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# VIBRATIONS OF AN ANISOTROPIC LATERALLY STIFFENED FLUID-FILLED CYLINDRICAL SHELL INHOMOGENEOUS IN THICKNESS AND CIRCUMFERENCE

**Z.M. Badirov**

*Azerbaijan University of Architecture and Construction, Baku, Azerbaijan, zbadirov@yahoo.com*

**Abstract-** In this paper we study free vibrations of an orthotropic, laterally stiffened, ideal fluid-filled cylindrical shell inhomogeneous in thickness and in circumferential direction. Using the Hamilton-Ostrogrdasky variational principle, the systems of equations of the motion of an orthotropic, ideal fluid-filled cylindrical shell stiffened in thickness and circumference, are constructed. In order to calculate inhomogeneity of the shell material in thickness and circumference, it is accepted that the Young modulus and the density of the shell material are the functions of normal and circumferential coordinates. Frequency equations are constructed and free vibrations of an orthotropic, ideal fluid-filled, laterally stiffened cylindrical shell inhomogeneous in thickness and in circumference are numerically implemented. The characteristically dependence curves were constructed.

**Keywords:** Reinforced Shell, Variational Principle, Ideal Fluid, Free Oscillation, Anisotropic Shell.

## 1. INTRODUCTION

Stability, vibrations and strength analysis of medium-contacting, shell-type thin-walled structural elements plays an important role in designing modern apparatus, machines and constructions. Such constructions may be in contact with fluid and be subjected to dynamic loadings. For greater rigidity the shells are stiffened with various ribs.

However, behavior of inhomogeneous orthotropic thin-walled structural elements with ribs, accounting of their discrete location, influence of fluid and inhomogeneity were not studied enough. Therefore, development of mathematical models for studying behavior of stiffened inhomogeneous anisotropic shells, that completely take into account their work under dynamic loads and conducting researches of stability and vibrations on their basis and also choice of rational parameters of a fluid-contacting structure are urgent problems.

It should be noted that free vibrations of an anisotropic, longitudinally stiffened, moving fluid-contacting cylindrical shell inhomogeneous in thickness were studied in [1].

Ref. [2] was devoted to numerical analysis of axially-symmetric vibrations of a cylindrical shell inhomogeneous in length, with circular cross-section and lying on an inhomogeneous viscous-elastic foundation. It was shown that the inhomogeneity of the shell and foundation essentially influences on the vibration frequency. Paper [3] deals with natural vibrations of an inhomogeneous orthotropic rectangular plate with regard to external visco-elastic resistance. Papers [4, 5, 6] study parametric vibrations of a nonlinear rectilinear bar inhomogeneous in thickness using the Pasternak contact model. Free vibrations of an isotropic inhomogeneous, moving fluid-contacting cylindrical shell stiffened with cross system of ribs were studied in [7, 8].

## 2. PROBLEM STATEMENT

An orthotropic, laterally stiffened, fluid-filled shell is considered as a system consisting of the own shell, ribs rigidly connected with it along the contact lines and fluid. It is accepted that the stress-strain state of the shell may be completely determined within linear theory of elastic thin shells, based on Kirchoff-Liav conjecture, and theory of Kirchhoff-Liav curvilinear bars is applied to calculate the ribs. The system of coordinates is chosen so that the coordinate lines coincide with the lines of principal curvatures of shell's median surface. It is accepted that the ribs are located along circular coordinate lines. Furthermore, it is assumed that all the ribs form a regular system. Under the regular system of lateral ribs we understand such a system where the rigidities of all ribs, their mutual distances are equal and the distance from the edge of the shell to the nearest rib equals the distance between the ribs.

The strain state of the seething may be determined by three components of displacements of its median surface $u, \vartheta$ and $w$. This time the turning angles of normal elements $\varphi_1, \varphi_2$ with respect to coordinate lines $y$ and $x$ are expressed by $w$ and $\vartheta$ by means of dependences $\varphi_1 = -\frac{\partial w}{\partial x}$, $\varphi_2 = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right)$, where $R$ is the radius of the shell's median surface.

In order to describe the strain state of lateral ribs, in addition to three components of displacements of gravity center of their cross-section ($u_i, \vartheta_i, w_i$ of the *i*th longitudinal bar), it is necessary to determine the twisting angles $\varphi_{kpi}$.

Taking into account that according to the accepted conjectures we have constancy of radial deflections along the height of sections, and also equality of appropriate twisting angles following from the conditions of rigid connection of ribs with a shell, we write the following relations:

$$u_i(x) = u(x, y_i) + h_i\varphi_1(x, y_i)$$
$$\vartheta_i(x) = \vartheta(x, y_i) + h_i\varphi_2(x, y_i)$$
$$w_i(x) = w(x, y_i)$$
$$\varphi_i = \varphi_1(x, y_i)$$
$$\varphi_{kpi}(x) = \varphi_2(x, y_i)$$

where, $h_i = 0.5h + H_i^1$, $h$ is shell's height, $H_i^1$ is the distance from the axes of the *i*th longitudinal bar to the shell surface, $y_i$ are the coordinates of the conjugation lines of ribs and a shell, $\varphi_i, \varphi_{kpi}$ are turning and twisting angles of cross-sections of longitudinal bars.

For external actions it is supposed that the surface loads acting on a ridge shell as viewed from fluid, may be reduced to the components $q_x, q_y$ and $q_z$ applied to the shell's median surface.

We obtain differential equations of motion and natural boundary conditions for a longitudinally stiffened orthotropic cylindrical shell with ideal fluid based on Ostrogradsky-Hamilton variational principle. For that we write the potential and kinetic energies of the system.

For accounting the inhomogeneity of the cylindrical shell in thickness, we will procced from three-dimensional functional. In this case, the functional of the total energy of the cylindrical shell is of the form:

$$V = \frac{1}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\Big(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \theta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)dxdydx \qquad (1)$$

There exist various methods for calculating inhomogeneity of the shell material. One of them is that the Young modulus and density of the shell material are taken as functions of normal circumferential coordinate $z$ and $y: E = E(z, y),\ \rho = \rho(z, y)$ [10]. In this case the stress-strain ratio has the form:

$$\sigma_{11} = b_{11}(z,y)\varepsilon_{11} + b_{12}(z,y)\varepsilon_{22}$$
$$\sigma_{22} = b_{12}(z,y)\varepsilon_{11} + b_{22}(z,y)\varepsilon_{22} \qquad (2)$$
$$\sigma_{12} = b_{66}(z,y)\varepsilon_{12}$$

$$\varepsilon_{11} = \partial u / \partial x$$
$$\varepsilon_{22} = \partial\vartheta / \partial y + w \qquad (3)$$
$$\varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x}$$

Suppose that

$$b_{11}(z,y) = \tilde{b}_{11} f_1(z) f_2(y)$$
$$b_{22}(z,y) = \tilde{b}_{22} f_1(z) f_2(y)$$
$$b_{12}(z,y) = \tilde{b}_{12} f_1(z) f_2(y) \qquad (4)$$
$$b_{66}(z,y) = \tilde{b}_{66} f_1(z) f_2(y)$$
$$\rho(z,y) = \tilde{\rho} f_1(z) f_2(y)$$

where, $\tilde{b}_{11} = \frac{E_1}{1-\nu_1\nu_2}$; $\tilde{b}_{22} = \frac{E_2}{1-\nu_1\nu_2}$; $\tilde{b}_{12} = \frac{\nu_2 E_2}{1-\nu_1\nu_2}$; $\tilde{b}_{66} = G$ are basic module of elasticity of shell's homogeneous orthotropic material, $\tilde{\rho}$ is density of material of the homogeneous shell, $f_1(z), f_2(y)$ are homogeneity functions in the direction of normal and circumference of the shell, respectively, $\nu_1, \nu_2$ are Poisson ratios, $E_1, E_2$ are Young modulus of the shell material in coordinate directions of the axes $x, y$, respectively.

Taking into account (4) in (2), we get:

$$\sigma_{11} = \left(\tilde{b}_{11}\varepsilon_{11} + \tilde{b}_{12}\varepsilon_{22}\right) f_1(z) f_2(y)$$
$$\sigma_{22} = \left(\tilde{b}_{12}\varepsilon_{11} + \tilde{b}_{22}\varepsilon_{22}\right) f_1(z) f_2(y) \qquad (5)$$
$$\sigma_{12} = \tilde{b}_{66}\varepsilon_{12} f_1(z) f_2(y)$$

Allowing for (5), the functional of the total energy of the cylindrical shell is of the form:

$$V = \frac{1}{2}\int_{-h/2}^{h/2} f_1(z)dz\iint\left\{\tilde{b}_{11}\varepsilon_{11}^2 + 2\tilde{b}_{12}\varepsilon_{11}\varepsilon_{22} + \tilde{b}_{22}\varepsilon_{22}^2 + \tilde{b}_{66}\varepsilon_{12}^2\right\} f_2(y)dxdy + \int_{-h/2}^{h/2} f_1(z)dz\iint\left(\tilde{\rho}\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \theta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right) f_2(y)dxdy \qquad (6)$$

The expressions for potential kinetic energy of elastic deformation of ribs are as follows [11]:

$$V_j = \frac{1}{2}\sum_{j=1}^{k_2}\int_{y_1}^{y_2}\left[E_j F_j\left(\frac{\partial\vartheta_j}{\partial y} - \frac{w_j}{R}\right)^2 + E_j J_{xj}\left(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2}\right)^2 + E_j J_{zj}\left(\frac{\partial^2 u_i}{\partial y^2} - \frac{\varphi_{\kappa pj}}{R}\right)^2\right]dy +$$
$$K_j = \sum_{j=1}^{k_2}\rho_j F_j\int_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2 + \left(\frac{\partial\vartheta_j}{\partial t}\right)^2 + \left(\frac{\partial w_j}{\partial t}\right)^2 + \frac{J_{\kappa pj}}{F_j}\left(\frac{\partial\varphi_{\kappa pj}}{\partial t}\right)^2\right]dy \qquad (7)$$

In expressions (7) $F_j, J_{xj}, J_{zj}, J_{kpj}$ are area and inertia moments of the *j*th lateral bar with respect to the axis $Ox$ and the axis parallel to the axis $Oz$ and passing through the gravity center of the section and also its inertia moment at torsion, respectively; $E_j$ is an elasticity modulus of the *j*th lateral bar, $\rho_j$ is density of the material of the *j*th lateral bar.

The potential energy of external surface loads acting as viewed from ideal fluid and applied to the shell is determined as a wok performed by these loads when taking the system from the deformed system to the initial undeformed state and is represented in the form:

$$A_0 = -\int_0^L \int_0^{2\pi} q_z w dx dy \quad (8)$$

The total energy of the system is equal to the sum of energies of elastic deformations of the shell and longitudinal ribs and also potential energies of all external loads acting as viewed from ideal fluid:

$$J = V + V_j + K_j + A_0 \quad (9)$$

Assuming that the main flow rate equals $U$ and deviations from this rate are small, we use a wave equation for the potential of perturbed rates $\varphi$ with respect to [9, 12]:

$$\Delta\varphi - \frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2} + 2U\frac{\partial^2\varphi}{R\partial\xi\partial t} + U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right) = 0 \quad (10)$$

The expression of the total energy of system (9), the equation of motion of fluid (10) are complemented with contact conditions. On the contact surface a shell-fluid, continuity of radial rates and pressures is observed. The condition of impermeability or smooth flow at the shell wall is of the form [12]:

$$\vartheta_r\big|_{r=R} = \frac{\partial\varphi}{\partial r}\bigg|_{r=R} = -\left(\omega_0\frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right) \quad (11)$$

Equality of radial pressures as viewed from fluid on the shell:

$$q_z = -p|_{r=R} \quad (12)$$

If in (10) and (11) we substitute $U = 0,$ then we obtain an equation of motion and condition of impermeability or smooth flow at the shell wall for fluid at rest. The frequency equation of a ridge inhomogeneous orthotropic shell with flowing fluid was obtained on the base of Ostrogradsky-Hamilton principle of stationarity of action:

$$\delta W = 0 \quad (13)$$

where, $W = \int_{t'}^{t''} J dt$ is Hamilton's action, $t'$ and $t''$ are the given arbitrary moments of time.

Complementing with contact conditions the total energy of system (9), the equation of motion of fluid (10), we arrive at a problem of natural vibrations of an orthotropic, longitudinally stiffened, fluid-filled cylindrical shell inhomogeneous in thickness and circumference. In other words, a problem on natural vibrations of an orthotropic, longitudinally stiffened, fluid-filled cylindrical shell inhomogeneous in thickness and circumference is reduced to joint integration of expressions for the total energy of system (9), the equation of motion of fluid (10) subject to conditions (11) and (12) on their contact surface.

## 3. PROBLEM SOLUTION

We look for the potential of perturbed velocities in the form:

$$\varphi(\xi, r, \theta, t_1) = f(r)\cos n\theta \sin\chi\xi \sin\omega_1 t_1 \quad (14)$$

Using (14), from conditions (11), (12) we have:

$$\varphi = -\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right)$$
$$p = \Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \quad (15)$$

where,

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta R), & M_1 < 1 \\ J_n(\beta_1 r)/J_n'(\beta_1 R), & M_1 > 1 \\ \dfrac{r^n}{nR^{n-1}}, & M_1 = 1 \end{cases} \quad (16)$$

where, $M_1 = \dfrac{U + \omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, $I_n$ is a modified first kind Bessel function of order $n$, $J_n$ is a first kind Bessel function of order $n$, $\omega_0 = \sqrt{E_0/\left(1-\nu^2\right)\rho_0 R^2}$, $\omega_1 = \omega/\omega_0$.

In expression (9), the quantities $u, \vartheta, w$ are variable quantities. These unknown quantities are approximated in the following way:

$$u = u_0 \cos\frac{\pi x}{l}\sin k\theta \sin\omega t$$
$$\vartheta = \vartheta_0 \sin\frac{\pi x}{l}\cos k\theta \sin\omega t \quad (17)$$
$$w = w_0 \sin\frac{\pi x}{l}\sin k\theta \sin\omega t$$

when simplifying (9) we accept the following dependences:

$$f_1(z) = 1 + \alpha\frac{z}{h}, \quad f_2(y) = 1 + \beta\frac{y}{2\pi R} \quad (18)$$

where, $\alpha, \beta$ are constant parameters of inhomogeneity in the direction along the normal and circumference of the shell, respectively, and $\alpha, \beta \in [0,1]$.

Substituting (17) and (18) in (9), after integrating we get a function of variables $u_0, \vartheta_0, w_0$. The stationary value of the obtained function is determined by the following system:

$$\begin{aligned} &1)\ \partial J/\partial u_0 = 0 \\ &2)\ \partial J/\partial \vartheta_0 = 0 \\ &3)\ \partial J/\partial w_0 = 0 \end{aligned} \quad (19)$$

Nontrivial solution of the system of third order algebraic equations (19) is possible only in the case when $\omega_1$ is the root of its determinant. The definition of $\omega_1$ is reduced to a transcendental equation as $\omega_1$ enters into the arguments of the Bessel function:

$$\det\|a_{ij}\| = 0, \quad i, j = 1,3 \quad (20)$$

## 4. CONCLUSIONS

Frequency Equation (20) was solved numerically under the following initial data:

$R = 160$ mm

$E_j = 6.67 \times 10^9$ Pa

$a_0 = 1430$ m/s

$\rho_j = 7.8$ g/cm$^3$

$L = 800$ mm

$v_1 = 0.11$

$v_2 = 0.19$

$h = 0.45$ mm

$J_{kpj} = 0.48$ mm$^4$

$J_{xj} = 19.4$ mm$^4$

$F_j = 5.75$ mm$^2$

$h_j = 1.39$ mm

The results of calculations were given in Figure 1 in the form of dependences of the frequency parameter on the amount of stiffening lateral bars $k_2$ on the shell surface, in Figure 2 in the form of dependence of the frequency parameter on inhomogeneity parameter in the circumferential direction $\beta$, in Figure 3 in the form of a frequency parameter on flow rate of fluid $U$. As can be seen from figure e 1, increasing the number of annular ribs, the value of the frequency parameter increases. When increasing inhomogeneity parameter in circumferential direction $\beta$, as can be seen from Figure 2, the value of the frequency parameter increases. Furthermore, the value of the frequency parameter increases with increasing orthotropic properties of a cylindrical shell and decreases with increasing the fluid flow rate (Figure 3).

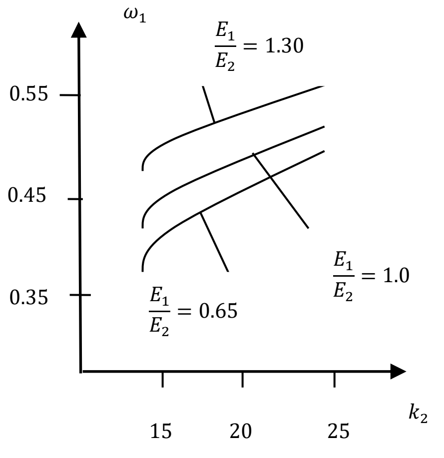


Figure 1. Dependence of frequency parameter on the amount of lateral ribs $k_2$

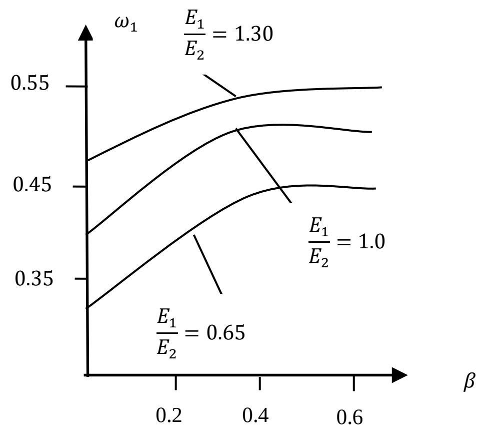


Figure 2. Dependence of frequency parameter on inhomogeneity parameter in circumferential direction $\beta$

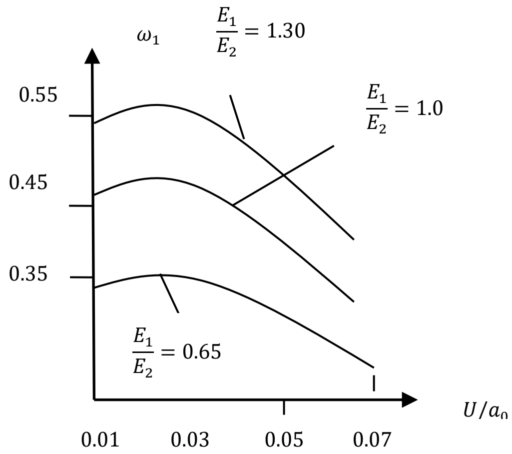


Figure 3. Dependence of frequency parameter on fluid rate

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## BIOGRAPHY

**Zamir Badirov** was born in Baku, Azerbaijan, 1978. He received the B.Sc. degree in International Relations, Economic and Public Administration from Istanbul University, Istanbul, Turkey in 2004. During 2005-2008, he worked as a Technician Engineer and Economist at the Tamay LLC, Baku, Azerbaijan. He received the B.Sc. and M.Sc. degrees in Industrial and Civil Constructions Engineering from Azerbaijan University of Architecture and Construction, Baku, Azerbaijan in 2013 and 2016, respectively. During 2008-2014, he worked as a Project Engineer at the Kon-Lay Project LLC, Baku, Azerbaijan. During 2014-2018, he worked a Director and Structural Engineer at Karkas-S LLC, Baku, Azerbaijan.