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OSCILLATIONS OF REINFORCED ORTHOTROPIC CYLINDRICAL SHELLS WITH LEAKING AMONG LIQUID

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Abstract- The development of modern technology is increasingly based on the achievements of fundamental and applied scientific research. Engineering structures and structures are becoming more complicated, therefore their design is difficult to imagine without a preliminary detailed calculation of the behavior of these structures or their elements in certain conditions. The study of oscillatory processes is of great importance for modern technology. Its development is associated with an increase in speed, pressure, temperature, with a continuous increase in the power and speed of machines and mechanisms, an increase in the aerodynamic effect of the flow of a flowing medium. However, there is a desire for better use of the bearing capacity of structures and reduce their weight. This entails an increase in the impact of dynamic loads on the elements of machines and structures.

Keywords: Variational Principle, Transversely Reinforced Cylindrical Shell, Elastic Medium, Fluid.

I. INTRODUCTION

Natural vibrations in an infinite elastic medium reinforced by a cross system of ribs of an isotropic cylindrical shell with a flowing fluid were considered in [1]. The work [2] investigated free vibrations of a transversely reinforced cylindrical shell in an infinite elastic medium with a flowing fluid. In [3], free oscillation of a longitudinally strengthened, orthotropic, moving fluid-contacting cylindrical shell inhomogeneous in thickness, is studied. Using the Hamilton-Ostrogradsky variational principle, the systems of equations of motion of a longitudinally strengthened, orthotropic moving-fluid-contacting cylindrical shell inhomogeneous in thickness, is constructed. In this paper the inhomogeneity was considered by accepting the Young modulus and the density of the material as a function of coordinate changing in thickness [4].

In this article, using the variational principle, the problem of the natural oscillation of a longitudinally supported orthotropic cylindrical shell in contact with an external medium and a flowing fluid is solved. The environmental influences are considered using the system of Lame equations in displacements.

We obtain differential equations of motion for a longitudinally supported orthotropic cylindrical shell in contact with the medium based on the Ostrogradsky-Hamilton variational principle. To apply the Ostrogradsky-Hamilton principle, we first write down the potential and kinetic energies of the system.

II. PROBLEM STATEMENT

As we know the potential energy an orthotropic cylindrical husk has the form:

$$V=\frac{hR}{2}\iint\left\{B_{11}\left(\frac{\partial u}{\partial x}\right)^2-2\left(B_{11}+B_{12}\right)\frac{w}{R}\frac{\partial u}{\partial x}+\right.$$
$$+\frac{w^2}{R^2}\left(B_{11}+2B_{12}+B_{22}\right)+\frac{B_{22}}{R^2}\left(\frac{\partial v}{\partial\theta}\right)^2- \qquad (1)$$
$$\left.-2\left(B_{12}+B_{22}\right)\frac{w}{R^2}\frac{\partial v}{\partial\theta}+2B_{12}\frac{1}{R^2}\frac{\partial u}{\partial x}\frac{\partial v}{\partial\theta}\right\}dxd\theta$$

where, R is radius of the middle surface of the shell, h is thickness of the husk, u, v and w are components of shifting of points of the middle appearance of husk.

$$B_{11}=\frac{E_1}{1-\nu_1\nu_2};\ B_{22}=\frac{E_2}{1-\nu_1\nu_2};\ B_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2}=\frac{\nu_1E_2}{1-\nu_1\nu_2}$$

The expressions for the potential energy of elastic deformation of the longitudinal longitudinal ribs are as follows [5]:

$$\Pi_i=\frac{1}{2}\int_{x_1}^{x_2}\left[E_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2+E_iJ_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+\right.$$
$$\left.+E_iJ_{zi}\left(\frac{\partial^2\vartheta_i}{\partial x^2}\right)^2+G_iJ_{\kappa pi}\left(\frac{\partial\varphi_{\kappa pi}}{\partial x}\right)^2\right]dx \qquad (2)$$

In Equations (1)-(2) the curves coordinate also rectilinear edges of the husk; the area and moment of inertia of the cross section of the longitudinal rod, respectively, relative to the axis and axis parallel to the axis and passing through the middle of gravity of the area , as well as its moment of inertia during torsion; are the elastic and shear moduli of the material of the longitudinal rod, respectively.

The potential energy of external surface also bound loads applied to the skin is defined as the work performed by these loads when the system is transferred from a deformed state to an initial undeformed state and is presented in the form

$$A_0 = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(q_x u + q_y \vartheta + \left(q_{zm}+q_{zc}\right)w\right)dxdy -$$
$$-\int_{y_1}^{y_2}\left(T_1 u + S_1\vartheta + Q_1 w + M_1\varphi_1\right)\Big|_{x=x_1}^{x=x_2} dy - \qquad (3)$$
$$-\int_{x_1}^{x_2}\left(S_2 u + T_2\vartheta + Q_2 w + M_2\varphi_2\right)\Big|_{y=y_1}^{y=y_2} dx$$

Similarly, the potential energies of external edge tons of applied to the ends of the corresponding longitudinal rod are determined by the following expressions (it is assumed that only edge loads are applied to the ribs):

$$A_i = -\left(T_i u_i + S_i \vartheta_i + Q_i w_i + M_i \varphi_i + \right.$$
$$\left. + M_{1i}\varphi_{zi} + M_{kpi}\varphi_{kpi}\right)\Big|_{x=x_1}^{x=x_2} \qquad (4)$$

The summary potential energy of the system is equal to the sum of the potential energies of the elastic deformations of the shell and ribs, as well as the potential energies of all external loads:

$$\Pi = \Pi_0 + \sum_{i=1}^{m}\Pi_i + A_0 + \sum_{i=1}^{m} A_i \qquad (5)$$

The kinetic energies of the shell and ribs are written as

$$K_0 = \frac{Eh}{2(1-\nu^2)}\int_0^{\xi_1}\int_0^{2\pi}\left[\left(\frac{\partial u}{\partial t_1}\right)^2 + \left(\frac{\partial \vartheta}{\partial t_1}\right)^2 + \left(\frac{\partial w}{\partial t_1}\right)^2\right] d\xi d\theta \qquad (6)$$

$$K_i = \rho_i F_i \int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \right.$$
$$\left. + \frac{J_{\kappa pi}}{F_i}\left(\frac{\partial \varphi_{\kappa pi}}{\partial t}\right)^2\right] dx \qquad (7)$$

where, t is the time coordinate, $t_1 = \omega_a t$, $\omega_a = \sqrt{\frac{E}{(1-\nu^2)\rho_s R^2}}$ and ρ_0, ρ_i are the density of the materials of which the shell is made, and the i-longitudinal rod, respectively.

Kinetic energy of a ribbed orthotropic shell

$$K = K_0 + \sum_{i=1}^{m} K_i \qquad (8)$$

The equations of motion of a ribbed orthotropic shell in contact with the medium are obtained based on the principle of stationarity of the Ostrogradsky-Hamilton action:

$$\delta W = 0 \qquad (9)$$

where, $W = \int_{t'}^{t''} L dt$ Hamilton action, $L = K - \Pi$ is Lagrange function t' and t'' are given arbitrary points in time.

Assuming that the main flow velocity is equal and the deviations from this velocity are small, we use the wave equation for the potential of perturbed velocities φ by [6]:

$$\Delta\varphi - \frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2} + 2U\frac{\partial^2\varphi}{R\partial\xi\partial t} + U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right) = 0 \qquad (10)$$

In the case when harmonic oscillations are considered, the equations of motion of the medium take the form [7]:

$$a_l^2\, graddiv\vec{u} - a_t^2\, rotrot\vec{u} + \omega^2\vec{u} = 0 \qquad (11)$$

The potential and kinetic energy of the shell (5), (8), the equation of motion of the liquid (10), and the medium (11) are supplemented by contact conditions.

On the contact surface of the shell-liquid, the continuity of radial velocities and pressures is observed. The condition of impermeability or smooth flow around the wall of the shell has the form:

$$\vartheta_r\big|_{r=R} = \frac{\partial\varphi}{\partial r}\bigg|_{r=R} = -\left(\omega_0\frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right) \qquad (12)$$

Equal radial pressure from the liquid to the shell

$$q_{zm} = -p_{|r=R} \qquad (13)$$

Assume that the contact between the shell and the medium is $r=R$ sliding i.e. at

$$w = s_z \qquad (14)$$

$$q_x = -\sigma_{rx} = 0,\ q_\theta = -\sigma_{r\theta} = 0,\ q_{zc} = -\sigma_{rr} \qquad (15)$$

Supplementing with the contact conditions (12)-(15) the expression for the potential and kinetic energy of the shell (5), (8), the equations of motion of the fluid (10) and medium (11), we arrive at the problem of natural vibrations in an infinite elastic medium longitudinally orthotropic cylindrical shell, with a flowing fluid. In other words, the problem of natural vibrations in an infinite elastic medium of a lengthways reinforced cylindrical husk with a waving liquid decrease to the joint integration of the equations of the theory of shells, the medium, and the fluid under the above conditions on the surface of their contact.

III. PROBLEM SOLUTION

The solution of the problem. We will look for shell movements in the form:

$$u = u_0.\sin\chi\xi.\cos n\theta.\sin\omega_1 t_1$$
$$\vartheta = \vartheta_0.\cos\chi\xi.\sin n\theta.\sin\omega_1 t_1 \qquad (16)$$
$$w = w_0.\cos\chi\xi.\cos n\theta.\sin\omega_1 t_1$$

Here are unknown constants; wave numbers in the longitudinal and circumferential directions, respectively. The solution of the equation of motion of the medium has the form [6]:

- in case without inertial medium

$$s_x = \left[\left(-kr\frac{\partial K_n(kr)}{\partial r} - 4(1-\nu_s)kK_n(kr)\right)A_s + \right.$$
$$\left. + kK_n(kr)B_s\right]\cos n\varphi \cos kx \sin \omega t$$
$$s_\theta = \left[-\frac{n}{r}K_n(kr)B_s - \frac{\partial K_n(kr)}{\partial r}C_s\right]\times \tag{17}$$
$$\times \sin n\varphi \sin kx \sin \omega t$$
$$s_r = \left[-k^2 r K_n(kr)A_s + \frac{\partial K_n(kr)}{\partial r}B_s + \right.$$
$$\left. + \frac{n}{r}K_n(kr)C_s\right]\cos n\varphi \sin kx \sin \omega t$$

- in case of inertial medium

$$s_x = \left[\tilde{A}_s k K_n(\gamma_e r) - \tilde{C}_s\frac{\gamma_t^2}{\mu_t}K_n(\gamma_t r)\right]\times$$
$$\times \cos n\varphi \cos kx \sin \omega t$$
$$s_\theta = \left[-\frac{\tilde{A}_s n}{r}K_n(\gamma_e r) - \frac{\tilde{C}_s nk}{r\mu_t}K_n(\gamma_t r) - \right.$$
$$\left. - \frac{\tilde{B}_s}{n}\frac{\partial K_n(\gamma_t r)}{\partial r}\right]\sin n\varphi \sin kx \sin \omega t \tag{18}$$
$$s_r = \left[\tilde{A}_s\frac{\partial K_n(\gamma_e r)}{\partial r} - \frac{\tilde{C}_s k}{\mu_t}\frac{\partial K_n(\gamma_t r)}{\partial r} + \right.$$
$$\left. + \frac{\tilde{B}_s n}{r}K_n(\gamma_t r)\right]\cos n\varphi \sin kx \sin \omega t$$

We look for the potential of perturbed velocities in the form:

$$\varphi(\xi, r, \theta, t_1) = f(r)\cos n\varphi \sin kx \sin \omega t \tag{19}$$

Using (19) from condition (12) and for from (13) we have:

$$\varphi = -\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial \xi}\right)$$
$$p = \Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \tag{20}$$

where,

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta r), & M_1 < 1 \\ J_n(\beta_1 r)/J_n'(\beta_1 r), & M_1 > 1 \\ \dfrac{R^n}{nR^{n-1}}, & M_1 = 1 \end{cases} \tag{21}$$

where, $M_1 = \dfrac{U + \omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, I_n are obtained using (19) from condition (12) and for from (13) we have:

$$p = \frac{\rho_m\Phi_{\alpha n}}{\rho_0\omega_0^2 h}\left(\omega_0^2\omega_1^2 + 2\omega_0\omega_1\chi U + \chi^2U^2\right)w \tag{22}$$

Using the contact conditions (14) and (15), the solution of the equation of motion of the medium (17) and (18), the formulas for the stresses [5], we can determine the contact pressure from the medium to the shell:

$$q_{zc} = \tilde{C}_{rr}w_0\cos\chi\xi\cos n\theta\sin\omega_1 t_1 \tag{23}$$

Using (19) from condition (12) and for from (13):

$$\tilde{C}_{rr} = -\mu_s\Delta^{-1}\left(q_{11}\Delta_1^{(3)} + q_{12}\Delta_2^{(3)} + q_{13}\Delta_3^{(3)}\right)$$
$$\Delta = k^{\bullet 2}n^2K_n''(k^\bullet)I_n^2(k^\bullet) - k^{\bullet 4}K_n'^3(k^\bullet) + 4n^2k^\bullet(1-\nu_s)K_n^3(k^\bullet) -$$
$$-4k^{\bullet 3}(1-\nu_s)K_n(k^\bullet)K_n'^2(k^\bullet) + k^{\bullet 4}K_n^2(k^\bullet)K_n'(k^\bullet)$$
$$q_{11} = \left(2(1-2\nu_s)K_n(k^\bullet) + 2k^\bullet K_n'(k^\bullet)\right)k^{\bullet 2};$$
$$q_{12} = -2k^{\bullet 2}K_n''(k^\bullet);$$
$$q_{13} = 2n\left(K_n(k^\bullet) - k^\bullet K_n'(k^\bullet)\right)$$
$$\Delta_1^{(3)} = \begin{vmatrix} 0 & l_{12} & l_{13} \\ 0 & l_{22} & l_{23} \\ w_0 & l_{32} & l_{33} \end{vmatrix}; \quad \Delta_2^{(3)} = \begin{vmatrix} l_{11} & 0 & l_{13} \\ l_{21} & 0 & l_{23} \\ l_{31} & w_0 & l_{33} \end{vmatrix};$$
$$\Delta_3^{(3)} = \begin{vmatrix} l_{11} & l_{12} & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & w_0 \end{vmatrix};$$
$$l_{11} = \left(k^\bullet K_n(k^\bullet) + k^\bullet K_n''(k^\bullet) + (5-4\nu_s)K_n'(k^\bullet)\right)k^{\bullet 2};$$
$$l_{12} = -2k^{\bullet 2}K_n'(k^\bullet);\ l_{13} = -nk^\bullet K_n(k^\bullet);$$
$$l_{21} = -nk^{\bullet 2}K_n(k^\bullet);\ l_{22} = 2n\left(k^\bullet K_n'(k^\bullet) - K_n(k^\bullet)\right);$$
$$l_{23} = k^{\bullet 2}K_n''(k^\bullet) - k^\bullet(k^\bullet) + n^2K_n(k^\bullet);$$
$$l_{31} = -k^{\bullet 2}K_n(k^\bullet);\ l_{32} = k^\bullet K_n'(k^\bullet);\ l_{33} = nK_n(k^\bullet)$$

After substituting (16), (22), (23) into (9), the problem reduces to a homogeneous system of linear algebraic equations of the third order

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0,\ (i = 1, 2, 3) \tag{24}$$

Elements have a bulky appearance, therefore not given here. A nontrivial solution to the system of linear algebraic equations (24) of the third order is possible only in the case when the root of its determinant. The definition reduces to a transcendental equation, since it is included in the arguments of the Bessel function:

$$\begin{vmatrix} \tilde{a}_{11} + \rho_1\omega_1^2 & a_{12} & a_{13} \\ a_{21} & \tilde{a}_{22} + \omega_1^2 & a_{23} \\ a_{31} & a_{32} & \tilde{a}_{33} - \varphi_1\omega_1^2 - \varphi_2\omega_1 \end{vmatrix} = 0 \tag{25}$$

It should be noted that with Eq. (25), it transfers to the frequency equation of free vibrations located in a limitless without inertial elastic medium, longitudinally supported by an orthotropic cylindrical shell filled with a liquid at rest. The last equation then goes over to the equation of free vibrations of a longitudinally reinforced cylindrical shell filled with a liquid at rest.

IV. CONCLUSIONS

Consider some of the results of calculations performed on the basis of the above dependencies using a computer. For the geometric and physical parameters characterizing the materials of the shell, the medium was adopted:

$E_c = 6.67\times10^9$ N/m², $\rho_0 = \rho_c = 0.26\times10^4$ Nsec²/m⁴,
$F_c = 3.4$ mm², $J_{yc} = 5.1$mm⁴, $h_c = 1.39$ mm,
$\rho_0 / \rho_m = 0.105$, $a_l = 2.25 a_t$, $a_t = 308$ m/sec, $m = 8$

In Figure 1 the dependences of the frequency parameter on the relative flow velocity at various values of and. It is seen that an increase in speed leads to a decrease in frequency. It is important to note the values at which the oscillation frequency vanishes. Obviously, there should be a loss of stability of the shell.

Finally, Figure 2 illustrates the effect of the number of longitudinal rods on the parameter of the frequency of oscillations of the considered system. It can be seen that with an increase in the parameter of the oscillation frequency of the system, it first increases, and then begins to decrease at a certain value. This is explained by the fact that, with increase, the weight of the rods increases and this leads to a significant effect of inertial properties.

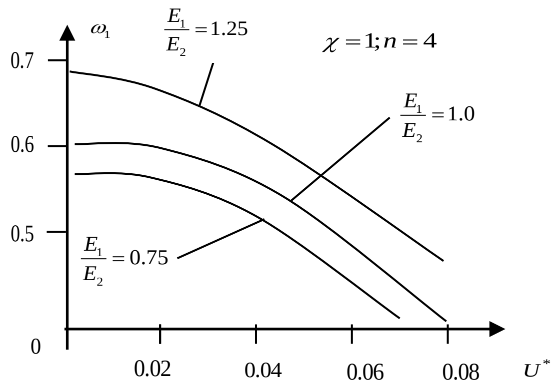

Figure 1. The dependence of the frequency parameter on the flow velocity for a longitudinally reinforced shell in an endless inertial-free medium with a moving fluid

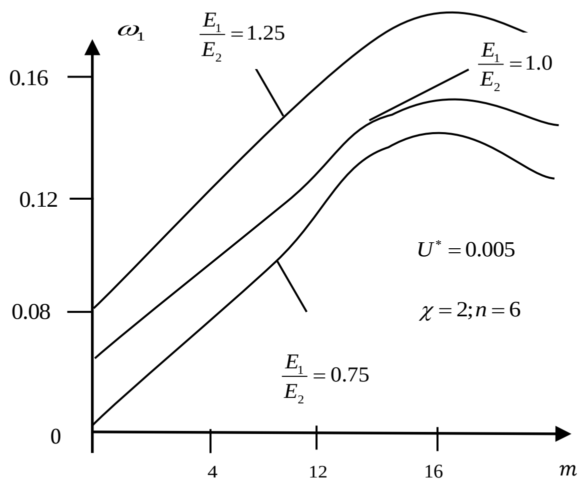

Figure 2. Dependence of the frequency parameter on the number of longitudinal rods of a longitudinally reinforced shell in an infinite inertia-free medium with a moving fluid

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BIOGRAPHY

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