left\{\frac{\tilde{q}\pi}{r_1^4}S_p+\frac{\tilde{q}_0\pi}{r_1^4}\left[2\sin^2\gamma\left(\frac{r_2^2 l}{2}+\frac{1}{2}r_2 l^2\sin\gamma+\right.\right.\right.$$
$$\left.+l^3\sin^2\gamma\left(\frac{1}{6}-\frac{1}{4m^2\pi^2}\right)+\frac{1}{l}\sin\gamma\times\right.$$
$$\times\left(-3r_2^2 l^2\sin\gamma+3r_2^2 l^3\sin^2\gamma+l^5\sin^3\gamma\left(-\frac{1}{2m\pi}+\frac{3}{4m^3\pi^3}\right)-\right.$$
$$\left.-\left(\frac{m\pi}{l}\right)^2 S_p\right\}\left(A_1^2(t)+A_3^2(t)+...\right)+$$
$$+\frac{\pi A_*}{r_1^4\sin\gamma}\left(\frac{1}{12}R_6-\frac{5}{2\alpha^2}R_4+\frac{30}{\alpha^4}R_2\right)\times$$
$$\times\frac{\Psi\sin^2\omega t-\omega\sin\omega t\cos\omega t}{\Psi^2+\omega^2}A_n^{*2}$$

Accepting, $z=z^*\sin\omega t, A_n=A_n^*\sin\omega t, B_n=B_n^*\sin\omega t$, $A_n=D_n^*\sin\omega t$, where ω is a natural frequency of vibrations of the system under consideration and also substitute these solutions in Equation (3), for the total energy we obtain the second order polynomial with respect to the unknowns z^*, A_n^*, B_n^*:

$$L_{bpd}=\phi_{11}\frac{\pi}{\omega}D_n^{*2}+\breve{\delta}_{22}(t)A_n^{*2}+\phi_{33}\frac{\pi}{\omega}B_n^{*2}+\phi_{44}\frac{\pi}{\omega}A_n^*D_n^*+$$
$$+\phi_{55}\frac{\pi}{\omega}B_n^*A_n^*+\left(z^*-\alpha_0A_n^*\right)^2\frac{\pi}{\omega}c-\frac{\pi\omega}{2}Mz^{*2}- \qquad (6)$$
$$-\frac{\pi\omega}{2}\phi_{66}A_n^{*2}-\frac{\pi\omega}{2}\phi_{77}D_n^{*2}-\frac{\pi\omega}{2}\phi_{88}B_n^{*2}$$

The coefficient $\phi_{11}, \tilde{\delta}_{22}, \phi_{33}, \phi_{44}, \phi_{55}, \phi_{66}, \phi_{77}, \phi_{88}$ are of the bulky form and also we don't give them here. Substitute the found expression for kinetic and also potential energy (7) in Lagrange's account order known Equation [13] we get system of linear equations with respect to unknown constants z^*, A_n^*, B_n^*, D_n^*:

$$\left(\frac{2c}{M}-\omega^2\right)z^*-\frac{2c\alpha_0}{M}A_n^*=0$$
$$-2c\alpha_0z^*+\left(2\left(\bar{\delta}_{22}+\tau_{22}\right)-2\phi_{66}\omega^2+2\alpha_0^2c\right)A_n^*+$$
$$+\phi_{44}D_n^*+\phi_{55}B_n^*=0 \qquad (7)$$
$$\left(2\phi_{11}-2\phi_{77}\omega^2\right)D_n^*+\phi_{44}A_n^*=0$$
$$\left(2\phi_{33}-2\phi_{88}\omega^2\right)B_n^*+\phi_{55}A_n^*=0$$

Some of the system (7) is homogenous, for the existent of in nontrivial solution,we equate the principal determinant of the system to zero. As a result, we obtain an equation allowing to define natural frequency vibrations of a valid conical shell with associated mass and also contacting with viscoelastic medium.

$$\begin{vmatrix}\left(\frac{2c}{M}-\omega^2\right) & -2c\alpha_0 & 0 & 0\\ 0 & \left(\left(2\bar{\delta}_{22}+2\tau_{22}\right)-2\phi_{66}\omega^2\right) & \phi_{55} & \phi_{44}\\ 0 & \phi_{44} & 0 & \left(2\phi_{33}-2\phi_{88}\omega^2\right)\\ 0 & \phi_{55} & \left(2\phi_{33}-2\phi_{88}\omega^2\right) & 0\end{vmatrix}=0 \qquad (8)$$

Equation (8) can be represented as:

$$\left(\frac{2c}{M}-\omega^2\right)\left[\phi_{44}^2\left(2\phi_{33}-2\phi_{88}\omega^2\right)+\right.$$
$$+\phi_{55}^2\left(2\phi_{11}-2\phi_{77}\omega^2\right)- \qquad (9)$$
$$-\left(\left(2\bar{\delta}_{22}+2\tau_{22}\right)-2\phi_{66}\omega^2\right)\times$$
$$\left.\times\left(2\phi_{11}-2\phi_{77}\omega^2\right)\left(2\phi_{33}-2\phi_{88}\omega^2\right)\right]=0$$

IV. NUMERICAL RESULTS

The natural frequency of vibrations of the system was determined by numerical solution of the (9). For the problem parameters were accepted $r_1=160\,\text{mm}$, $r_2=85\,\text{mm}$ longitudinal bar of the corner profile with sites $5\times5\times1$ (to mm), $k_1=32,\ m=1,\ A^*=0.034$, $\Psi=0.05$. The longitudinal bars were attached to the inner surface of the shell. The associated mass whose value varied in the research process was attached in the middle of the shell at diametrically opposite points. In (9), the desired ω is a complex value: $\omega=\omega_1+i\omega_2$, moreover ω_1 corresponds to the natural frequency of vibrations of the system, ω_2 characterizes the damping of vibrations of the system in time. Figure 2 depict the curve reflecting the dependence of the natural frequency $f^*=\omega_1/2\pi$ of vibrations of the shell with associated mass medium in the amount of waves *n* in peripheral direction. If it is seen that with increasing the number of waves *n* in peripheral direction, the frequency vibrations of associated mass medium shell at first decrease, attaining minimum it befits to increase.

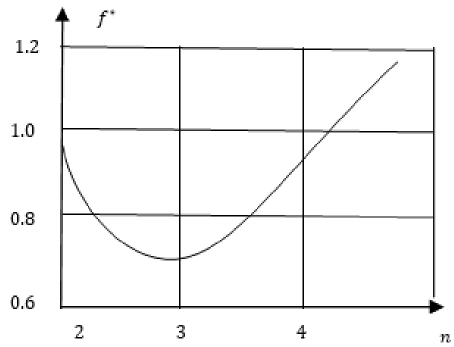

Figure 2. Dependence of the natural frequency of shell vibrations on the number of waves in the circumferential direction

Figure 3 depicts the curves reflecting the dependence of minimal natural frequency of vibrations of the system on the value of the associated mass with respect to the rigidity of springs $\bar{c}=c/D$ for $\bar{q}=q/D=0.1$ and also $q_0/q=0.5$.

The analysis of curves shows that the influence of the associated mass on minimal natural frequency of vibrations of the shell is very substantial. With decreasing rigidity of connections between the mass and also shell, this influence increases.

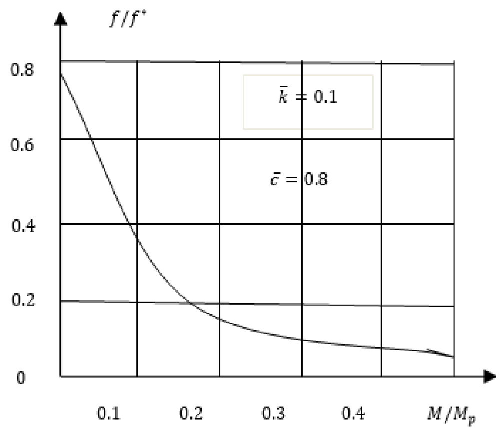

Figure 3. Dependence of the natural frequency of shell vibrations on the magnitude of the attached mass

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