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# STUDY TOE-IN ANGLES WHILE DRIVING

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**Abstract-** The article presents a mathematical model of the wheel unit taking into account its rigidity, while the wheel is installed with a certain toe-in angle. When driving toe-in angles change their initial values that were set on a stationary car in statics. The developed mathematical model that allows to explore the changing of toe-in angles at rectilinear motion of car depending on its speed. To test the obtained mathematical model, a research laboratory for road tests of the car was created on the basis of a car of category M1. This research laboratory for road tests of the car allows to carry out experimental and theoretical studies of changes in the toe-in angles while driving during and their impact on various operating indicators.

**Keywords:** Car, Wheel, Suspension, Toe-in Angles, Road, Speed, Mathematical Model, Oscillations.

## 1. INTRODUCTION

The main trend in the development of the modern automotive industry is aimed at providing fuel economy, traction and braking dynamics of active and passive driving safety, comfort and other indicators. Implementation of these properties are largely determined to some extend by the wheel interaction process with the road and as a result of it direct development of driving forces and moments occur [1, 2, 3]. In its turn, the parameters of wheels mounting, i.e. camber and toe-in angles, affect significantly the wheel interaction process with the road [4].

When driving of car toe-in angles are constantly changing. The reason for this phenomenon is the design features of the system (clearance in joints of steering roads and elasticity corresponding compounds) [5]. The influence on wheel vibrations in a horizontal plane of bumpy roads, wheel imbalance and defects in geometry of steering drive is not discussed in this article.

When choosing toe-in angles it is necessary to take into account that their value depends on operating and structural factors such as car suspension stiffness, motion modes, etc. It is necessary to consider the driving duration of different modes when choosing toe-in angles of steered wheels [6, 7].

Research of camber and toe-in angles and their impact on various car operating characteristics of was studied by such scientists as: H.B. Pacejka, S. Chatur, A.S. Litvinov, J. Reimpell [4-7], and others.

Most authors [4, 6, 7, 8], consider that the main purpose of wheel toe-in is to compensate the negative influence of camber angle by decreasing elementary side reactions acting in the tire contact with the road.

The purpose of this work is to develop a mathematical model that allows you to explore the changing of toe-in angles at rectilinear car motion depending on its speed, the initial choice of rational toe-in angles; determination of dynamic characteristics of possible shimmy vibration and analysis of the impact on these phenomena by specific design parameters of the system.

## 2. CHANGING TOE-IN ANGLES WHEN DRIVING

To achieve this purpose on the basis of M1 category vehicle it was set a scientific research laboratory for road car tests to determine changes of toe-in angles using different modes of the car motion.

### 2.1. Equipment for Research

Measuring and recording equipment was attached to the wheels and the car's wings some of the equipment has been installed on an aluminum plate with fastening it hard to the body of car on rear passenger seat (Figures 1-5).


Figure 1. Vehicle with research equipment, 1- measurement device of toe-in angles and turn; 2- "the fifth" wheel of measuring complex


Figure 2. Car for research


Figure 3. Equipment for measuring side force on the steered wheels, 1- measuring ring; 2- strain gauge with a protective layer of sealant; 3- adjustment board; 4- power supply


Figure 4. Measuring - recording equipment for testing car, 1- gyroscope CGV-5; 2- transducer of angular accelerations AVS-45 AC; 3- accelerometer MP-95; 4- MtPro-2 measuring complex; 5- central control panel; 6- remote control; 7- power supply

There are the following parameters while conducting experimental studies:
- Toe-in angle;
- Side force on the steered wheels;
- Turn angle of the steered wheels;
- Turn angle of steering wheel;
- Effort on steering wheel;
- Acceleration of the car in three planes;
- Angles of inclination of the car body in three planes;
- Angular speed of the turn (rotation) of the vehicle longitudinal axle;
- Distance travelled by the car;
- Vehicle speed.

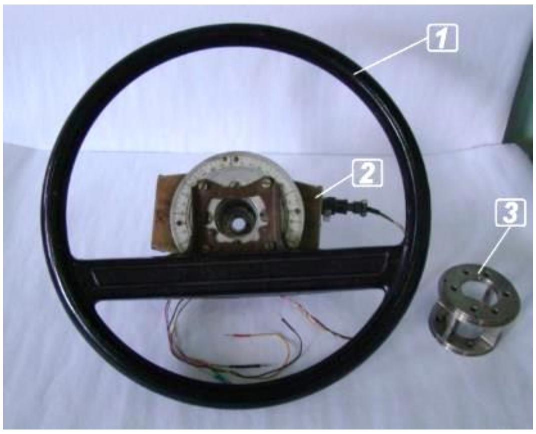


Figure 5. Steering wheel for testing car, 1- steering wheel; 2- measuring unit; 3- measuring element

This presented research laboratory for road car testing allows to carry out experimental and theoretical studies of toe-in angles changes while driving and their impact on various operating indicators.

### 2.2. Mathematical Model for Research

When driving toe-in angles change their initial values that were set on a stationary car in statics which will be associated with the presence of the elastic characteristics of the model - torsional and longitudinal, where $\varphi$, $y$ are common coordinates. Let's consider the motion of steered wheels along the straight line at constant speed. The forces acting on the wheel contact with the road are shown by diagram in Figure 6.

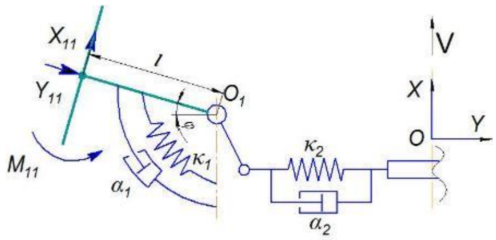


Figure 6. Schematic diagram of wheel assembly with toe-in angle

The system of forces ($X_{11}$, $Y_{11}$) acts on the wheel which is given to the center of the wheel contact with the road, and the main $M_{11}$ slip time (stabilizing moment) tries to turn the wheel about a vertical axle to reduce the slip angle. Changing the toe-in angle occurs with respect to the vertical axle passing through the center of $O_1$ rotation (turn).

The equations of the perturbed motion of the wheel module, namely the equation of rotational motion with respect to the center of $O_1$ rotation and the equation of the lateral displacement of the center of wheel rotation under certain simplifications are (1):

$$\ddot{\varphi} = \frac{\dfrac{lX_{11}^*}{\cos\varphi} + \lambda Y_{11} - M_{11} - \kappa_1(\varphi - \varphi_0) - \alpha_1\dot{\varphi}}{I_1 + m_1(l^2 + \lambda^2)}$$
$$\ddot{y} = -\frac{\kappa_2 y + \alpha_2\dot{y} + Y_{11}\cos\varphi + X_{11}^*\tan\varphi}{m_1} \qquad (1)$$

where,
$X_{11}^{*}$ - the longitudinal force component (projection on the $x$-axis), acting on the wheel;
$Y_{11}$ - lateral force acting on the wheel;
$M_{11}$ - moment due to the lateral force;
$l$ - the length of the wheel knuckle;
$\lambda$ - the wheel removal;
$\kappa_1$ - torsional stiffness of suspension;
$\varphi$ - toe-in angle;
$\varphi_0$ - the initial toe-in angle;
$\alpha_1$ - damping on the angle;
$\dot{\varphi}$ - angular speed of toe-in angle change;
$I_1$ - axle moment of wheel inertia;
$m_1$ - mass of the wheel;
$y$ - lateral displacement of axle of wheel rotation;
$\dot{y}$ - speed of lateral displacement of wheel rotation center;
$\kappa_2$ - lateral stiffness of the suspension for the displacement of the wheel rotation center;
$\alpha_2$ - damping the lateral displacement of wheel rotation center.

These equations describe the oscillations of one wheel with respect to the rotation center of the wheel and its elastic displacement about a pivot axis under the action of lateral and longitudinal forces. Interaction of the wheel with the supporting surface in the lateral direction can be described by reaction of the roadway as a function of slip angle "slip nonlinear hypothesis" [4, 9, 10], i.e.:

$$Y_{11} = k\delta_{11}\sqrt{1+\left(\frac{k\delta_{11}}{f_{ii}Z}\right)^2} \tag{2}$$

where,
$k$ - coefficient of slip resistance;
$\delta_{11}$ - slip angle;
$f_{ii}$ - traction coefficient;
$Z$ - vertical reaction at the wheel contact with the road.

$$Z = \frac{1}{2}\left(\frac{mgb}{a+b} - \frac{2X_{11}^{*}h}{a+b}\right) \tag{3}$$

where,
$m$ - the weight of the car;
$a$ - the position of mass center from the front axle;
$b$ - the position of mass center from the rear axle;
$h$ - metacenter height.

In accordance with a kinematic diagram in Figure 1, the slip angle is determined by the expression (4):

$$\delta_{11} = \arctan\left(\frac{v\sin\varphi - \lambda\dot{\varphi} + \dot{y}}{v\cos\varphi + l\dot{\varphi}}\right) \tag{4}$$

Stabilizing moment of slip force can be approximated by the dependence on (5) coefficients having empirical origin [10]:

$$M_{11} = \frac{\sigma_1\delta_{11}}{39122.65\times\delta_{11}^{\ 4} + 71.45\times\delta_{11}^{\ 2} + 1} \tag{5}$$

where, $\sigma_1$ is the coefficient that determines the linear stabilizing moment.

To determine the constant values of the toe-in angles as a function of the car speed parameter, iterative numerical methods for solving finite equations were used. Analysis of system dynamics was based on the numerical integration of the differential equations of perturbed motion (was realized with help of 18 Maple software).

### 2.3. Research Results

Using a mathematical model and the experimental data graphs of fixed toe-in angles were plotted changing the longitudinal car speed, which are shown in Figure 7.

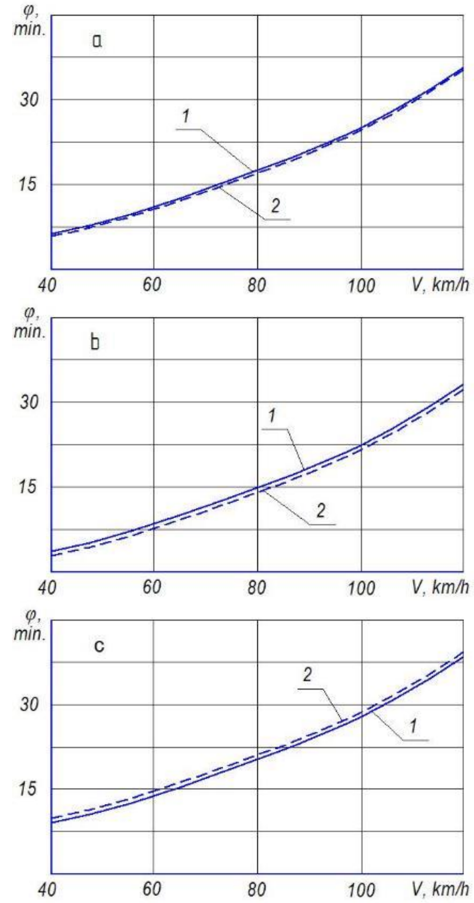


Figure 7. Changing toe-in angle of the car speed, 1,2- theoretical and experimental curves, a- $\varphi_0$ = 20 min, b- $\varphi_0$ = 0 min, c- $\varphi_0$ = -20 min

The comparative analysis of experimental testing on the road, curve "e" in Figure 7 and theoretical calculated data, the curve "a" showed that they are qualitatively similar and quantitatively different from each other by an average of 12-14%.

Thus, it can be argued that the theoretically obtained results of calculations of changing toe-in angle for at a constant speed, while driving correspond the results of the experiment.

On the basis of the numerical integration of the equation system (1) the stability intervals and the area of oscillatory instability were determined by speed parameters where a self oscillation mode is realized. The amplitude of the oscillations depends both on design parameters of the system (damping parameters) and speed parameter. When increasing these parameters shimmy self-oscillations do not occur in operating range of speed. Figures 8-10 show such self-oscillations appearing when car is moving at speed of 60 km/h, 75 km/h and 90 km/h.

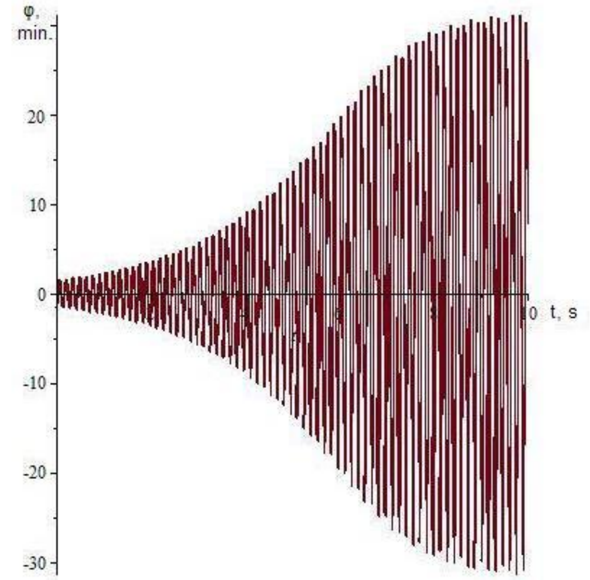


Figure 8. The amplitude of self-oscillation in the angle of toe-in at a speed of 60 km/h

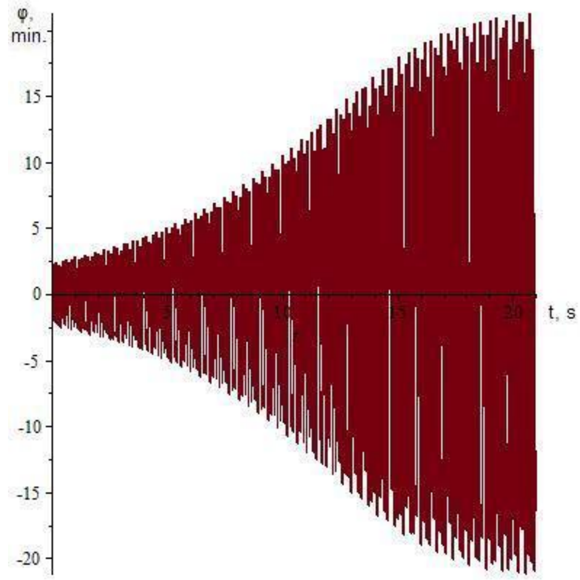


Figure 9. The amplitude of self-oscillation in the angle of toe-in at a speed of 75 km/h

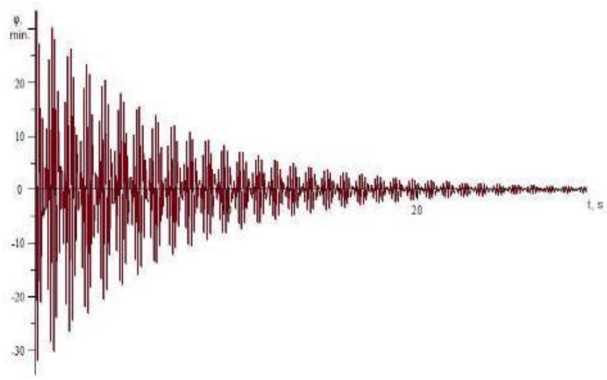


Figure 10. The amplitude of self-oscillation in the angle of toe-in at a speed of 90 km/h

These graphs allow to predict the maximum value of the dynamic toe-in angle (amplitude) and lateral displacement of the wheel rotation center when driving on the straight sections of road.

The most favorable initial value of toe-in angle was the value of minus 20 minutes, which provided in the range of cruise speed the oscillation symmetry of phase coordinates according to their zero value. It does not result in asymmetric wear of the tire tread and reduces the resistance to motion.

Using a mathematical model based on vibrations of controlled wheel allows to determine the indicators of toe-in angles of sustainability while driving on the straight sections of road.

Analysis of the stability of the stationary states of the system, namely, the toe-in angle stability at different speed, was based on the numerical eigenvalues analysis of the linearized system of equations of perturbed motion. It is necessary to determine stationary modes of toe-in angle and lateral displacement of rotation center for discrete parameters of speed, i.e. to find a final solution of the system (in the system (6) all the derivatives must be zero):

$$\begin{aligned} \ddot{\varphi} &= f_1(y, \varphi, \dot{y}, \dot{\varphi}) \\ \ddot{y} &= f_2(y, \varphi, \dot{y}, \dot{\varphi}) \end{aligned} \qquad (6)$$

After linearization of the system (6) for each of the stationary mode and a system of equations of the perturbed motion of the following form is built:

$$\begin{aligned} \dot{\varphi} &= a_{11}\varphi + a_{12}\dot{\varphi} + a_{13} \cdot y + a_{14} \cdot \dot{y} \\ \ddot{\varphi} &= a_{21}\varphi + a_{22}\dot{\varphi} + a_{23} \cdot y + a_{24} \cdot \dot{y} \\ \dot{y} &= a_{31}\varphi + a_{32}\dot{\varphi} + a_{33} \cdot y + a_{34} \cdot \dot{y} \\ \ddot{y} &= a_{41}\varphi + a_{42}\dot{\varphi} + a_{43} \cdot y + a_{44} \cdot \dot{y} \end{aligned} \qquad (7)$$

Appropriate characteristic equation and its roots were obtained for the system of equations (7) (the appearance of at least one root with a positive real part corresponds to the loss of stability of the corresponding stationary conditions).

Thus, when the vehicle is moving at the speed of 60 km/h, the following characteristic equation is obtained:

$$\mu^4 + 62.08\mu^3 + 8040.15\mu^2 + 1.18\times10^5\mu + 1.21\times10^7 = 0 \qquad (8)$$

The solution of the characteristic equation (eigenvalues) is shown in Figure 11. So, the steady value of toe-in angles can be unstable.

The analysis of stability for stationary conditions which was carried out confirmed the results of calculations obtained by numerical integration of the system of Equations (1), namely, shimmy zone occurs within a speed range of 14.95 km/h to 82.65 km/h.

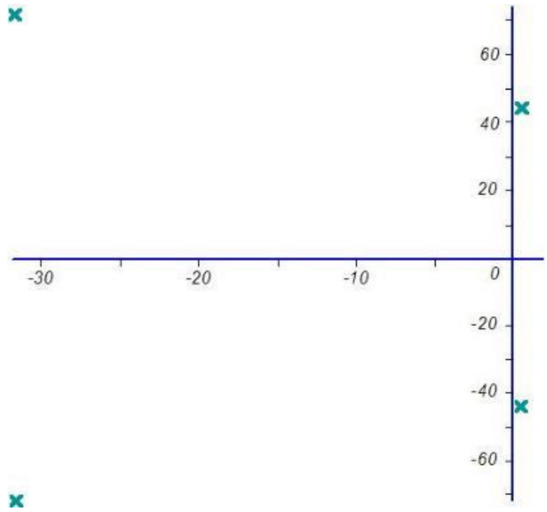


Figure 11. Schematic diagram of characteristic equation roots

## 3. CONCLUSIONS

With this mathematical model (1)-(8) were analyzed oscillation amplitude of the toe-in angles, depending on the possible initial values of these angles. Based on the analysis performed for this vehicle, the most optimal starting toe-in angles were determined for various speeds. These data are presented in Figure 12.

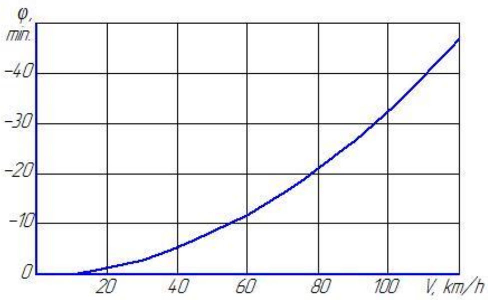


Figure 12. The optimum toe-in angles for various speeds of car

It should be noted that this mathematical model allows us to predict the maximum dynamic values of the toe-in angle (amplitude) when the car moves in straight sections.

As can be seen from Figure 12, the most optimal initial toe-in angle values for this vehicle were minus 12 minutes at a speed of 60 km/h, minus 18 minutes at 75 km/h, and minus 26 minutes at 90 km/h.

At given speeds, these values of toe-in angles provide the symmetry of the oscillations of the phase coordinates relative to their zero value. Fulfillment of this condition avoids asymmetric wear of the tire tread and reduces the resistance to movement.

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