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VIBRATIONS OF A MOVING FLUID-CONTACTING THREE-DIMENSIONAL CYLINDER STIFFENED WITH RODS

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Abstract- In many cases, the use of thick-walled cylinders in oil and gas transportation makes necessary to use more exact methods in their calculation. In the present paper based just on this direction, in the calculation of a cylinder we use a system consisting of the equations of three-dimensional elasticity theory. In solving the problem, the Hamilton-Ostrogradsky principle was used.

Keywords: Three-Dimensional Cylinder, Elasticity Theory, Thick-Walled Cylinder, Stiffened Cylinder, Ideal Fluid.

1. INTRODUCTION

In calculating stiffened, cylindrical form structural elements the two-dimensional model is preferable. In [1], free vibrations of an orthotropic shell-solid medium fluid system stiffened with rings, were studied. Free vibrations of a viscous fluid and soil-contacting orthotropic cylindrical shell stiffened with rings were researched in [2]. Ref. [3] deals with vibrations of an orthotropic, moving fluid-contacting viscous-elastic cylindrical shell stiffened with cross ribs.

Ref. [4] was devoted to vibrations of an orthotropic, moving fluid-contacting viscous-elastic cylindrical shell stiffened with rods. The vibrations of an anisotropic cylindrical shell stiffened with cross system of ribs were considered in [5].

The conducted analysis shows that the problem of vibrations of a moving fluid-contacting, three-dimensional cylinder has not been studied enough.

2. PROBLEM STATEMENT

In this paper, vibrations of a flowing fluid-contacting three-dimensional cylinder stiffened with rods, were studied. In solving the problem, the Ostrogradsky-Hamilton variational principle was used.

To apply the mentioned method, the total energy of the system consisting of a three-dimensional cylinder, fluid and rods (Figure 1) is written. To write the total energy of a three-dimensional cylinder, we use a three-dimensional functional:

$$V=\iiint\frac{1}{2}(\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}+\sigma_{12}\varepsilon_{12}+ \\ +\sigma_{13}\varepsilon_{13}+\sigma_{21}\varepsilon_{23}+\sigma_{33}\varepsilon_{33}+\rho_s\left(\left(\frac{\partial s_x}{\partial t}\right)^2+\right. \\ \left.\left.+\left(\frac{\partial s_\theta}{\partial t}\right)^2+\left(\frac{\partial s_r}{\partial t}\right)^2\right)\right)rdxd\theta dx \tag{1}$$

where, $\varepsilon_{11},\varepsilon_{12},\varepsilon_{13},\varepsilon_{22},\varepsilon_{23},\varepsilon_{33}$ are deformations of the cylinder's points, s_x,s_θ,s_r are displacements of the cylinder's points, $\sigma_{11},\sigma_{22},\sigma_{12},\sigma_{13},\sigma_{22},\ \sigma_{23},\sigma_{33}$ are stresses of the cylinder's points, $x,\ r,\theta$ are longitudinal, radial, annular coordinates, ρ_s is density of the cylinder's material.

The deformations and stresses in expression (1) are expressed in cylindric coordinates by means of displacements of cylindrical points:

$$\varepsilon_{rr}=\frac{\partial s_x}{\partial r}$$

$$\varepsilon_{\theta\theta}=\frac{1}{r}\frac{\partial s_\theta}{\partial\theta}+\frac{s_r}{r}$$

$$\varepsilon_{xx}=\frac{\partial s_x}{\partial x}$$

$$\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial s_r}{\partial\theta}+\frac{\partial s_\theta}{\partial r}-\frac{s_\theta}{r}\right)$$

$$\varepsilon_{x\theta}=\frac{1}{2}\left(\frac{\partial s_\theta}{\partial x}+\frac{1}{r}\frac{\partial s_x}{\partial\theta}\right)$$

$$\varepsilon_{xr}=\frac{1}{2}\left(\frac{\partial s_x}{\partial r}+\frac{1}{r}\frac{\partial s_r}{\partial x}\right)$$

$$\sigma_{rx}=\mu_s\left(\frac{\partial s_x}{\partial r}+\frac{\partial s_r}{\partial x}\right)$$

$$\sigma_{r\varphi}=\mu_s\left[r\frac{\partial}{\partial r}\left(\frac{s_\varphi}{r}\right)+\frac{1}{r}\frac{\partial s_r}{\partial\varphi}\right]$$

$$\sigma_{rr}=\lambda_s\left(\frac{\partial s_x}{\partial x}+\frac{1}{r}\frac{\partial(rs_r)}{\partial r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi}\right)+2\mu_s\frac{\partial s_r}{\partial r}$$

The expressions for the potential energy of elastic deformation of ith longitudinal ribs are as follows [18]:

$$\prod_i = \frac{1}{2}\left[E_i F_i \left(\frac{\partial u_i}{\partial x}\right)^2 + E_i J_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + \right.$$
$$\left. + E_i J_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + G_i J_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right]dx \qquad (2)$$

The kinetic energies of ribs are written in form [8]:

$$K_i = \rho_i F_i \int_0^l \left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \frac{J_{\kappa pi}}{F_i}\left(\frac{\partial \varphi_{\kappa pi}}{\partial t}\right)^2\right]dx \qquad (3)$$

In expressions (2) and (3) u_i, ϑ_i, w_i are the displacements of rod's points used in stiffening, F_i are the areas of cross-sections of the ith rod stiffened to the shell in the direction of the generatrix, $\tilde{E}_i$ is the modulus of elasticity at tension of the ith rod stiffened to the cylindrical shell in the direction of generatrix, J_{yi}, J_{zi} are inertia moments of the ith rod with respect to the axis passing through the gravity center of the cross-section, J_{kpi} are inertia moments at torsion of the ith rod, t is time, ρ_i is density of the material of the ith rods, $\varphi_i(x)$, $\varphi_{\kappa pi}(x)$ are turning and twisting angles of the rod's cross section.

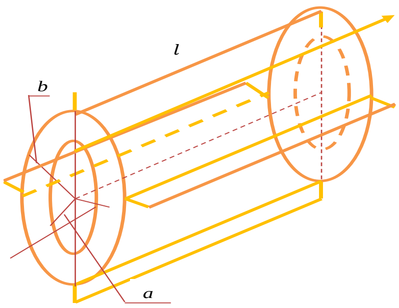

Figure 1. Three-dimensional cylinder stiffened with rods

Potential energy of external surface loads acting as viewed from ideal fluid and applied to the shell is determined as a work performed by these loads when taking the system from the deformed state to the initial undeformed state and is represented as follows:

$$A_0 = -\int_0^l \int_0^{2\pi a} q_z w dx dy \qquad (4)$$

The total energy of the system equals the sum of the energy of elastic deformation of the shell and all longitudinal ribs and also potential energies of external loads acting as viewed from ideal fluid:

$$J = V + \sum_{i=1}^{k_1}\left(\prod{}_i + K_i\right) + A_0 \qquad (5)$$

where, k_1 is the amount of longitudinal ribs.

Assuming that the main flow rate equals U and deviations from this rate are small, we use a wave equation for the potential of perturbed rates φ with respect to [7]:

$$\Delta\varphi - \frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2} + 2U\frac{\partial^2\varphi}{R\partial x\partial t} + U^2\frac{\partial^2\varphi}{\partial x^2}\right) = 0 \qquad (6)$$

where, a_0 is velocity of sound propagation in fluid.

The equation of motion of the cylinder takes form [8]:

$$a_t^2 \,\mathrm{graddivs} - a_e^2 \,\mathrm{rotrots} = \partial^2 s / \partial t^2 \qquad (7)$$

where, $a_t = \sqrt{\frac{\lambda_s + 2\mu_s}{\rho_s}}$, $a_e = \sqrt{\frac{\mu_s}{\rho_s}}$, $\vec{s}(s_x, s_\theta, s_r)$ is a displacement vector of shell points, λ_s, μ_s are Lame coefficients for the cylinder material, ρ_s is density of the cylinder material.

The solution of the motion of a three-dimensional cylinder has the form [8]:

a) in the case for inertia less cylinder

$$s_x = \left[\left(-kr\frac{\partial I_n(kr)}{\partial r} - 4(1-v_s)kI_n(kr)\right)A_s + \right.$$
$$+ kI_n(kr)B_s + \left(-kr\frac{\partial K_n(kr)}{\partial r} - 4(1-v_s)kK_n(kr)\right)A_s +$$
$$\left. + kK_n(kr)B_s\right] \times \cos n\theta \cos\chi\xi \sin\omega t \qquad (8)$$
$$s_\theta = \left[-\frac{n}{r}I_n(kr)B_s - \frac{\partial I_n(kr)}{\partial r}C_s - \right.$$
$$\left. - \frac{n}{r}K_n(kr)B_s - \frac{\partial K_n(kr)}{\partial r}C_s\right]\sin n\theta \sin\chi\xi \sin\omega t$$
$$s_r = \left[-k^2 r I_n(kr)A_s + \frac{\partial I_n(kr)}{\partial r}B_s + \right.$$
$$\left. + \frac{n}{r}I_n(kr)C_s - k^2 r K_n(kr)A_s + \frac{\partial K_n(kr)}{\partial r}B_s + \frac{n}{r}K_n(kr)C_s\right] \times$$
$$\times \cos n\theta \sin\chi\xi \sin\omega t$$

b) in the case of inertial cylinder

$$s_x = \left[A_s k I_n(\gamma_e r) - C_s\frac{\gamma_t^2}{\mu_t}I_n(\gamma_t r) + A_s k K_n(\gamma_e r) - C_s\frac{\gamma_t^2}{\mu_t}K_n(\gamma_t r)\right] \times$$
$$\times \cos n\varphi \cos\chi\xi \sin\omega_1 t_1$$
$$s_\theta = \left[-\frac{A_s n}{r}I_n(\gamma_e r) - \frac{C_s nk}{r\mu_t}I_n(\gamma_t r) - \frac{B_s}{n}\frac{\partial I_n(\gamma_t r)}{\partial r} - \right.$$
$$\left. - \frac{A_s n}{r}K_n(\gamma_e r) - \frac{C_s nk}{r\mu_t}K_n(\gamma_t r) - \frac{B_s}{n}\frac{\partial K_n(\gamma_t r)}{\partial r}\right] \times$$
$$\times \sin n\varphi \sin\chi\xi \sin\omega_1 t_1 \qquad (9)$$
$$s_r = \left[A_s\frac{\partial I_n(\gamma_e r)}{\partial r} - \frac{C_s k}{\mu_t}\frac{\partial I_n(\gamma_t r)}{\partial r} + \frac{B_s n}{r}I_n(\gamma_t r) + \right.$$
$$+ A_s\frac{\partial I_n(\gamma_e r)}{\partial r} - \frac{C_s k}{\mu_t}\frac{\partial I_n(\gamma_t r)}{\partial r} + \frac{B_s n}{r}I_n(\gamma_t r) +$$
$$\times \cos n\varphi \sin\chi\xi \sin\omega_1 t_1$$

In expressions (8), (9), $A_s, B_s, C_s, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s$ are constants, I_n, K_n are modified first and second kind Bessel functions of nth order, $\gamma_e^2 = k^2 - \mu_e^2$, $\gamma_t^2 = k^2 - \mu_t^2$.

The expression of the total energy of the system (5), the equation of motion of fluid (6) and cylinder (7) are complemented with contact conditions. On the contact surface a cylinder-fluid, we observe continuity of radial velocities and pressures. The impermeability and smooth flow condition at the cylinder's wall have the form [8]:

$$\vartheta_r\Big|_{r=a}=\frac{\partial\varphi}{\partial r_{r=a}} \tag{10}$$

where ϑ_r is a radial velocity of fluid points, $\omega_0=\sqrt{\dfrac{E_1}{\left(1-v_1^2\right)R^2\tilde{\rho}}}$, $t_1=\omega_0 t$.

Equality of external surface loads of radial pressure acting as viewed from fluid on the shell wall:

$$q_z=-p\big|_{r=R} \tag{11}$$

where, the pressure p through the potential φ is determined by the formula (ρ_m is fluid's density) [9]:

$$p=-\rho_m\left(\frac{\partial\varphi}{\partial t}+U\frac{\partial\varphi}{\partial x}\right) \tag{12}$$

It is assumed that the rigid contact conditions between the shell and rods were satisfied:

$$\begin{aligned}&u_i(x)=s_x\left(x,\mathrm{b},\theta_i\right)\\&\vartheta_i(x)=s_\theta\left(x,\mathrm{b},\theta_i\right)\\&w_i\left(x\right)=w\left(x,\mathrm{b},\theta_i\right)\end{aligned} \tag{13}$$

It is supposed that in the sections $x=0$ and $x=l$ $\sigma_{xx}=0,\ s_\varphi=s_r=0$ are fulfilled and the lateral surface of the cylinder is free from loadings, i.e. for $r=b$

$$\begin{aligned}&\sigma_{rx}=0\\&\sigma_{r\theta}=0\\&\sigma_{rr}=0\end{aligned} \tag{14}$$

The frequency equation of the ridge inhomogeneous shell with flowing fluid is obtained based on Ostrogradsky-Hamilton principle of stationarity of action:

$$\delta W=0 \tag{15}$$

where $W=\int_{t'}^{t''}Jdt$ is Hamilton's action, t' and t'' are the given arbitrary moments of time.

Complementing with contact conditions the total energy of the system (5), the equation of motion of fluid (6) and cylinder we arrive at a problem of natural vibrations of a flowing fluid three-dimensional cylinder stiffened with longitudinal ribs. In other words, the problem of natural vibrations of a flowing-fluid three-dimensional cylinder stiffened with longitudinal ribs is reduced to joint integration of expressions for total energy of the system (5), equation of motion of fluid (6) subject to conditions (10)-(14) on their contact surface.

3. PROBLEM SOLUTION

We look for the potential of perturbed velocities φ in the form:

$$\varphi\left(\xi,r,\theta,t_1\right)=f(r)\cos n\theta\sin\chi\xi\sin\omega_1 t_1 \tag{16}$$

where, $\omega_1=\omega/\omega_0,\chi,n$ are wave numbers in longitudinal and circumferential directions, respectively, $\xi=x/R,\ \chi=kR=\dfrac{m\pi R}{l}$, ω is a desired frequency.

Proceeding from condition (12), using (16) we have:

$$\varphi=-\frac{f(r)}{f'(R)}\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right) \tag{17}$$

To determine the unknown function $f(r)$, we substitute the solution (16) in (6).

As a result, we get a Bessel equation whose solutions a Bessel functions. Depending on the parameter $M_1=\dfrac{U+\omega_0\omega_1/k}{a_0}$ for $M_1<1$ its solution will be first and second kind nth order modified Bessel functions $I_n\left(\beta_2 r\right)$ and $-K_n\left(\beta_2 r\right)$, for $M_1>1$ the first and second kind, nth order Bessel functions $J_n\left(\beta_1 r\right)$ and $Y_n\left(\beta_1 r\right)$, for $M_1=1$ the functions r^n and r^{-n} where $\beta_1^2=-\beta_2^2=R^{-2}\left(M_1^2-1\right)k^2$.

In all the cases for $r=0$ the modified Bessel function $K_n\left(\beta_2 r\right)$ and the Bessel functions $Y_n\left(\beta_1 r\right)$ are infinitely large. Therefore, in the solutions of the Bessel equation the modified first kind Bessel function of order n, $I_n\left(\beta_2 r\right)$, and the first order, nth order Bessel function $I_n\left(\beta_1 r\right)$ and also the function r^n will figure. We introduce the denotation:

$$\varphi_{\alpha n}=\begin{cases}\dfrac{I_n\left(\beta_2 r\right)}{I_n'\left(\beta_2 R\right)}, & M_1<1\\[2ex] \dfrac{J_n\left(\beta_1 r\right)}{J_n'\left(\beta_1 R\right)}, & M_1>1\\[2ex] \dfrac{r^n}{nR^{n-1}}, & M_1=1\end{cases} \tag{18}$$

Then expressions (12) and (17) will be rewritten in the form

$$\begin{aligned}&\varphi=-\varphi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right)\\&p=\tilde{\varphi}_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2}+2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1}+U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right)\end{aligned} \tag{19}$$

where, $\tilde{\varphi}_{\alpha n}=\varphi_{\alpha n}\big|_{r=R}$.

If we calculate the integrals contained in expression (5), and use conditions (14), we get a two-dimensional polynomial with respect the constants A_s,B_s,C_s:

$$J=\varphi_{11i}A_s^2+\varphi_{22i}B_s^2+\varphi_{33i}C_s^2+\varphi_{44i}A_sB_s+\varphi_{55i}A_sC_s+$$
$$+\varphi_{66i}B_sC_s$$

If we vary Π the expression by constants A_s,B_s,C_s and equate the coefficients of independent variations to zero, we obtain the following system of homogeneous algebraic equations

$$\begin{cases} 2\varphi_{11i}A_s + \varphi_{44i}B_s + \varphi_{55i}C_s = 0 \\ \varphi_{44i}A_s + 2\varphi_{22i}B_s + \varphi_{66i}C_s = 0 \\ \varphi_{55i}A_s + \varphi_{66i}B_s + 2\varphi_{33i}C_s = 0 \end{cases} \quad (20)$$

Since system (20) is a homogeneous system of linear algebraic equations, a necessary and sufficient condition for the existence of its nonzero solution is the equality of its principal determinant to zero. As a result, we obtain the following frequency equation

$$\begin{vmatrix} 2\varphi_{11i} & \varphi_{44i} & \varphi_{55i} \\ \varphi_{44i} & 2\varphi_{22i} & \varphi_{66i} \\ \varphi_{55i} & \varphi_{66i} & 2\varphi_{33i} \end{vmatrix} = 0 \quad (21)$$

We write equation (21) in the form:

$$\begin{aligned} &4\varphi_{11i}\varphi_{22i}\varphi_{33i} + \varphi_{44i}\varphi_{55i}\varphi_{66i} - \\ &-\varphi_{55i}^2\varphi_{22i} - \varphi_{66i}^2\varphi_{11i} - \varphi_{44i}^2\varphi_{33i} = 0 \end{aligned} \quad (22)$$

4. NUMERICAL RESULTS

Equation (22) was calculated numerically. The parameters contained in the solution of the problem were adopted:

$a_l = 2.25a_t$

$a_t = 308$ m/sec

$a_0 = l = 2$ m

$E_i = 6.67 \times 10^9$ N/m^2

$\rho_s = \rho_i = 0.26 \times 10^4$ Nsec^2/m^2

$F_i = 3.4$ mm^2

$J_{yi} = 5.1$ mm^4

$h_i = 1.39$ mm

The results of calculations were given in Figure 2 in the form of dependence of frequency parameter on the rate of fluid flow, in Figure 3 on the thickness of cylinder, in Figure 4 on the amount of rods.

As can be seen from Figure 2, with increasing the fluid rate, the value of the frequency parameter at first weakly decreases, and then after certain value a sharp decrease is observed. As the thickness $h = b - a$ of the cylinder increases, the value of the frequency parameter at first increases, and then after certain value decreases.

This is explained by the fact that at the initial increases in the cylinder's thickness, its rigidity increases and this increases natural vibration frequency of the system. Increase in the cylinder's thickness causes increase in its mass and this in its turn the increase of natural vibration frequency of the system. This result was given graphically in Figure 3.

As can be seen from Figure 4 the first increase in the amount of rods causes increase of natural vibration frequency of the system, the subsegment increase causes decrease in natural vibration frequency of the system.

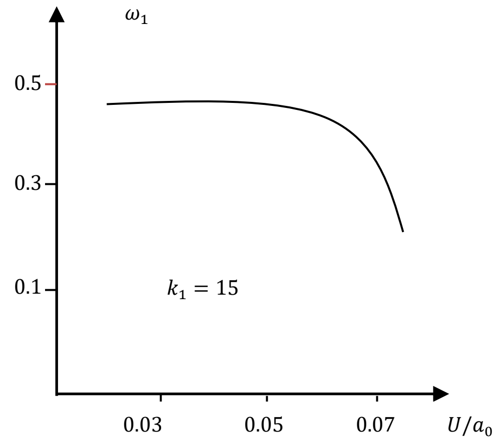

Figure 2. Dependence of frequency on flow rate of fluid

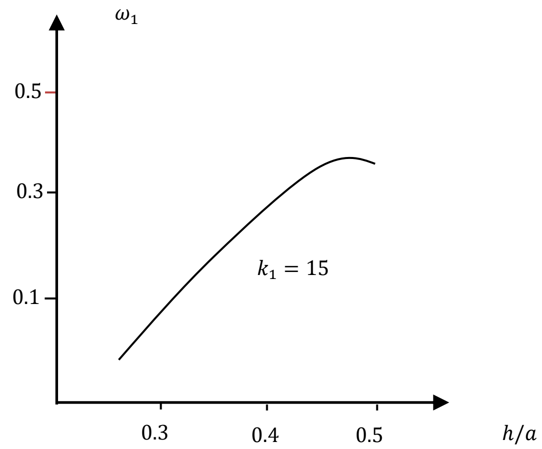

Figure 3. Dependence of frequency parameter of cylinder's thickness

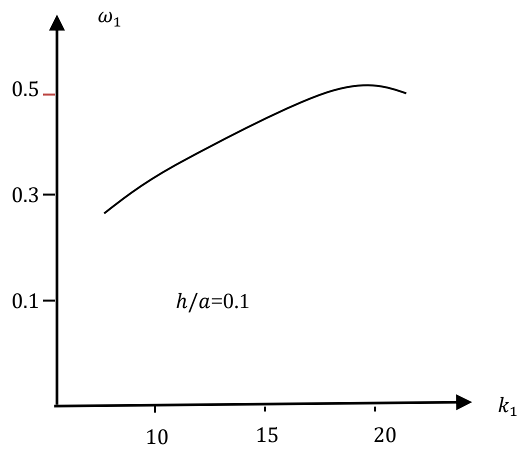

Figure 4. Dependence of frequency parameter on the amount of rods

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BIOGRAPHY

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