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# FREE VIBRATIONS OF AN ORTHOTROPIC CYLINDRICAL SHELL STIFFENED WITH CROSS SYSTEMS OF RIBS, WITH MEDIUM AND FLUID CROSS SYSTEMS

**F.T. Mamedrzayeva**

*Azerbaijan University of Architecture and Construction, Baku, Azerbaijan, firaseyidova@gmail.com*

**Abstract-** The present paper is devoted to the study of an orthotropic cylindrical shell stiffened with cross system of ribs and contacting with hollow filler and flowing fluid in the channel of the filler. In the paper, a rigid contact between the shell and filler is considered. When solving the problem, the Hamilton-Ostrogradsky variational-principle is used. The motion of the filler is described by the system of elasticity theory equations in displacements. Using the contact conditions between the shell and filler, between the filler and fluid, a frequency equation for finding natural frequencies of the vibrations of the system under consideration, is found. The frequency equation is implemented numerically. In calculations the properties of the Bessel functions are used. The curves of dependence of natural frequencies of vibrations of the system on the radius of the channel of the filler and on the fluid density, are constructed.

**Keywords:** Orthotropic Shell, Fluid, Solid Medium, Frequency Equation, Stiffened Shell.

## 1. INTRODUCTION

Structural materials are widely used in different fields of machine-building, aircraft building, shipbuilding, etc. This reduces to necessity of complete account of peculiarities of materials and constructions or rational construction and reliable strength analysis. For more complete description of the load-bearing capacity of the construction, it is appropriate to take into account the external force effects as viewed from medium. One of such effects is its contact with a filler and fluid. Note that the solutions described in references predominantly relate to the stiffened, isotropic medium less isotropic cylindrical shells [1].

Vibrations of smooth cylindrical shells with medium were completely studied in [2, 3]. Behavior of deformable smooth shells with flowing fluid was considered in the monographs [4, 5]. Vibrations of laterally stiffened cylindrical shells with flowing fluid in medium, were studied in [6].

Natural vibrations of an isotropic cylindrical shell with flowing ideal fluid and stiffened with cross system of ribs were considered in [7]. Ref. [8] deals with forced axially-symmetric vibrations of a fluid-filled cylindrical shell stiffened and loaded with axial compressive force.

As it follows from this review, there are almost no works devoted to free vibrations of anisotropic ridge shells with a filler having a fluid-filled central channel. Therefore, investigation of one of principle dynamical characteristics of the elastic system, the frequency of edge anisotropic cylindrical shells with a filler having a fluid-filled channel is of great practical interest.

## 2. PROBLEM STATEMENT

We consider free vibrations of orthotropic edge shells with a filler having a central channel filled with ideal moving fluid. Study of the problem of natural vibrations of a cylindrical shell with a filler and flowing fluid and stiffened with cross system of ribs is reduced to joint integration of the equations of theory of shells, medium and fluid subject to indicated conditions on their contact surface. Equations of motion of a ridge shell has form:

$$\left[\left(a_1+\gamma_c^{(1)}\right)\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\varphi^2}\right]u+\left(1+a_{12}\right)\frac{\partial^2\vartheta}{\partial\xi\partial\varphi}-$$

$$-\left(a_{12}\frac{\partial}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)w-\rho_1\frac{\partial^2 u}{\partial t_1^2}=\frac{R^2q_x}{G_{12}h}$$

$$\left(1+a_{12}\right)\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{\frac{1-\nu}{2}\frac{\partial^2}{\partial\xi^2}+\left[1+\left(1-\frac{h_s}{R}\right)^2\gamma_s^{(2)}+a_2\right]\frac{\partial^2}{\partial\theta^2}\right\}\vartheta-$$

$$-\left[a_2+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial w}{\partial\theta}-\rho_2\frac{\partial^2\vartheta}{\partial t_1^2}=\frac{R^2q_y}{G_{12}h}$$

$$-\left(a_{12}\frac{\partial}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)u-\left\{\left[a_2+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]-\right.$$

$$\left.\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\frac{\partial^3}{\partial\theta^3}\right\}\vartheta+\left\{a_2+\gamma_s^{(2)}+\eta_{s1}^{(2)}+\right.$$

$$+2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)\frac{\partial^2}{\partial\theta^2}+a^2\left(a_1+\eta_c^{(1)}\right)\frac{\partial^4}{\partial\xi^4}+$$

$$\left.+\left[2\left(a_{12}+2\right)\frac{\partial^4}{\partial\xi^2\partial\theta^2}+\left(a_{12}+\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)\frac{\partial^4}{\partial\theta^4}\right]\right\}+$$

$$+\rho_3\frac{\partial^2 w}{\partial t_1^2}=\frac{R^2}{G_{12}h}\left(q_z+q_{zz}\right)$$

where,

$\rho_1 = 1 + \bar{\rho}_c \bar{\gamma}_c^{(1)}$

$\rho_2 = 1 + \bar{\rho}_s \bar{\gamma}_s^{(2)}$

$\rho_3 = 1 + \bar{\rho}_c \bar{\gamma}_c^{(2)} + \bar{\rho}_s \bar{\gamma}_s^{(2)}$

$\bar{\gamma}_c^{(1)} = \varphi_1^1$

where, $\varphi_1^1$ is the ratio of the weight of all ribs to the weight of the shell, $\bar{\gamma}_s^{(2)} = \frac{F_s}{L_1 h}$,

$\delta_s^{(2)} = \frac{h_s}{R}\bar{\gamma}_s^{(2)}$, $\gamma_s^{(2)} = \frac{E_s\left(1-\nu^2\right)}{G_{12}}\bar{\gamma}_s^{(2)}$, $\bar{\rho}_c = \frac{\rho_c}{\rho_0}$, $\bar{\rho}_s = \frac{\rho_s}{\rho_o}$

where, $\rho_0$, $\rho_c$, $\rho_s$ are densities of shell and rib materials, respectively), $\delta_c^{(1)} = \frac{h_c}{r}\gamma_c^{(1)}$, $\gamma_c^{(1)} = \frac{E_c}{E}\left(1-\nu^2\right)\bar{\gamma}_c^{(1)}$, $E$, $\nu$ are modulus of elasticity and the Poisson ratio of the shell material, $R$ is a median surface radius of the shell, $E_s$, $E_c$ are elasticity modulus of the rib material, $a^2 = \frac{h}{12R^2}$, $\Delta = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\theta^2}$, $\delta_s^{(2)} = \frac{h_s}{R}\bar{\gamma}_s^2$,

$\eta_c^{(1)} = \frac{E_c\left(J_{yc} + h^2 F_c\right)}{2\pi R^3 hE}\left(1-\nu_{12}^2\right)$, $\eta_{s1}^{(2)} = \frac{E_s J_{xs}\left(1-\nu_{12}^2\right)}{EL_1 R^2 h}$,

$\eta_{s2}^{(2)} = \frac{E_s\left(1-\nu_{12}^2\right)}{E}\bar{\eta}_s^{(2)}$, $\bar{\eta}_s^{(2)} = \left(\frac{h_s}{R}\right)^2 \bar{\gamma}_s^{(2)}$, $L_1$ is the length of the shell, $J_{xs}$ is inertia moment of the cross-section of the rib with respect to the axis $ox$, $F_c$, $F_s$, $J_{yc}$ are area and inertia moment of the cross-section of the rib with respect to the axis $oz$, $\xi = \frac{x}{R}$, $\theta = \frac{y}{R}$, $u, \vartheta, w$ are the displacement parameters of the median surface of the shell, $t_1 = \omega_0 t$, $\omega_0 = \sqrt{\frac{G_{12}}{\left(1-\nu_{12}^2\right)\rho_0 R^2}}$, $q_x, q_\phi, q_z, q_{zz}$ is a pressure as viewed from medium and fluid on the shell, respectively. Here the indices "$c$" belongs to longitudinal, the index "$s$" to the lateral ribs.

The equation of motion of the medium is of the form [2, 3]:

$$\left(\lambda+2\mu\right)\frac{\partial\theta}{\partial r} - \frac{2\mu}{r}\frac{\partial\omega_x}{\partial\varphi} + 2\mu\frac{\partial\omega_\varphi}{\partial x} - \rho\frac{\partial^2 s_x}{\partial t^2} = 0$$

$$\left(\lambda+2\mu\right)\frac{1}{r}\frac{\partial\theta}{\partial\varphi} - 2\mu\frac{\partial\omega_r}{\partial x} + 2\mu\frac{\partial\omega_x}{\partial x} - \rho\frac{\partial^2 s_\varphi}{\partial t^2} = 0 \qquad (2)$$

$$\left(\lambda+2\mu\right)\frac{\partial\theta}{\partial x} - \frac{2\mu}{r}\frac{\partial}{\partial r}\left(r\omega_\varphi\right) + \frac{2\mu}{r}\frac{\partial\omega_r}{\partial\varphi} - \rho\frac{\partial^2 s_r}{\partial t^2} = 0$$

where, $s_x$, $s_\varphi$, $s_r$ are the components of displacement vector of medium; $\lambda_s$ and $\mu_s$ are Lame elasticity modulus, and $\rho$ is fluid density.

Volume expansion $\theta$ and rotation components $\omega_x$, $\omega_\varphi$, $\omega_r$ are determined from the expressions:

$$\theta = \frac{\partial s_r}{\partial r} + \frac{s_r}{r} + \frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi} + \frac{\partial s_x}{\partial x};\ 2\omega_x = \frac{1}{r}\left[\frac{\partial\left(rS_\varphi\right)}{\partial r} - \frac{\partial s_r}{\partial\varphi}\right]$$

$$2\omega_\varphi = \frac{\partial s_r}{\partial x} - \frac{\partial s_x}{\partial r};\ 2\omega_r = \frac{1}{r}\frac{\partial s_x}{\partial\varphi} - \frac{\partial s_\varphi}{\partial x}$$

The stress tensor components $\sigma_{rx}, \sigma_{r\varphi}, \sigma_{rr}$ are determined as follows [2]:

$$\sigma_{rx} = \mu\left(\frac{\partial s_x}{\partial r} + \frac{\partial s_r}{\partial x}\right)$$

$$\sigma_{r\varphi} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{s_\varphi}{r}\right) + \frac{1}{r}\frac{\partial s_r}{\partial\varphi}\right] \qquad (3)$$

$$\sigma_{rr} = \lambda\left(\frac{\partial s_x}{\partial x} + \frac{1}{r}\frac{\partial\left(rs_r\right)}{\partial r} + \frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi}\right) + 2\mu\frac{\partial s_r}{\partial r}$$

It is assumed that the main part of flow equals $U$ and deviations from this velocity are small, we use the wave equation for the potential of perturbed velocities $\varphi$ with respect to [4]:

$$\Delta\varphi - \frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2} + 2U\frac{\partial^2\varphi}{R\partial\xi\partial t} + U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right) = 0 \qquad (4)$$

The equation of motion of the shell (1) of medium (2) and fluid (4) are complemented with contact conditions. Suppose that the contact between the shell and medium is rigid, i.e. for $r = R$ the equality of displacements

$$s_x = u, s_\varphi = \upsilon,\ s_r = w\ (r = R) \qquad (5)$$

the equality of pressures

$$q_x = -\sigma_{rx},\ q_\varphi = -\sigma_{r\varphi},\ q_r = -\sigma_{rr}\ \ (r = R) \qquad (6)$$

On the contact surface a medium-fluid we observe continuity of radial velocities and pressures. The permeability or smoothness conditions at the medium wall have the form [4]:

$$\vartheta_r\big|_{r=a} = \frac{\partial\varphi}{\partial r}\big|_{r=a} = -\left(\omega_0\frac{\partial s_r}{\partial t_1} + U\frac{\partial s_r}{R\partial\xi}\right) \qquad (7)$$

Equality of radial pressures as viewed from fluid on medium

$$\sigma_{rx} = 0,\quad \sigma_{r\theta} = 0,\quad \sigma_{rr} = -p\quad (r = a) \qquad (8)$$

where, $a$ is a radius of the medium channel.

To contact conditions (5)-(8) we add boundary conditions. It is supposed that the shell is highly supported by the edges, i.e. for $\xi = 0$ and $\xi = \xi_1\ (\xi_1 = L_1 / R)$ it is fulfilled

$$\vartheta = w = 0,\ T_1 = M_1 = 0 \qquad (9)$$

and for the medium

$$\sigma_{xx} = 0;\ \ s_\varphi = s_r = 0 \qquad (10)$$

Supplementing with contact conditions (5)-(8) the equations of motion of the shell (1), of medium (2) and fluid (4), we get a problem of natural vibrations of a cylindrical shell with elastic medium and flowing flow and stiffened with cross system of ribs. In other words, the problem of natural vibrations of the shell with elastic medium and with flowing fluid and stiffened with cross system of ribs is reduced to joint integration of the equations of theory of shells, medium and fluid subject to conditions on their contact surface.

## 3. PROBLEM SOLUTION

We look for the potential of perturbed velocities $\varphi$ in the form:

$$\varphi(\xi,r,\theta,t_1)=f(r)\cos n\varphi\sin\chi\xi\sin\omega_1 t_1 \quad (10)$$

Using (10), from conditions (7), (8), we have:

$$\begin{aligned}&\varphi=-\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right)\\&p=\Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2}+2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1}+U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right)\end{aligned} \quad (11)$$

where

$$\Phi_{\alpha n}=\begin{cases}I_n(\beta r)/I_n'(\beta r), & M_1<1\\ J_n(\beta_1 r)/J_n'(\beta_1 r), & M_1>1\\ \dfrac{R^n}{nR^{n-1}}, & M_1=1\end{cases} \quad (12)$$

where, $M_1=\dfrac{U+\omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2=R^{-2}\left(1-M_1^2\right)\chi^2$, $\beta_1^2=R^{-2}\left(M_1^2-1\right)\chi^2$, $I_n$ is a first kind modified Bessel function of order $n$, $J_n$ is a first kind Bessel function of order $n$, $\omega_1=\omega/\omega_0$.

We will look for the perturbations of the shell in the form:

$$\begin{aligned}&u=u_0\sin\chi\xi\cos n\varphi\sin\omega_1 t_1\\&\vartheta=\vartheta_0\cos\chi\xi\sin n\varphi\sin\omega_1 t_1\\&w=w_0\cos\chi\xi\cos n\varphi\sin\omega_1 t_1\end{aligned} \quad (13)$$

where, $u_0,\vartheta_0,w_0$ are unknown constants; $\chi,n$ are wave numbers in longitudinal and peripheral directions, respectively.

Then in (1) as $q_{zz}$ we should take the quantity $q_{zz}=-p$, where $p$ is a pressure according to (11). Allowing in (13) we can represent pressure $p$ in form:

$$p=\frac{\rho_m\Phi_{\alpha n}}{\rho_0\omega_0^2 h}\left(\omega_0^2\omega_1^2+2\omega_0\omega_1\chi U+\chi^2U^2\right)w \quad (14)$$

We will look for the solution of the equations of the motion of elastic filler (2) in two variants: a) assuming that medium inertia influence on the vibration process is negligible; b) assuming that influence of medium motion inertia on the vibrations process is significant and it cannot be neglected.

For displacements of the filler we have [2]:

in the case a)

$$\begin{aligned}s_x=&\left[\left(-kr\frac{\partial I_n(kr)}{\partial r}-4(1-v_s)kI_n(kr)\right)A_s+\right.\\&+\left(-kr\frac{\partial K_n(kr)}{\partial r}-kI_n(kr)B_s+4(1-v_s)kK_n(kr)\right)\times\\&\left.\times A_s+kK_n(kr)B_s\right]\times\cos n\phi\cos\chi\xi\sin\omega_1 t_1\end{aligned} \quad (15)$$

$$s_x=\left[-\frac{n}{r}I_n(k)B_s-\frac{\partial I_n(kr)}{\partial r}C_s-\frac{n}{r}K_n(kr)B_s-\frac{\partial I_n(kr)}{\partial r}C_s\right]\times$$
$$\times\sin n\varphi\sin\chi\xi\sin\omega_1 t_1$$

$$s_x=\left[-k^2rI_n(kr)A_s+\frac{\partial I_n(kr)}{\partial r}B_s+\frac{n}{r}I_n(kr)C_s-k^2rK_n(kr)A_s+\right.$$
$$\left.+\frac{\partial K_n(kr)}{\partial r}B_s+\frac{n}{r}K_n(kr)C_s\right]\cos n\phi\sin\chi\xi\sin\omega_1 t_1$$

in the case b)

$$s_x=\left[A_skI_n(\gamma_e r)-C_s\frac{\gamma_t^2}{\mu_t}I_n(\gamma_t r)+A_skK_n(\gamma_e r)-C_s\frac{\gamma_t^2}{\mu_t}K_n(\gamma_t r)\right]\times$$
$$\times\cos n\varphi\cos\chi\xi\sin\omega_1 t_1$$

$$s_\theta=\left[-\frac{A_sn}{r}I_n(\gamma_e r)-\frac{C_snk}{r\mu_t}I_n(\gamma_t r)-\frac{B_s}{n}\frac{\partial I_n(\gamma_t r)}{\partial r}-\frac{A_sn}{r}K_n(\gamma_e r)-\right.$$
$$\left.-\frac{C_snk}{r\mu_t}K_n(\gamma_t r)-\frac{B_s}{n}\frac{\partial K_n(\gamma_t r)}{\partial r}\right]\sin n\varphi\sin\chi\xi\sin\omega_1 t_1 \quad (16)$$

$$s_\theta=\left[A_s\frac{\partial I_n(\gamma_e r)}{\partial r}-\frac{C_sk}{\mu_t}\frac{\partial I_n(\gamma_t r)}{\partial r}+\frac{B_sn}{r}I_n(\gamma_t r)+A_s\frac{\partial K_n(\gamma_e r)}{\partial r}-\right.$$
$$\left.-\frac{C_sk}{\mu_t}\frac{\partial K_n(\gamma_t r)}{\partial r}+\frac{B_sn}{r}K_n(\gamma_t r)\right]\cos n\varphi\sin\chi\xi\sin\omega_1 t_1$$

where, $k$, $n$, $\gamma_e$, $\gamma_t$ are wave numbers corresponding to compression and shear wave, and we have the dependences $\gamma_e^2=k^2-\mu_e^2$, $\gamma_t^2=k^2-\mu_t^2$.

Using (15), (13), (3) and contact conditions (5), (6), we get a system of algebraic equations with respect to the constants $u_0,\vartheta_0,w_0$, $A_s,B_s,C_s,\tilde{A}_s,\tilde{B}_s,\tilde{C}_s$. This system has a bulky form, and we do not give it here. By means of this system the constants $A_s,B_s,C_s,\tilde{A}_s,\tilde{B}_s,\tilde{C}_s$ are denoted by $u_0,\vartheta_0,w_0$.

$$\begin{aligned}&A_s=\Delta^{-1}(\Delta_1^{(1)}u_0+\Delta_1^{(2)}\vartheta_0+\Delta_1^{(3)}w_0)\\&B_s=\Delta^{-1}(\Delta_2^{(1)}u_0+\Delta_2^{(2)}\vartheta_0+\Delta_2^{(3)}w_0)\\&C_s=\Delta^{-1}\left(\Delta_3^{(1)}u_0+\Delta_3^{(2)}\vartheta_0+\Delta_3^{(3)}w_0\right)\\&A_s=\Delta^{-1}\left(\Delta_4^{(1)}u_0+\Delta_4^{(2)}\vartheta_0+\Delta_4^{(3)}w_0\right)\\&B_s=\Delta^{-1}\left(\Delta_5^{(1)}u_0+\Delta_5^{(2)}\vartheta_0+\Delta_5^{(3)}w_0\right)\\&C_s=\Delta^{-1}\left(\Delta_6^{(1)}u_0+\Delta_6^{(2)}\vartheta_0+\Delta_6^{(3)}w_0\right)\end{aligned} \quad (17)$$

where, $\Delta$ is a principle determinant, $\Delta_i^{(j)}$ are auxiliary determinants of the mentioned system.

Substituting (17) in expression (3), for the stresses we get:

$$\sigma_{rx}=-\mu_s\Delta^{-1}\left[\left(q_{11}\Delta_1^{(1)}+q_{12}\Delta_2^{(1)}+q_{13}\Delta_3^{(1)}+\right.\right.$$
$$\left.+q_{14}\Delta_4^{(1)}+q_{15}\Delta_5^{(1)}+q_{16}\Delta_6^{(1)}\right)\cdot u_0+$$

$$+\left(q_{11}\Delta_1^{(2)}+q_{12}\Delta_2^{(2)}+q_{13}\Delta_3^{(2)}+q_{14}\Delta_4^{(2)}+q_{15}\Delta_5^{(2)}+q_{16}\Delta_6^{(2)}\right)\cdot\vartheta_0+$$
$$+\left(q_{11}\Delta_1^{(3)}+q_{12}\Delta_2^{(3)}+q_{13}\Delta_3^{(3)}+q_{14}\Delta_4^{(3)}+q_{15}\Delta_5^{(3)}+q_{16}\Delta_6^{(3)}\right)\cdot w_0\Big]\times$$
$$\times\cos n\varphi\cos\chi\xi\sin\omega_1 t_1 \quad (18)$$

$$\sigma_{r\varphi}=-\mu_s\Delta^{-1}\Big[\left(p_{11}\Delta_1^{(1)}+p_{12}\Delta_2^{(1)}+p_{13}\Delta_3^{(1)}+p_{14}\Delta_4^{(1)}+p_{15}\Delta_5^{(1)}+p_{16}\Delta_6^{(1)}\right)\cdot u_0$$
$$+\left(p_{11}\Delta_1^{(2)}+p_{12}\Delta_2^{(2)}+p_{13}\Delta_3^{(2)}+p_{14}\Delta_4^{(2)}+p_{15}\Delta_5^{(2)}+p_{16}\Delta_6^{(2)}\right)\cdot\vartheta_0+$$
$$+\left(p_{11}\Delta_1^{(3)}+p_{12}\Delta_2^{(3)}+p_{13}\Delta_3^{(3)}+p_{14}\Delta_4^{(3)}+p_{15}\Delta_5^{(3)}+p_{16}\Delta_6^{(3)}\right)\cdot w_0\Big]$$
$$\times\sin n\varphi\sin\chi\xi\sin\omega_1 t_1$$

$$\sigma_{rr}=-\mu_s\Delta^{-1}\Big[\left(r_{11}\Delta_1^{(1)}+r_{12}\Delta_2^{(1)}+r_{13}\Delta_3^{(1)}+r_{14}\Delta_4^{(1)}+r_{15}\Delta_5^{(1)}+r_{16}\Delta_6^{(1)}\right)\cdot u_0$$
$$+\left(r_{11}\Delta_1^{(2)}+r_{12}\Delta_2^{(2)}+r_{13}\Delta_3^{(2)}+r_{14}\Delta_4^{(2)}+r_{15}\Delta_5^{(2)}+r_{16}\Delta_6^{(2)}\right)\cdot\vartheta_0+$$
$$+\left(r_{11}\Delta_1^{(3)}+r_{12}\Delta_2^{(3)}+r_{13}\Delta_3^{(3)}+r_{14}\Delta_4^{(3)}+r_{15}\Delta_5^{(3)}+r_{16}\Delta_6^{(3)}\right)\cdot w_0\Big]$$
$$\times\cos n\varphi\sin\chi\xi\sin\omega_1 t_1$$

where,

$$q_{11}=\left(\chi I_n(\chi)+\chi I_n''(\chi)+(5-4\nu_s)I_n'(\chi)\right)\chi^2$$
$$q_{12}=-2\chi^2 I_n'(\chi);\; q_{13}=-n\chi I_n(\chi)$$
$$q_{14}=\left(\chi K_n(\chi)+\chi K_n''(\chi)+(5-4\nu_s)K_n'(\chi)\right)\chi^2$$
$$q_{15}=-2\chi^2 K_n'(\chi);\; q_{16}=-n\chi K_n(\chi)$$
$$p_{11}=-n\chi^2 I_n(\chi);\; p_{12}=2n\left(\chi I_n'(\chi)-I_n(\chi)\right)$$
$$p_{13}=\chi^2 I_n''(\chi)-\chi I_n'(\chi)+n^2 I_n(\chi);\; p_{14}=-n\chi^2 K_n(\chi)$$
$$p_{15}=2n\left(\chi K_n'(\chi)-K_n(\chi)\right)$$
$$p_{16}=\chi^2 K_n''(\chi)-\chi K_n'(\chi)+n^2 K_n(\chi)$$
$$r_{11}=\left(2(1-2\nu_s)I_n(\chi)+2\chi I_n'(\chi)\right)\chi^2$$
$$r_{13}=2n\left(I_n(\chi)-\chi I_n'(\chi)\right);\; r_{12}=-2\chi^2 I_n''(\chi)$$
$$r_{14}=\left(2(1-2\nu_s)K_n(\chi)+2\chi K_n'(\chi)\right)\chi^2$$
$$r_{15}=-2\chi^2 K_n''(\chi);\; r_{16}=2n\left(I_n(\chi)-\chi I_n'(\chi)\right)$$

Using (18) and contact conditions (6), we can determine contact stresses $q_x, q_\theta, q_r$. We represent them in the form

$$q_x=\left(C_{x1}A+C_{x2}B+C_{x3}C\right)\cos n\varphi\cos\chi\xi\sin\omega_1 t_1$$
$$q_\varphi=\left(C_{\theta 1}A+C_{\theta 2}B+C_{\theta 3}C\right)\cos n\varphi\cos\chi\xi\sin\omega_1 t_1 \quad (19)$$
$$q_r=\left(C_{r1}A+C_{r2}B+C_{r3}C\right)\cos n\varphi\cos\chi\xi\sin\omega_1 t_1$$

Substituting (13) and (19) in (1), we get a system of homogeneous algebraic equations with respect to the constants $u_0, \vartheta_0, w_0$. Nontrivial solution of this system is possible only in the case when $\omega_1$ is the root of its determinant. As a result, we get the frequency equation

$$\det\left\|\alpha_{ij}\right\|=0,\quad i,j=1,2,3 \quad (20)$$

In the case b) the obtained frequency equation for finding the parameter of the vibration frequency, formally coincides with Equation (20) and as Equation (20) is transcendental with respect to $\omega_1$, since the sought for parameter of vibration frequency enters into the argument of the Bessel function.

## 4. NUMERICAL RESULTS

Let us consider some results of calculations performed proceeding from the above dependences. For geometrical and physical parameters characterizing the system, the followings were adopted:

$E_c=E_s=6.67\times10^9$ N/m$^2$, $\rho_0=\rho_c=\rho_s=0.26\times10^4$ Ns$^2$/m$^4$

$F_c=3.4$ mm$^2$, $J_{yc}=5.1$ mm$^4$, $h_c=1.39$ mm, $R=160$ mm

$h=0.45$ mm, $h_s=1.95$ mm, $I_{xh}=19.9$ mm$^4$

$I_{kp.s}=0.48$ mm$^4$, $\rho=1000$ kg/m$^3$

$a_l=2.25a_t$, $a_t=308$ m/sec

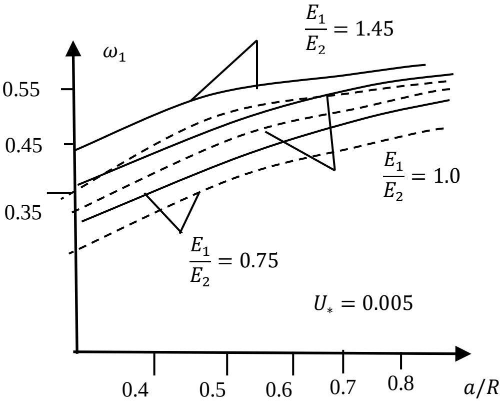


Figure 1. Dependence of the frequency of natural vibrations of the system on solid medium channel

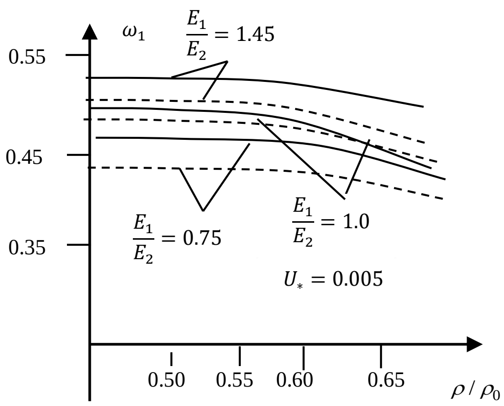


Figure 2. Dependence of the frequency of natural vibrations of the system on fluid density

The results of calculations were represented in Figure 1 and Figure 2. In Figures 1 and 2 the dotted lines correspond to the case b), entire curves to the case a). In Figure 1 dependence of $\omega_1$ on the radius of medium channel, in Figure 2 dependence of $\omega_1$ on fluid density for different ratios of the elasticity modulus of the shell material, was given.

Calculation shows that with increasing the radius of the medium channel the frequencies of the studied system increase while with increasing fluid density, the frequencies of the studied system decrease. It can be seen from the figures that accounting of inertial properties of the medium reduces to decrease of the value of natural frequency of the studied construction. Furthermore, with increasing the ratio $E_1 / E_2$ the frequencies of natural vibrations of the system increase.

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## BIOGRAPHY

**Firangiz Tofiq Mammadrzayeva** was born in Baku, Azerbaijan, in 1985. She graduated Bachelor degree in the field of Hydraulic Structures from Water Farm and Engineering Communication Systems Faculty, Azerbaijan Architectural and Construction University (Baku, Azerbaijan) in 2008. She graduated with honors Master degree in the field of River and Underground Hydraulic Structures from the same faculty and university in 2011. Currently, she is a Doctoral student of Amelioration and Water Farm Construction Faculty in the same university. She is also working as Senior laboratory assistant for Foundations and Underground Structures Department, Azerbaijan Architecture and Construction University (Baku, Azerbaijan).