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OSCILLATIONS OF MEDIUM-CONTACTING INHOMOGENEOUS CYLINDRICAL SHELLS STRENGTHENED WITH TRANSVERSE RIBS SUBJECTED TO AXIAL COMPRESSION

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Abstract- In the paper we study one of the dynamical strength characteristics, the frequency of natural vibrations of a fiberglass medium-contacting cylindrical shell inhomogeneous in thickness and along the generator and strengthened with annular ribs subjected to the axial compression under Navier conditions. The motion of the medium is described by the Lame equations in displacements. Using the Hamilton Sharp Urban Variational Principle, frequency equations are constructed to calculate the natural frequencies of the oscillations of the system under study. In the process of computing linear laws are adopted for the heterogeneity function. When studying free oscillations of a medium - contacting cylindrical shell inhomogeneous in thickness and along the generator, strengthened with transverse system of ribs, we consider two cases: a) the effects of inertial actions of the medium on the oscillation process can be neglected; b) the influence of the inertial properties of the medium on the oscillation process is significant. In both cases, the frequency equations are constructed and implemented numerically. The calculation results of the natural vibration frequencies are presented in the form of a dependence on the inhomogeneity parameter, on the number of transverse ribs for various values of the wave formation parameters and various elastic moduli of the medium material. Characteristic curves of dependence are constructed.

Keywords: Strengthened Shell, Variational Principle, Solid Medium, Free Oscillation, Heterogeneity.

I. INTRODUCTION

Composites based on polymer, carbon, metal, and organic and porous aluminum are widely used in various branches of technology. To create heterogeneity in the load-bearing structures, diffusion or other technologies introduce another material with high strength characteristics into its surface layers, resulting in technological heterogeneity in the structure. There is a need to develop methods for calculating such inhomogeneous shells and to study the effect of heterogeneity on the frequencies of their natural vibrations. We need algorithms for determining resonant frequencies, leading to the destruction of inhomogeneous shells. To give greater rigidity, the thin-walled part of the shell is reinforced with ribs, which significantly increases its strength with a slight increase in the mass of the structure, even if the ribs have a small height.

Note that the works [1-3] are devoted to the study of the parametric oscillation of a rectilinear rod nonlinear and non-uniform in thickness in a viscoelastic medium using the Pasternak contact model. The influence of the main factors - elasticity of the base, damage to the material of the rod and shell, the dependence of the shear coefficient on the vibration frequency on the characteristics of the longitudinal oscillation of the points of the rod in a viscoelastic medium is studied. In all cases studied, the dependences of the zone of dynamic stability of rod oscillations in a viscoelastic medium on structural parameters on the load-frequency plane are constructed. In [4], free oscillation of a longitudinally strengthened, orthotropic, moving fluid-contacting cylindrical shell inhomogeneous in thickness, is studied. Using the Hamilton Sharp Urban Variational Principle, systems of equations of motion of a longitudinally strengthened, orthotropic moving-fluid-contacting cylindrical shell inhomogeneous in thickness, is constructed.

The thickness inhomogeneity of the shell material was taken into account, assuming that the Young's modulus and the density of the shell material are functions of the normal coordinate. Frequency equations are constructed and implemented numerically. In the process of calculation, linear and parabolic laws are adopted for the inhomogeneity function. Characteristic curves of dependence are constructed. If the shell has geometric and physical nonlinearity, the equations describing its stress-strain state become complex nonlinear partial differential equations and for solving it in [5] the method of successive loading is constructed. The derivation of these equations is given in [6, 7]. To reduce the linearization error of the equation and reduce the counting time, a two-step method of sequential parameter perturbation has been developed [8]. The influence of the contour support condition on the stability of polymer concrete shells was studied in [9]. In this paper [10] the inhomogeneity was taken into account by accepting the Young modulus and density of the material as a function of coordinate changing in thickness.

II. PROBLEM STATEMENT

To apply the variational principle of Hamilton Sharp Urban, we write the total energy of the structure under study since the structure under study consists of a heterogeneous cylindrical shell and reinforcing ring elements, the numbers of which vary. In addition, the studied structure is in contact with a solid medium (Figure 1(a)).

To take into account the inhomogeneity in the thickness of the cylindrical shell, we will proceed from the three-dimensional functional. In this case, the functional of total energy of cylindrical shell has form:

$$V=\frac{1}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}+\sigma_{12}\varepsilon_{12}+\right.$$
$$\left.+\rho\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dxdydx \quad (1)$$

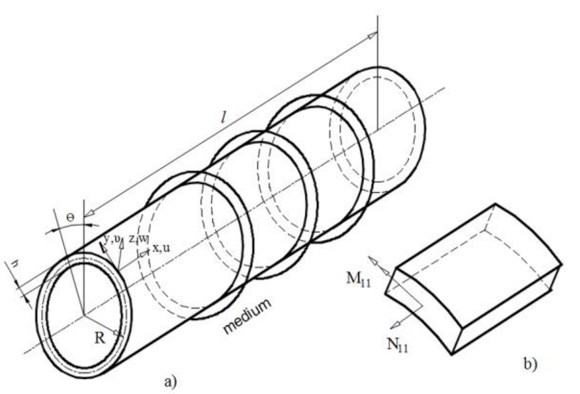

Figure 1. Strengthened inhomogeneous cylindrical shell

There are various ways to account for the heterogeneity of the shell material. One of them is that the Young's modulus and the density of the shell material are accepted as functions of the normal and longitudinal coordinates [11]. It is assumed that the Poisson's ratio is constant. In this case, the strain-stress ratio has the form:

$$\sigma_{11}=\frac{E(x,z)}{1-v^2}\left(\varepsilon_{11}+v\varepsilon_{22}\right);\ \sigma_{22}=\frac{E(x,z)}{1-v^2}\left(\varepsilon_{22}+v\varepsilon_{11}\right);\quad (2)$$
$$\sigma_{12}=G\varepsilon_{12}$$

$$\varepsilon_{11}=\frac{\partial u}{\partial x};\ \varepsilon_{22}=\frac{\partial \vartheta}{\partial y}+\frac{w}{R};\ \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x} \quad (3)$$

Assume that

$$E(x,z)=E_0 f_1(z)f_2(x);\ \rho(z,x)=\rho_0 f_1(z)f_2(x) \quad (4)$$

Considering (4) in (2), we get:

$$\sigma_{11}=\frac{E_0}{1-v^2}\left(\varepsilon_{11}+v\varepsilon_{22}\right)f_1(z)f_2(x)$$
$$\sigma_{22}=\frac{E_0}{1-v^2}\left(\varepsilon_{22}+v\varepsilon_{11}\right)f_1(z)f_2(x) \quad (5)$$
$$\sigma_{12}=G\varepsilon_{12}=\frac{E_0}{2(1+v)}\varepsilon_{12}f_1(z)f_2(x)$$

where, E_0 is the elastic modulus of the homogeneous shell material, ρ_0 is the density of the material of the homogeneous shell.

The functional of the total energy of the cylindrical shell, taking into account (5), has the form:

$$V=\frac{E_0}{2(1-v^2)}\int_{-h/2}^{h/2}f_1(z)dz\times$$
$$\times\iint\left\{\varepsilon_{11}^2+2(1-v)\varepsilon_{11}\varepsilon_{22}+\varepsilon_{22}^2+\varepsilon_{12}^2\right\}\times$$
$$\times f_2(x)dxdy+\int_{-h/2}^{h/2}f_1(z)dz\times \quad (6)$$
$$\times\iint\left(\rho_0\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\right)f_2(x)Rdxd\theta$$

The expression for the potential energy of elastic deformation of the *j*th transverse rib is as follows:

$$\Pi_j=\frac{1}{2}\int_0^{2\pi}\left[\tilde{E}_jF_j\left(\frac{\partial \vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+\tilde{E}_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right.$$
$$\left.+\tilde{E}_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y^2}-\frac{\varphi_{\kappa pj}}{\mathrm{R}}\right)^2+\tilde{G}_jJ_{\kappa pj}\left(\frac{\partial \varphi_{\kappa pi}}{\partial y}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]Rd\theta \quad (7)$$

The kinetic energies of the ribs are written as:

$$K_j=\rho_jF_j\int_0^{2\pi}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial \vartheta_j}{\partial t}\right)^2+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{\kappa pj}}{F_j}\left(\frac{\partial \varphi_{\kappa pj}}{\partial t}\right)^2\right]Rd\theta \quad (8)$$

In expressions (7) and (8), $F_j, J_{zj}, J_{yj}, J_{kpj}$ are the area and moments of inertia of the cross section of the *j*th transverse rod, respectively, with respect to the axis Oz and axis Oy parallel to the axis and passing through the center of gravity of the section, as well as its moment of inertia during torsion; $\tilde{E}_j, \tilde{G}_j$ are the elastic and shear moduli of the material of the transverse rod, respectively, ρ_j is the density of the materials from which the *j*th transverse rod was made.

The potential energy of external surface loads acting from the medium applied to the shell is defined as the work performed by these loads when the system is transferred from a deformed state to an initial undeformed state and is represented as:

$$A_0=-R\int_0^l\int_0^{2\pi}\left(q_xu+q_\theta\vartheta+q_rw\right)dxd\theta \quad (9)$$

We write the potential energy in the shell from the compressive stress σ_x:

$$\Pi=-\frac{\sigma_xhR^2}{2}\int_0^l\int_0^{2\pi R}\left(\frac{\partial w}{\partial x}\right)^2dxdy \quad (10)$$

The total energy of the system is equal to the sum of the energy of elastic deformations of the shell and all the transverse ribs, as well as the potential energies of external loads acting from the medium:

$$J=V+\sum_{j=1}^{k_2}\left(\Pi_j+K_j\right)+A_0+\Pi \quad (11)$$

where, k_2 is the number of transverse ribs.

The equation of motion of the medium takes the form [11]:

$$a_t^2 graddiv\vec{s} - a_e^2 rotrot\vec{s} = \frac{\partial^2 \vec{s}}{\partial t^2} \quad (12)$$

where, $a_t = \sqrt{\frac{\lambda_s + 2\mu_s}{\rho_s}}$, $a_e = \sqrt{\frac{\mu_s}{\rho_s}}$, $\vec{s}(s_x, s_\theta, s_r)$ is the displacement vector, λ_s, μ_s are the Lame coefficients for the medium material, and ρ_s is the density of the medium material.

The solution of the equation of motion of the medium (12) has the form [12]:

a) in the case of no inertial medium

$$s_x = \left[\left(-kr\frac{\partial K_n(kr)}{\partial r} - 4(1-\nu_s)kK_n(kr)\right)A_s + \right.$$
$$\left. + kK_n(kr)B_s\right]\cos n\varphi \cos kx \sin \omega t$$
$$s_\theta = \left[-\frac{n}{r}K_n(kr)B_s - \frac{\partial K_n(kr)}{\partial r}C_s\right]\times$$
$$\times \sin n\varphi \sin kx \sin \omega t \quad (13)$$
$$s_r = \left[-k^2 r K_n(kr)A_s + \frac{\partial K_n(kr)}{\partial r}B_s + \right.$$
$$\left. + \frac{n}{r}K_n(kr)C_s\right]\cos n\varphi \sin kx \sin \omega t$$

b) in the case of an inertial medium

$$s_x = \left[A_{s1} k K_n(\gamma_e r) - C_{s1}\frac{\gamma_t^2}{\mu_t}K_n(\gamma_t r)\right]\times$$
$$\times \cos n\varphi \cos kx \sin \omega t$$
$$s_\theta = \left[-\frac{A_{s1} n}{r}K_n(\gamma_e r) - \frac{C_{s1} nk}{r\mu_t}K_n(\gamma_t r) - \right.$$
$$\left. - \frac{B_{s1}}{n}\frac{\partial K_n(\gamma_t r)}{\partial r}\right]\sin n\varphi \sin kx \sin \omega t \quad (14)$$
$$s_r = \left[\frac{A_{s1}\partial K_n(\gamma_e r)}{\partial r} - \frac{C_{s1} k}{\mu_t}\frac{\partial K_n(\gamma_t r)}{\partial r} + \right.$$
$$\left. + \frac{B_{s1} n}{r}K_n(\gamma_t r)\right]\cos n\varphi \sin kx \sin \omega t$$

In expressions (13), (14) $A_s, B_s, C_s, A_{s1}, B_{s1}, C_{s1}$ are constants, K_n is a modified Bessel of the second kind function of the nth kind, $\gamma_e^2 = k^2 - \mu_e^2$, $\gamma_t^2 = k^2 - \mu_t^2$.

The expression of the total energy of system (11), the equation of motion of the medium (12) are supplemented by contact conditions. On the contact surface of the shell - the medium is observed continuity of displacement and pressure $(r = R)$:

$$s_x = u,\ s_\theta = \vartheta,\ s_r = w \quad (15)$$

$$q_x = \sigma_{rx},\ q_\theta = \sigma_{r\theta},\ q_r = \sigma_{rr} \quad (16)$$

It is considered that the conditions of hard contact between the shell and the rods are satisfied:

$$u_j(y) = u(x_j, y) + h_j\varphi_1(x_j, y)$$
$$\vartheta_j(x) = \vartheta(x_j, y) + h_j\varphi_2(x_j, y)$$
$$w_j(x) = w(x_j, y);\ \varphi_j = \varphi_2(x_j, y)$$
$$\varphi_{kpj}(x) = \varphi_1(x_j, y);\ h_j = 0.5h + H_j^1$$

where, H_j^1 is the distance from the axes of the jth rod to the surface of the cylindrical shell, φ_j, φ_{kpj} are the angles of rotation and twist of the cross section of the jth rod, through the displacements of the shell are expressed as follows:

$$\varphi_j(y) = \varphi_2(x_j, y) = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{r}\right)\Big| x = x_j$$

$$\varphi_{kpj}(y) = \varphi_1(x_j, y) = -\frac{\partial w}{\partial x}\Big|_{x=x_j}$$

It is generally accepted that on the lines $x = 0$ and $x = l$ the Navier boundary conditions are satisfied:

$$\vartheta = 0,\ w = 0,\ N_{11} = 0,\ M_{11} = 0 \quad (17)$$

where, l is the shell length, T_{11}, M_{11} are the forces and moments acting on the cross sections of the cylindrical shell (Figure 1(b)).

The frequency equation of a ribbed inhomogeneous shell with a flowing fluid is obtained on the basis of the principle of stationarity of the Hamilton Sharp Urban action:

$$\delta W = 0 \quad (18)$$

where, $W = \int_{t'}^{t''} J dt$ is the Hamiltonian action, t' and t'' are the given arbitrary moments of time.

Supplementing by contact conditions (5) and (6) the total energy of the system (11), the equations of motion of fluid (12), we arrive at the problem of natural vibrations of a medium-contacting inhomogeneous cylindrical shell strengthened with annular ribs. In other words, the problem of natural oscillations of a medium-contacting inhomogeneous cylindrical shell strengthened with annular ribs is reduced to the joint integration of expressions for the total energy of the system (11), the equation of medium (12) subject to the conditions (15) and (16) on their contact surface and boundary conditions (17).

III. PROBLEM SOLUTION

In expression (11), the variable quantities are $u,\ \vartheta,\ w$. We approximate these unknown quantities as follows:

$$u = u_0 \cos \chi\xi \cos n\theta \sin \omega_1 t_1$$
$$\vartheta = \vartheta_0 \sin \chi\xi \sin n\theta \sin \omega_1 t_1 \quad (19)$$
$$w = w_0 \sin \chi\xi \cos n\theta \sin \omega_1 t_1$$

where, u_0, ϑ_0, w_0 are unknown constants; χ, n are wave numbers in the longitudinal and circumferential directions, respectively, $\xi = x/R$, $\chi = kR = m\pi R/l$, $t_1 = \omega_0 t$, ω is the desired frequency.

To calculate work (9) using (16), we find the contact surface forces q_x, q_θ, q_r. The constants, which are included in their expression $A_s, B_s, C_s, A_{s1}, B_{s1}, C_{s1}$ using contact conditions (15), u_0, ϑ_0, w_0 are expressed through shell constants.

With simplification (11), the following dependences are accepted:

$$f_1(z) = 1 + \alpha \frac{z}{t},\ f_2(x) = 1 + \beta \frac{x}{l} \tag{20}$$

where, α, β are constant parameters of heterogeneity in the direction of normal and along the generatrix of the shell, respectively, and $\alpha, \beta \in [0,1]$.

Substituting solution (20) into (11), taking into account expression (19) for the total energy (11), we obtain a second-order polynomial with respect to constants u_0, ϑ_0, w_0:

$$J_i = \varphi_{11i} u_0^2 + \varphi_{22i} \vartheta_0^2 + \varphi_{33i} w_0^2 + \varphi_{44i} u_0 \vartheta_0 + \varphi_{55i} u_0 w_0 + \varphi_{66i} \vartheta_0 w_0$$

Since the expressions of the coefficients φ_{11i}, φ_{22i}, φ_{33i}, φ_{44i}, φ_{55i}, φ_{66i} are cumbersome, so we do not give them. In them, $i = 1$ corresponds to option a), and $i = 2$ corresponds to option b).

If we vary Π the expression by constants u_0, ϑ_0, w_0 and equate the coefficients of independent variations to zero, we obtain the following system of homogeneous algebraic equations

$$\begin{cases} 2\varphi_{11i} u_0 + \varphi_{44i} \vartheta_0 + \varphi_{55i} w_0 = 0 \\ \varphi_{44i} u_0 + 2\varphi_{22i} \vartheta_0 + \varphi_{66i} w_0 = 0 \\ \varphi_{55i} u_0 + \varphi_{66i} \vartheta_0 + 2\varphi_{33i} w_0 = 0 \end{cases} \tag{21}$$

Since system (20) is a homogeneous system of linear algebraic equations, a necessary and sufficient condition for the existence of its nonzero solution is the equality of its principal determinant to zero. As a result, we obtain the following frequency equation

$$\begin{vmatrix} 2\varphi_{11i} & \varphi_{44i} & \varphi_{55i} \\ \varphi_{44i} & 2\varphi_{22i} & \varphi_{66i} \\ \varphi_{55i} & \varphi_{66i} & 2\varphi_{33i} \end{vmatrix} = 0 \tag{22}$$

We write Equation (22) in the form:

$$\begin{gathered} 4\varphi_{11i}\varphi_{22i}\varphi_{33i} + \varphi_{44i}\varphi_{55i}\varphi_{66i} - \\ -\varphi_{55i}^2\varphi_{22i} - \varphi_{66i}^2\varphi_{11i} - \varphi_{44i}^2\varphi_{33i} = 0 \end{gathered} \tag{23}$$

IV. CONCLUSIONS

Equation (23) was calculated numerically. The parameters contained in the solution of the problem were adopted:

$\rho_0 = \rho_j = 1850$ kg/m^3, $\tilde{E}_j = 6.67 \times 10^9$ N/m^2, $m = 1$, $n = 8$, $h = 1.39$, $R = 160$ cm, $I_{kpj} = 0.48$ mm^4, $I_{xj} = 19.9$ mm^4, $F_j = 0.45$ mm^2, $h_i = 0.45$ mm^2, $\nu = 0.35$, $h_i = 0.45$ mm^2, $\frac{l}{R} = 3$, $\frac{h}{R} = \frac{1}{6}$, $\alpha = 0.4$

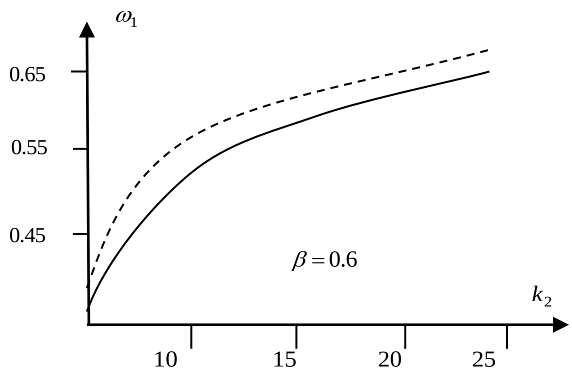

Figure 2. The dependence of the frequency parameter on k_2

The calculation results were shown in Figure 2 in the form of the dependence of the frequency parameter on the number of reinforcing rods k_2. On the shell surface, in Figure 3 in the form of the dependence of the frequency parameter on the heterogeneity parameter in the direction of the generatrix of the shell β. As can be seen from Figure 3, with an increase in the number of transverse ribs, the value of the frequency parameter increases. In these figures, dashed curves correspond to option a), and solid ones option b). It can be seen from the figures that taking into account the inertial actions of the medium leads to a decrease in the eigenfrequencies of the oscillations of the system under study compared to the oscillation frequency of the same system when the medium is inertia-free.

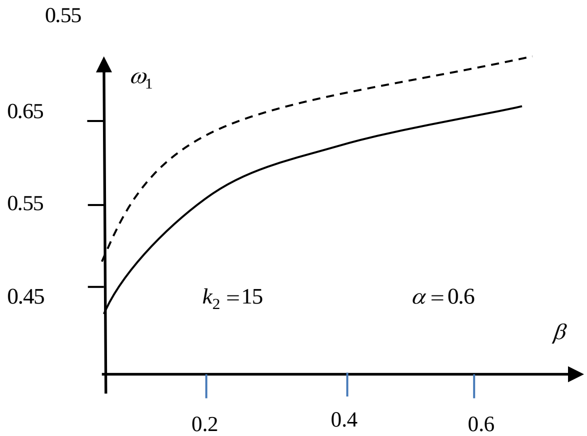

Figure 3. The dependence of the frequency parameter on β

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