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UNSTABLE THERMOMAGNETIC WAVES IN ANISOTROPIC MEDIUM OF ELECTRONIC TYPE OF CHARGE CARRIERS

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Abstract- It is proved that in anisotropic conductive medium it is possible to excite several thermomagnetic waves. Depending on the ratio of the numerical values of the conductivity tensor, these waves increase. Analytical formulas for frequency and for the increment of excited waves are obtained. Frequency and increment values are in some cases the same. In one case, the frequency values are greater than the increment of the wave rise σ_{ik} .

Keywords: Frequency, Increment, Oscillations, Capture, Recombination, Generation.

1. INTRODUCTION

In works [1-3] it is proved that in the presence of hydrodynamic movements of charge carriers in plasma and in isotropic conductive medium with constant temperature gradient, so-called thermomagnetic waves, even without an external magnetic field. In these works, it is proved that the excited thermomagnetic wave in a solid body does not interact with small fluctuations of the lattice, i.e. sound waves. However, the conditions of excitation of thermomagnetic waves and their instability in anisotropic solid bodies have not been investigated. Conditions of excitation of thermomagnetic waves in impurities semiconductors taking into account recombination and generation of charge carriers are theoretically investigated in work [3]. In this theoretical work we will investigate conditions of excitation of thermomagnetic waves in anisotropic conductive medium without external permanent magnetic field.

2. BASIC EQUATIONS OF THE PROBLEM

In the presence of a temperature gradient $\nabla T = \text{const}$, there is a concentration gradient ∇n and hydrodynamic movements of charge carriers with speed $v(\vec{r},t)$.

Under the influence of an external electric field E there is a variable magnetic field inside the medium. Given the above, you can write the following expressions for the density of the H'

$$j' = \sigma \vec{E}^* + \sigma' \left[\vec{E}^* \vec{H}^* \right] - \alpha \vec{\nabla} T - \alpha' \left[\vec{\nabla} T H' \right] \quad (1)$$

where,

$$\vec{E}^* = \vec{E} + \frac{\left[\vec{\upsilon}^* H' \right]}{c} + \frac{T}{e} \frac{\vec{\nabla} n}{n};\ e > 0 \quad (2)$$

In (1) does not take into account diffusion members, which are much smaller than $\sigma \vec{E}^*$. In (2) $\vec{E}$ is electric field, $\frac{\left[\vec{\upsilon}^* H' \right]}{c}$ is electric field created by hydrodynamic movements, $\frac{T}{e} \frac{\vec{\nabla} n}{n}$ is electric field due to the gradient of concentrations of charge carriers, e is positive elementary charge, and T is the temperature of the lattice in the ergs. Substituting (2) in (1) determine the electric field $\vec{E}$. The definition $\vec{E}$ of (1)-(2) is reduced to the solution of the vector equation

$$\vec{x} = \vec{a} + \left[\vec{b}, \vec{x} \right] \quad (3)$$

where, $\vec{x}$ is an unknown vector. From (3)

$$\vec{b}\vec{x} = \vec{b}\vec{a} \quad (4)$$

so, $\left(\vec{b} \left[\vec{b}\vec{x} \right] \right) = 0$.

Given (4) of (3) we can easily get:

$$\vec{x} = \vec{a} + \left[\vec{b}\vec{a} \right] + \left(\vec{b} \left[\vec{b}\vec{x} \right] \right) \quad (5)$$

and

$$\vec{x} = \vec{a} + \left[\vec{b}\vec{a} \right] + \left(\vec{a}\vec{b} \right) \vec{b} \quad (6)$$

Applying (6) in (1) and taking Maxwell's equation $\text{rot}\vec{H} = \frac{4\pi}{cj}$, we get the following expressions for the electric field

$$\vec{E} = -\frac{\left[\vec{v}\vec{H}' \right]}{c} - \Lambda' \left[\nabla \vec{T} \vec{H}' \right] + \frac{c}{4\pi\sigma} \text{rot}\vec{H} - \frac{c\sigma'}{4\pi\sigma^2} \left[\text{rot}\vec{H}', \vec{H}' \right] + \frac{T}{e} \frac{\nabla \rho}{\rho} + \Lambda \nabla T \quad (7)$$

where, $\Lambda=\frac{\alpha}{\sigma}$, $\Lambda'=\frac{\alpha'\sigma-\alpha\sigma'}{\sigma}$, σ is the coefficient of conductivity, Λ is the differential thermoelectric, and Λ' is the coefficient of the Nernsta-Ettipgausen effect.

In anisotropic conductive medium, all these values are tensors. In the isotropic crystal, the full electric field has the following kind of

$$\vec{E}=\varsigma\vec{j}+\varsigma'\left[\vec{j}\vec{H}\right]+\varsigma''\left(\vec{j}\vec{H}\right)H+\Lambda\nabla T+ +\Lambda'\left[\nabla TH\right]+\Lambda''\left(\left[\nabla TH\right]\right)H \quad (8)$$

where, $\varsigma\vec{j}$ is electric field directed at current, $\varsigma'\left[\vec{j}\vec{H}\right]$ is electric field perpendicular to $\vec{j}$ and on $\vec{H}$, $\varsigma''\left(\vec{j}\vec{H}\right)H$ is electric field directed at $\vec{H}$, $\Lambda\nabla T$ is electric field directed at ∇T, $\Lambda'\left[\nabla TH\right]$ is electric field directed perpendicular at ∇T and $\vec{H}$, and $\Lambda''\left(\left[\nabla TH\right]\right)H$ is electric field, directed to ∇T and $\vec{H}$.

In anisotropic conductive medium all physical quantities are tensors. Then from (8) you can write

$$E_i=\varsigma_{im}j'_m+\varsigma'_{im}\left[jH\right]_m+\varsigma''_{im}\left[jH\right]H_m+ +\Lambda_{im}\nabla_m T+\Lambda'_{im}\left[\nabla TH\right]_m+\Lambda''_{im}\left[\nabla TH\right]H_m \quad (9)$$

where, $\varsigma_{im}=\frac{1}{\sigma_{im}}$ is the tensor of the inverse magnitude of the ohmic conductivity, Λ_{im} is the tensor of the differential of electromotive difference of potential, Λ'_{im} is the tensor of the Nernsta-Emminhausens coefficient.

We consider excitation of thermomagnetic waves in anisotropic conductive medium in the presence of an external electric field and at $\nabla T=\text{const}$. Then in equations (9) the members containing $\varsigma'_{im},\varsigma''_{im},\Lambda'_{im},\Lambda_{im}$ are equal to zero. Thus, for teaching the tensor of the electric field, we get the following equations

$$E'_i=\varsigma_{im}j'_m+\Lambda'_{im}\left[\nabla TH'\right]_m$$

$$\text{rot}\vec{H}'=\frac{4\pi}{c}\vec{j}'+\frac{1}{c}\frac{\partial\vec{E}'}{\partial t} \quad (10)$$

$$\text{rot}\vec{E}'=-\frac{1}{c}\frac{\partial\vec{H}'}{\partial t}$$

where, c is speed of light.

Suppose that all variables have the character of monochromatic waves.

$$E'\sim e^{i(\vec{k}\vec{r}-wt)},H'\sim e^{i(\vec{k}\vec{r}-wt)} \quad (11)$$

where, k is wave vector, and w is frequency of oscillation). Then from (10) we can easily get

$$E'_i=\varsigma_{im}j'_m+\Lambda'_{im}\left[\nabla TH\right]_m$$

$$j'_m=\frac{ic^2}{4\pi w}\left[\vec{k}\left[\vec{k}\vec{E}'\right]\right]_m+\frac{iw}{4\pi}E'_m \quad (12)$$

where, $i,m=1,2,3$ dimensionless numbers that determine the direction of the wave inside the conductive medium.

3. THEORY

To obtain the dispersion equation from (12), we have to select the coordinate system defining directions of the wave vector. We choose the coordinate system

$$k_1\neq 0,\ k_2=0,\ k_3=0 \quad (13)$$

Then,

$$\frac{\partial T}{\partial x_1}\neq 0,\ \frac{\partial T}{\partial x_2}\neq 0,\ \frac{\partial T}{\partial x_3}=0 \quad (14)$$

Taking into account (13)-(14) of (12) it is easy to obtain tensor equation for determining the electric field inside anisotropic conductive medium of the following kind

$$E'_i=\left[A\varsigma_{il}k_lk_m+B\varsigma_{im}+\frac{c\Lambda'_{il}}{w}k_l\frac{\partial T}{\partial x_m}-\frac{c\Lambda'_{im}}{w}\left(\vec{k}\vec{\nabla}T\right)\right]\times \times E'_m=R_{im}E'_m \quad (15)$$

where, $A=\frac{ic^2}{4\pi w};B=i\frac{w^2-c^2k^2}{4\pi w}$.

Considering that

$$E'_m=\delta_{im}E'_i \quad (16)$$

From (15) we get $\left(R_{im}-\delta_{im}\right)E'_i=0$, ($\delta_{im}=1$ at $i=m$, $\delta_{im}=0$ at $i\neq m$). The differential equation has the form of

$$\left|R_{im}-\delta_{im}\right|=0 \quad (17)$$

Uncovering the determinant (17) we get the following dispersion equations

$$\left(R_{11}-1\right)\left(R_{22}-1\right)\left(R_{33}-1\right)+ +R_{12}R_{31}R_{23}+R_{21}R_{32}R_{13}- -R_{31}R_{13}\left(R_{22}-1\right)-R_{32}R_{23}\left(R_{11}-1\right)- -R_{21}R_{12}\left(R_{33}-1\right)=0 \quad (18)$$

Tensors R_{im} have the following values

$$R_{11}=\frac{iw}{4\pi}\varsigma_{11},R_{12}=\Omega\varsigma_{12}+\frac{w_{11}-w_{12}}{w}$$

$$R_{13}=\Omega\varsigma_{13}-\frac{w_{13}}{w}$$

$$R_{21}=\frac{iw}{4\pi}\varsigma_{21},R_{22}=\Omega\varsigma_{22}+\frac{w_{21}-w_{22}}{w} \quad (19)$$

$$R_{23}=\Omega\varsigma_{23}-\frac{w_{23}}{w}$$

$$R_{31}=\Omega\varsigma_{31},R_{32}=\Omega\varsigma_{32},R_{33}=\Omega\varsigma_{33}-\frac{w_{33}}{w}$$

$$\Omega=i\frac{w^2-c^2k^2}{w}$$

where the characteristic frequencies of thermomagnetic waves are the following

$$w_{11}=ck\Lambda'_{11}\nabla_2 T,\ w_{12}=ck\Lambda'_{12}\nabla_1 T$$

$$w_{13}=ck\Lambda'_{31}\nabla_1 T$$

$$w_{21}=ck\Lambda'_{21}\nabla_2 T,\ w_{22}=ck\Lambda'_{22}\nabla_1 T \quad (20)$$

$$w_{23}=ck\Lambda'_{23}\nabla_1 T,$$

$$w_{33}=ck\Lambda'_{31}\nabla_1 T$$

From (20) it is clear, that frequencies w_{31} and w_{32} are equal to zero. This was due to the choice of the direction of the wave vector (13). Another choice of the direction of the wave vector will result in different frequency values for the excited thermomagnetic waves. Frequencies of excited thermomagnetic waves depend on the values of tensors R_{ik}, which are included in the dispersion equation (18). Analysis of the dispersion equation (18) shows that different thermomagnetic waves with different values of frequency are excited in the conductive medium at different values of tensors R_{ik}. We will investigate excited thermomagnetic waves in the next selection of tensor values R_{ik}

$$R_{22} = R_{33}, R_{13} = R_{23}$$

i.e.

$$\varsigma_{33} - \varsigma_{22} = \frac{w_{33}}{w_{13} - w_{23}} \left(\varsigma_{13} - \varsigma_{23} \right) \tag{22}$$

Given (21) of (18) we can easily get:

$$\begin{aligned} &\left(R_{11} - 1 \right) \left[\left(R_{22} - 1 \right)^2 - R_{32} R_{23} \right] + \\ &+ R_{12} \left[R_{31} R_{23} - R_{21} \left(R_{22} - 1 \right) \right] + \\ &+ \left[R_{21} R_{32} - R_{31} \left(R_{22} - 1 \right) \right] R_{23} = 0 \end{aligned} \tag{23}$$

It is easy to make sure that (23) is satisfied if

$$R_{31} = R_{13} \tag{24}$$

By supplying values R_{31} and R_{13} from (19) we get

$$i \left(w^2 - c^2 k^2 \right) \left(\varsigma_{13} - \varsigma_{31} \right) = w_{13} \tag{25}$$

or

$$\begin{aligned} &w = w_0 + i w_1 \\ &\left(i w_0^2 - i w_1^2 - 2 w_0 w_1 - c^2 k^2 \right) \left(\varsigma_{13} - \varsigma_{31} \right) = w_{13} \end{aligned} \tag{26}$$

For the validity of the (26) should be

$$w_0 = w_1 \tag{27}$$

Then

$$\left(2 w_1^2 + c^2 k^2 \right) \left(\varsigma_{31} - \varsigma_{13} \right) = w_{13}$$

$$w_1^2 = \frac{w_{11}}{2 \left(\varsigma_{31} - \varsigma_{13} \right)} - \frac{c^2 k^2}{2} \tag{28}$$

Analysis (28) shows that if $\sigma_{13} < \sigma_{31}$ the excited wave attenuates, if $\sigma_{13} > \sigma_{31}$ the excited wave in the above anisotropic conductive medium increases by increment

$$w_1 = \left(\frac{w_{31} \sigma_{31}}{2} \right)^{\frac{1}{2}} \tag{29}$$

Frequency of this wave

$$w_0 = w_1 \tag{30}$$

i.e. the frequency and increment of the excited wave have the same values at

$$R_{31} = R_{21}, \; R_{23} = R_{32} \tag{31}$$

is satisfied (23). Then from (31) we can easily get:

$$w_0 = \left(2 \pi w_{23} \sigma_{32} \frac{\sigma_{21}}{\sigma_{31}} \right)^{\frac{1}{2}} = w_1 \tag{32}$$

Consider the following case

1) $R_{11} = R_{12} = R_{21} = R_{22} = R_{31} = R_{32}, \; w_{11} = w_{12} = w_{21} = w_{22}$

2) $R_{13} = R_{23} = R_{33}$

Taking into account (33) of (18) it is easy to get

$$2 R_{11} = R_{33} = 1 \tag{34}$$

Delivering R_{11} and R_{33} from (19) to (34) we get the following equations to determine the frequency of excited waves

$$\frac{i w}{2 \pi \sigma_{11}} + i \frac{w^2 - c^2 k^2}{4 \pi w \sigma_{33}} - \frac{w_{33}}{w} = 1 \tag{35}$$

From the solution (35) we get

$$\begin{aligned} &w_{1,2} = -i \frac{c^2 k^2}{2 \pi \sigma_{33}} \pm ck \left(\frac{w_{33}}{2 \pi \sigma_{33}} \right)^{\frac{1}{2}} (i - 1) \\ &\frac{1}{4 \sigma_{11}} + \frac{1}{4 \sigma_{33}} = \frac{c^2 k^2}{\sigma_{33}} \end{aligned} \tag{36}$$

From (36) we find frequencies and increment of rising waves

$$\begin{aligned} &w_0 = -ck \left(\frac{w_{33}}{2 \pi \sigma_{33}} \right)^{\frac{1}{2}} \\ &w_1 = ck \left[\left(\frac{w_{33}}{2 \pi \sigma_{33}} \right)^{\frac{1}{2}} - \frac{ck}{2 \pi \sigma_{33}} \right] \end{aligned} \tag{37}$$

From (37) it is clear that for the rise of excited waves the following inequalities are required

$$w_{33} \left(\sigma_{33} + \sigma_{11} \right) > 4 \pi \sigma_{11} \sigma_{33} \tag{38}$$

If tensors R_{ik} have the following values

$$R_{11} = R_{21} = R_{31} = R_{32}, \; R_{12} = R_{22}, \; R_{13} = R_{23} = R_{33} \tag{39}$$

Given (39) of (18) we get

$$\left(1 - R_{33} \right) \left(R_{22} + R_{33} \right) = 1 \tag{40}$$

By supplying values $R_{22,} R_{33}$ from (19) we can easily get the following equations

$$\begin{aligned} &w^2 + [i (\frac{\varsigma_{22} c^2 k^2}{4 \pi} - \frac{\varsigma_{33} c^2 k^2}{8 \pi} + 2 \pi \sigma_{22}) - \\ &- \frac{3}{2} w_{33}] w - c^2 k^2 - i \frac{\sigma_{22} w_{33}}{2 \pi} = 0 \end{aligned} \tag{41}$$

From result of solving problem (41) at $\sigma_{22} = 2 \sigma_{33}$ we get

$$\begin{aligned} &w_{1,2} = \frac{3 w_{33}}{4} - i \pi \sigma_{22} \pm \frac{3 w_{33}}{2} \times \\ &\times \left[1 - \frac{4 \pi^2 w_{22}^2}{9 w_{33}^2} + \frac{4 c^2 k^2}{9 w_{33}^2} - i \frac{2}{3} \frac{\sigma_{22}}{w_{33}} \right]^{\frac{1}{2}} \end{aligned} \tag{42}$$

From (42) we will define σ_{22}

$$\sigma_{22} = \frac{3 w_{33}}{2 \pi} \left(1 + \frac{4 c^2 k^2}{9 w_{33}^2} \right)^{\frac{}{2}} \tag{43}$$

Then taking (43) of (42) we get

$$w_1 = \frac{3}{4} w_{33} (1 - 2\alpha) + i \frac{w_{33}}{2} (\alpha - 1)$$
$$w_2 = \frac{3}{4} w_{33} (1 - 2\alpha) - i \frac{w_{33}}{2} (\alpha + 1) \quad (44)$$
$$\sigma_{22} = \frac{ck}{\pi}, \alpha = \frac{2ck}{2w_{33}}$$

A wave with a frequency w_2 is damping and a wave with a frequency w_1 is increasing if $ck > \frac{3}{2} w_{33}$.

4. CONCLUSIONS

In anisotropic conductive medium in the presence of an external electric field and a constant temperature gradient $\nabla T = \text{const}$ there is a variable magnetic field. In the medium there is redistribution of charge carriers and due to the concentration gradient ∇n, thermomagnetic waves are excited in different directions (20). The frequencies of these thermomagnetic waves have different meanings. The solution of the dispersion equation (19) shows that, depending on the conductivity values σ_{ik}, the excited waves are increasing.

At $\sigma_{13} > \sigma_{31}$ the excited wave increases. In this case, the rise increment and wave frequency have the same values (formula 30). At $\sigma_{31} = \sigma_{32}, \sigma_{23} = \sigma_{32}$ and $w_{23} = 0$ wave is excited by frequency and increment (formula 32). The frequency of these waves depends on the frequency of thermomagnetic waves in a nonlinear way. When performing inequality (38), a purely thermomagnetic wave is excited. This wave is increasing and the rise increment is less than the frequency of that wave. Such anisotropic conductive medium with increasing waves becomes a source of energy, i.e. in this state the medium emits energy from itself. We investigated the dispersion equation (18) in some extreme cases. Of course, there is excitation of some waves in other extreme cases.

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