# FREE VIBRATIONS OF AN ORTHOTROPIC, VISCO-ELASTIC AND MEDIUM-CONTACTING CYLINDRIC SHELL STRENGTHENED WITH RINGS AND ELASTIC SYMMETRY AXIS FORMS OF ANGLE WITH COORDINATE AXIS 

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#### Abstract

In engineering, transmission systems, thinwalled constructions or structural elements are widely used in the field of civil construction. In the operation process, these construction and structural elements are affected by dynamical forces. Therefore, the problems related to the study and choice of optimal variants of dynamical strength characteristics of such constructions and structural elements are still urgent. In this paper, natural vibrations frequency of the system is studied that consisting of a solid medium-filled elastic-plastic orthotropic cylindrical shell strengthened with discretely distributed rings established on a plane perpendicular to its axis. Utilizing the Hamilton-Ostrogradsky principle, a frequency equation for determining vibration frequencies of the system following consideration was created; its roots were obtained by mathematical method.


Keywords: Orthotropic, Elastic-Plastic, Cylindrical Shell, Medium, Free Vibration, Strengthened with Rings.

## I. INTRODUCTION

In [1], free vibrations of a strengthened elastic isotropic cylindrical shell under the action of compressive force and whose elastic symmetry axis coincides with the coordinate axis were considered. In [2, 6], for finding free vibrations of an anisotropic, viscous-elastic, cylindrical shell interacting dynamically with soil and fluid and strengthened with discretely distributed ribs, physical and mathematical model of the problem was constructed. Based on this model and applying the HamiltonOstrogradsky variation principle the vibrations frequency of the construction and optimization parameter of the considered construction a frequency equation was written. To find natural vibrations frequency and optimization parameters of the considered construction the analytic expressions were obtained and the influence of physical, mechanical and geometrical parameters characterizing this system anisotropy and viscosity character of cylindrical shell material soil and fluid on these quantities was studied. Three types of strengthening were considered. 1) with ribs located along with the generator of an anisotropic, viscous-elastic cylindrical shell; 2) with the
ribs located in the plane perpendicular to the axis of the anisotropic, viscous-elastic cylindrical shell; 3) with the ribs forming an orthogonal lattice on the surface of the anisotropic viscous elastic cylindrical shell.

In these three cases, a frequency equation was constructed and its roots were found for finding natural vibrations frequency of the system. The influence of geometrical, physical and mechanical characteristics characterizing the system, the material of the cylinder and viscosity properties on these frequencies and optimization parameter was studied.

The number of ribs and optimal variant of geometrical sizes of the shell for strengthening anisotropic viscouselastic cylindrical shell was found using the optimization parameter, and influence of physical, mechanical, geometrical parameters characterizing the system, anisotropy and viscosity character of the cylindrical shell material on this quantity was studied.

Choice of the optimal variant of parameters of a medium-contacting elastic cylindrical shell strengthened with discretely distributed ribs and subjected to the action of the compressive force and influence of quantities characterizing the medium i.e., compressive force, amount of ribs on optimization parameter was studied in [7, 8]. In [9-11], based on the variational principle, stability and vibrations of a viscous-elastic medium-contacting viscouselastic cylindrical shells were considered and the critical value of the parameter characterizing the critical state of the construction was found by the numerical method.

In the present paper, we study natural vibrations frequency of the system consisting of a solid mediumfilled elastic-plastic orthotropic cylindrical shell strengthened with discretely distributed rings located on a plane perpendicular to its axis and whose elastic symmetric axis forms a certain angle with coordinate axis.

Using the Hamilton-Ostrogradsky principle, a frequency equation for finding vibration frequencies of the system under consideration was constructed; its roots were found by numerical method and studied depending on physical and geometrical parameters characterizing the system.

## II. PROBLEM STATEMENT

In deformation process, the total energy of the system consisting of an elastic-plastic, orthotropic, solid-mediumfilled cylindrical shell located on a plane perpendicular to the axis, with elastic symmetry axis forming a certain angle with the coordinate axis and strengthened with discretely distributed rings, is written as follows:
$J=\frac{1}{2} R^{2} \int_{x_{1}}^{x_{2}} \int_{y_{1}}^{y_{2}}\left\{N_{11} \varepsilon_{11}+N_{22} \varepsilon_{22}+N_{12} \varepsilon_{12}-\right.$
$\left.-M_{11} \chi_{11}-M_{22} \chi_{22}-M_{12} \chi_{12}\right\} d x d y+$
$+\frac{1}{2} \sum_{j=1}^{k_{2}} \int_{y_{1}}^{y_{2}}\left[E_{j} F_{j}\left(\frac{\partial \vartheta_{j}}{\partial y}-\frac{w_{j}}{R}\right)^{2}+\right.$
$\left.+E_{j} J_{x j}\left(\frac{\partial^{2} w_{j}}{\partial x^{2}}+\frac{w_{j}}{R^{2}}\right)^{2}+E_{j} J_{z j}\left(\frac{\partial^{2} u_{i}}{\partial y^{2}}-\frac{\varphi_{\kappa p \mathrm{j}}}{\mathrm{R}}\right)^{2}\right] d x+$
$+\frac{1}{2} \sum_{j=1}^{k_{2}} \int_{y_{1}}^{y_{2}}\left[E_{j} F_{j}\left(\frac{\partial \vartheta_{j}}{\partial y}-\frac{w_{j}}{R}\right)^{2}+E_{j} J_{x j}\left(\frac{\partial^{2} w_{j}}{\partial x^{2}}+\frac{w_{j}}{R^{2}}\right)^{2}+\right.$
$\left.+E_{j} J_{z j}\left(\frac{\partial^{2} u_{i}}{\partial y^{2}}-\frac{\varphi_{k p \mathrm{j}}}{\mathrm{R}}\right)^{2}+G_{j} J_{k p j}\left(\frac{\partial \varphi_{k p \mathrm{i}}}{\partial y}+\frac{1}{R} \frac{\partial u_{j}}{\partial y}\right)^{2}\right] d y+$
$+\rho_{0} h \int_{x_{1}}^{x_{2}} \int_{y_{1}}^{y_{2}}\left[\left(\frac{\partial u}{\partial t}\right)^{2}+\left(\frac{\partial v}{\partial t}\right)^{2}+\left(\frac{\partial w}{\partial t}\right)^{2}\right] d x d y+$
$+\sum_{j=1}^{k_{2}} \rho_{j} F_{j} \int_{y_{1}}^{y_{2}}\left[\left(\frac{\partial u_{j}}{\partial t}\right)^{2}+\left(\frac{\partial \vartheta_{j}}{\partial t}\right)^{2}+\left(\frac{\partial w_{j}}{\partial t}\right)^{2}-\right.$
$\left.-\frac{J_{k p j}}{F_{j}}\left(\frac{\partial \varphi_{\kappa p j}}{\partial t}\right)^{2}\right] d y-\int_{x_{1}}^{x_{2}} \int_{y_{1}}^{y_{2}}\left(q_{x} u+q_{y} \vartheta+q_{z} w\right) d x d y$
The following relations are taken for internal force and moments:
$N_{i j}=\int_{-h / 2}^{h / 2}\left(\sigma_{i j}+z w_{i j}\right) d z, M_{i j}=-\int_{-h / 2}^{h / 2}\left(\sigma_{i j}+z w_{i j}\right) z d z$
$w_{11}=B_{11} x_{11}+B_{12} x_{22}, w_{22}=B_{12} x_{11}+B_{22} x_{22}$
$w_{21}=w_{12}=B_{66} x_{12}$
the stresses $\sigma_{i j}$ contained in (2) are expressed by deformations $\varepsilon_{i j}$ as follows:
$\sigma_{11}=B_{11} \varepsilon_{11}+B_{12} \varepsilon_{22}$
$\sigma_{22}=B_{12} \varepsilon_{11}+B_{22} \varepsilon_{22}, \sigma_{12}=b_{66} \varepsilon_{12}$
In expressions (2), (3):
$B_{11}=b_{11} \cos ^{4} \varphi+b_{22} \sin ^{4} \varphi+\left(b_{66}+0,5 b_{12}\right) \sin ^{2} 2 \varphi$
$B_{22}=b_{11} \sin ^{4} \varphi+b_{22} \cos ^{4} \varphi+\left(b_{66}+0,5 b_{12}\right) \sin ^{2} 2 \varphi$
$B_{12}=\left(b_{11}+b_{22}-4 b_{66}\right) \sin ^{2} \varphi \cos ^{2} \varphi+$
$+b_{12}\left(\sin ^{4} \varphi+\cos ^{4} \varphi\right)$
$B_{66}=-\left(b_{11}+b_{22}-2 b_{12}\right) \sin ^{2} \varphi \cos ^{2} \varphi+b_{66} \cos ^{2} 2 \varphi$
$B_{26}=1 / 2\left(b_{22} \cos ^{2} \varphi-b_{11} \sin ^{2} \varphi\right) \sin 2 \varphi-$
$-1 / 4\left(b_{12}+2 b_{66}\right) \sin 4 \varphi$
$B_{16}=1 / 2\left(b_{22} \sin ^{2} \varphi-b_{11} \cos ^{2} \varphi\right) \sin 2 \varphi$ -
$-1 / 4\left(b_{12}+2 b_{66}\right) \sin 4 \varphi$
where, $b_{11}, b_{22}, b_{12}, b_{66}$ is the main elasticity modulus of appropriate orthotropic material in the direction of the coordinate axis, Young's modulus $\tilde{E}_{1}, \tilde{E}_{2}$ are expressed by the Poisson ratios $v_{1}, v_{2}$ as follows:
$b_{11}=\frac{\tilde{E}_{1}}{1-v_{1} v_{2}}, b_{22}=\frac{\tilde{E}_{2}}{1-v_{1} v_{2}}$
$b_{12}=\frac{v_{2} \tilde{E}_{1}}{1-v_{1} v_{2}}=\frac{v_{1} \tilde{E}_{2}}{1-v_{1} v_{2}}, b_{66}=G_{12}=G$
where, $\varphi$ is an orientation angle. We will take deformations as follows:
$\varepsilon_{i j}=\tilde{\varepsilon}_{i j}+\int_{-\infty}^{t} \Gamma(t-\tau) \varepsilon_{i j}(\tau) d \tau$
In expressions (1)-(4):
$\varepsilon_{11}=\frac{\partial u}{\partial x}, \varepsilon_{22}=\frac{\partial v}{\partial y}+w, \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$
$\chi_{11}=\frac{\partial^{2} w}{\partial x^{2}}, \chi_{22}=\frac{\partial^{2} w}{\partial y^{2}}, \chi_{12}=-2 \frac{\partial^{2} w}{\partial x \partial y}, \Gamma(t)=A e^{-\psi t}$
where, $u, \vartheta, w$ are displacements of the shell, $R, h$ are radius and thickness of the cylindrical shell, respectively, $E_{j}$ is an elasticity modulus of the $j$ th ring, $F_{j}$ is the area of the cross-section of the $j$ th ring, $I_{x j}, I_{y j}, I_{k p j}$ are inertia moments of the cross-section of the $j$ th ring. $q_{x}, q_{y}, q_{z}$ are the components of the pressure force acting on the cylindrical shell the medium, $k_{2}$ is the amount of rings, $G_{j}$ is the elasticity modulus of the $j$ th ring in shear.

The equations of motion of medium in cylindric coordinates are written as follows [12]:
$\left(\lambda_{s}+2 \mu_{s}\right) \frac{\partial \theta_{1}}{\partial r}-\frac{2 \mu_{s}}{r} \frac{\partial \omega_{x}}{\partial \theta}+2 \mu_{s} \frac{\partial \omega_{\theta}}{\partial x}-\rho_{s} \frac{\partial^{2} s_{x}}{\partial t^{2}}=0$
$\left(\lambda_{s}+2 \mu_{s}\right) \frac{1}{r} \frac{\partial \theta_{1}}{\partial \theta}-2 \mu_{s} \frac{\partial \omega_{r}}{\partial x}+2 \mu_{s} \frac{\partial \omega_{x}}{\partial x}-\rho_{s} \frac{\partial^{2} s_{\theta}}{\partial t^{2}}=0$
$\left(\lambda_{s}+2 \mu_{s}\right) \frac{\partial \theta_{1}}{\partial x}-\frac{2 \mu_{s}}{r} \frac{\partial}{\partial r}\left(r \omega_{\theta}\right)+\frac{2 \mu_{s}}{r} \frac{\partial \omega_{r}}{\partial \theta}-\rho_{s} \frac{\partial^{2} s_{r}}{\partial t^{2}}=0$
where, $s_{x}, s_{\varphi}, s_{r}$ are the displacement vector components of the medium $\lambda_{s}, \mu_{s}$ are Lame coefficients of the medium $\rho_{s}$ is the density of the medium $x, r, \varphi$ are longitudinal radial circular coordinates.

Volume extension components $\theta, \omega_{x}, \omega_{\varphi}, \omega_{r}$ are calculated utilizing the following expressions:
$\theta=\frac{\partial s_{r}}{\partial r}+\frac{s_{r}}{r}+\frac{1}{r} \frac{\partial s_{\varphi}}{\partial \varphi}+\frac{\partial s_{x}}{\partial x}$
$2 \omega_{x}=\frac{1}{r}\left[\frac{\partial\left(r s_{\varphi}\right)}{\partial r}-\frac{\partial s_{r}}{\partial \varphi}\right]$
$2 \omega_{\varphi}=\frac{\partial s_{r}}{\partial x}-\frac{\partial s_{x}}{\partial r}$
$2 \omega_{r}=\frac{1}{r} \frac{\partial s_{x}}{\partial \varphi}-\frac{\partial s_{\varphi}}{\partial x}$
the stresses in the medium are expressed by displacements $s_{x}, s_{\varphi}, s_{r}$ as follows:
$\sigma_{r x}=\mu_{s}\left(\frac{\partial s_{x}}{\partial r}+\frac{\partial s_{r}}{\partial x}\right)$
$\sigma_{r \theta}=\mu_{s}\left[r \frac{\partial}{\partial r}\left(\frac{s_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial s_{r}}{\partial \theta}\right]$
$\sigma_{r r}=\lambda_{s}\left(\frac{\partial s_{x}}{\partial x}+\frac{1}{r} \frac{\partial\left(r s_{r}\right)}{\partial r}+\frac{1}{r} \frac{\partial s_{\theta}}{\partial \theta}\right)+2 \mu_{s} \frac{\partial s_{r}}{\partial r}$
When studying vibrations of viscous-elastic cylindrical shell with medium we will assume that we can not ignore the influence of inertia of medium in studying the vibrations process.

The solution of system (5) is written as follows [12]:
$s_{x}=\left[A_{s} k I_{n}\left(\gamma_{e} r\right)-\frac{C_{s} \gamma_{t}^{2}}{\mu_{t}} I_{n}\left(\gamma_{t} r\right)\right] \times$
$\times \cos n \varphi \cos k x \sin \omega t$
$s_{\theta}=\left[-\frac{A_{s} n}{r} I{ }_{n}\left(\gamma_{e} r\right)-\frac{C_{s} n k}{r \mu_{t}} I_{n}\left(\gamma_{t} r\right)-\right.$
$\left.-\frac{B_{s}}{n} \frac{\partial I_{n}\left(\gamma_{t} r\right)}{\partial r}\right] \sin n \varphi \sin k x \sin \omega t$
$s_{r}=\left[A_{s} \frac{\partial I_{n}\left(\gamma_{e} r\right)}{\partial r}-\frac{C_{s} k}{\mu_{t}} \frac{\partial I_{n}\left(\gamma_{t} r\right)}{\partial r}+\right.$
$\left.+\frac{B_{s} n}{r} I_{n}\left(\gamma_{t} r\right)\right] \cos n \varphi \sin k x \sin \omega t$
To the system (5) of equations of motion of medium, we add the contact conditions. We will assume that the tangential surface of the cylindrical shell and medium slides freely with respect to each other, but in the deformation process, they are not separated from each other. For a medium to stay interior to the cylindrical shell, a membrane rigid in its plane and with very weak resistance to bending out of the plane should be attached to its ends. In this case, in the sections $x=x_{1}$ and $x=x_{2}$ the conditions $\sigma_{x x}=0, s_{\theta}=s_{r}=0$ should be fulfilled.

The condition of equality of normal components of displacements:
$s_{r}=w \quad(r=R)$
The conditions of equality of pressure forces:
$q_{x}=0, \quad q_{y}=0, \quad q_{z}=-\sigma_{r r} \quad(r=R)$

It is assumed that the following rigid contact conditions between the shell and rings are satisfied:
$u_{j}(y)=u\left(x_{j}, y\right)+h_{j} \varphi_{1}\left(x_{j}, y\right)$
$\vartheta_{j}(x)=\vartheta\left(x_{j}, y\right)+h_{j} \varphi_{2}\left(x_{j}, y\right)$
$w_{j}(x)=w\left(x_{j}, y\right), \varphi_{j}=\varphi_{2}\left(x_{j}, y\right), \varphi_{k p j}(x)=\varphi_{1}\left(x_{j}, y\right)$
Thus, by means of the obtained expressions, one can define the forces acting on the cylinder by the medium. As a result, the solution of the stated problem is reduced to joint integration of total energy (1) of a system consisting of a cylindrical shell with medium- filled internal domain and strengthened with discretely distributed rings, of the system of motion equations (5) of the medium within boundary conditions (8) and (9). We represent the expression of the pressure component $q_{z}$ in the form:
$q_{z}=q_{z}^{(0)} C \cos n \varphi \sin k x \sin \omega t$
Using of contact conditions (8) and (9) the system of motion Equations (5) of the medium for $q_{z}^{(0)}$ we get the expression:

$$
\begin{align*}
& q_{z}^{(0)}=\frac{E_{s}}{1+v_{s}} I_{n}\left(\gamma_{l}^{*}\right)\left[\left(\frac{I_{n}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)}-\gamma_{l}^{*} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)}+\gamma_{l}^{* 2}+\right.\right. \\
& \left.+n^{2}-\frac{v_{s}}{1-2 v_{s}} \mu_{l}^{* 2}\right) \frac{-n^{2} k^{* 2} \mu_{t}^{*}+\frac{R^{4} k^{* 3} \gamma_{t}^{* 2}}{\mu_{t}^{*}} \frac{I_{n}^{\prime 2}\left(\gamma_{t}^{*}\right)}{I_{n}^{2}\left(\gamma_{t}^{*}\right)}}{-\frac{k^{* 3} \gamma_{l}^{*} \gamma_{t}^{* 2}}{\mu_{t}^{*}} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)} \frac{I_{n}^{\prime 2}\left(\gamma_{t}^{*}\right)}{I_{n}^{2}\left(\gamma_{t}^{*}\right)}}+ \\
& +\frac{2 n k^{*} \gamma_{l}^{*} \mu_{t}^{*} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)}+\frac{2 n k^{* 3} \gamma_{t}^{*}}{\mu_{t}^{*}} \frac{I_{n}^{\prime}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{t}^{*}\right)}}{-\frac{k^{* 3} \gamma_{l}^{*} \gamma_{t}^{* 2}}{\mu_{t}^{*}} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)} \frac{I_{n}^{\prime 2}\left(\gamma_{t}^{*}\right)}{I_{n}^{2}\left(\gamma_{t}^{*}\right)}} \times  \tag{11}\\
& \times\left(-n^{2}+n \gamma_{t}^{*} \frac{I_{n}^{\prime}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{t}^{*}\right)}+\frac{v_{s}}{1-2 v_{s}} n \gamma_{t}^{*}\left(\gamma_{t}^{*}-\gamma_{t}^{*} \frac{I_{n}^{\prime}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{t}^{*}\right)}\right)\right)+ \\
& +\left(\frac{k^{*} \gamma_{t}^{*}}{\mu_{t}^{*}} \frac{I_{n}^{\prime}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{t}^{*}\right)}+\gamma_{t}^{* 2}+n^{2}-\frac{v_{s}}{1-2 v_{s}} \frac{2 k^{*} \gamma_{t}^{* 2}}{\mu_{t}^{*}}\right) \times \\
& \left.\times \frac{\left[-\frac{k^{* 3} \gamma_{l}^{*} \gamma_{t}^{* 2}}{\mu_{t}^{*}} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)} \frac{I_{n}^{\prime 2}\left(\gamma_{t}^{*}\right)}{I_{n}^{2}\left(\gamma_{t}^{*}\right)}\right]}{2 k^{* 2} \gamma_{l}^{*} \gamma_{t}^{*} \frac{I_{n}^{\prime}\left(\gamma_{l}^{*}\right)}{I_{n}\left(\gamma_{l}^{*}\right)} \frac{I_{n}^{\prime}\left(\gamma_{t}^{*}\right)}{I_{n}\left(\gamma_{t}^{*}\right)}-2 n^{2} k^{* 2}}\right]
\end{align*}
$$

In expressions (7)-(11), $A_{s}, B_{s}, C_{s}$ are unknown constants $k, n, \gamma_{e}, \gamma_{t}$ are wave numbers, $I_{n}$ is a modified $n$th order, first kind Bessel function, $\gamma_{e}^{2}=k^{2}-\mu_{e}^{2}, \gamma_{t}^{2}=k^{2}-\mu_{t}^{2}, k^{*}=k R$.

## III. TASK SOLUTION

We will look for displacements of the shell in the following form:
$u=A \cdot \cos n \varphi \cdot \cos k x \cdot \sin \omega t$
$v=B \cdot \sin n \varphi \cdot \sin k x \cdot \sin \omega t$
$w=C \cdot \cos n \varphi \cdot \sin k x \xi \cdot \sin \omega t$
where, $A, B, C$ are unknown constants, $\omega$ is an unknown frequency.

Using the Ostrogradsky-Hamilton stationarity action condition, solutions (12), if we vary the obtained square polynomial with respect to independent constant $A, B, C$, and equate the coefficients of independent variations to zero, we get a system of homogeneous algebraic equations.

As the obtained system is a system of a linear homogeneous system of algebraic equations, the necessary and sufficient condition for the existence of its non-zero solution is the equality of its principal determinant to zero. As a result, we get the following frequency equation:

$$
\left|\begin{array}{ccc}
2\left(\breve{\varphi}_{11}-\psi_{11} \omega^{2}\right) & \breve{\varphi}_{44} & \breve{\varphi}_{55}  \tag{13}\\
\breve{\varphi}_{44} & \left(\breve{\varphi}_{22}-\psi_{22} \omega^{2}\right) & \breve{\varphi}_{66} \\
\breve{\varphi}_{55} & \breve{\varphi}_{66} & 2\left(\breve{\varphi}_{33}-\psi_{33} \omega^{2}\right)
\end{array}\right|=0
$$

As the expressions of quantities participating in Equation (13) are bulky, we do not give them here. We note that this equation is a transcendental equation with respect to $\omega$ and its roots are complex. The real parts of these roots correspond to vibrations frequency. The imaginary part characterizes damping of vibrations in time.


Figure 1. Dependence of vibrations frequency on the number of pivots

## IV. NUMERICAL RESULTS

The roots of Equation (13) was calculated by the numerical method. In the calculation, the following values were taken for the parameters characterizing the medium, shell, and bars:
$E_{j}=6.67 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}, h_{j}=1.39 \mathrm{~mm}, R=160 \mathrm{~mm}$
$A=0.1615, \beta=0.05, L_{1}=800 \mathrm{~mm}$
$\frac{F_{i}}{2 \pi R h}=0.1591 \times 10^{-1}$
$\frac{I_{y i}}{2 \pi R^{3} h}=0.8289 \times 10^{-6}, h=0.45 \mathrm{~mm}$
$F_{j}=5.75 \mathrm{~mm}^{2}, J_{x j}=19.9 \mathrm{~mm}^{4}, J_{k p . j}=0.48 \mathrm{~mm}^{4}$
$a_{l}=2.25 a_{t}, a_{t}=308 \mathrm{~m} / \mathrm{s}$
$v_{1}=0.11, v_{2}=0.19$
$b_{11}=18.3 \mathrm{KPa}, b_{22}=25.2 \mathrm{KPa}$
$b_{66}=3.5 \mathrm{KPa}, b_{12}=2.77 \mathrm{KPa}$


Figure 2. dependence of natural vibrations frequency of the system on orientation angle

## V. CONCLUSIONS

The result of calculations the dependence of the frequency $\operatorname{Re} \omega$ on the number of rings was given in Figure 1, the dependence on the orientation angle as depicted in Figure 2. In Figure 1, the ahead lines correspond to the case when the material of the cylindrical shell is viscous, the entire line to the case when the material of the cylindrical shell is not viscous. As is seen from Figure 1 as the number of rings increases minimal frequencies of the system at first increases and then decreases after a certain value of the number of rings. This is explained by the fact that an increase in the number of rings after certain value causes increases in the mass of the system and this in its turn to decreases of natural vibrations frequency of the system. The account of the viscosity of the material of the cylindrical shell also increases the frequency of the natural vibrations of the system. Strengthening of orthotropic character of the material of the cylindrical shell also increases the natural vibrations frequency of the system. It becomes clear from Figure 2 that in small values of ratio $R / L$ natural vibrations frequency of the system is weakly dependent on rotation angle $\varphi$ when $R / L=0.1$ the dependence of the value of the critical force on the rotation angle $\varphi$ is of complicated character.

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