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VIBRATIONS OF FLUID-FILLED INHOMOGENEOUS CYLINDRICAL SHELLS STRENGTHENED WITH LATERAL RIBS

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Abstract- In this paper we study one of dynamical strength characteristics, the frequency of natural variations of a fluid-filled cylindrical shell made of a fiber-glass and strengthened with annular ribs heterogeneous in thickness and along the generatrix under the Navier boundary conditions. Using Hamilton-Ostrogradsky's variational principle, the frequency equations for calculating natural vibrations of the system under consideration, are constructed. In the calculation process, the linear laws for heterogeneity function were accepted. Frequency equations were constructed and numerically implemented. The results of calculations of natural frequency of vibrations were represented in the form of dependence on homogeneity parameter, on the number of lateral ribs for different values of wave formation parameters. Characteristic curves of dependence were constructed.

Keywords: Strengthened Shell, Variational Principle, Fluid, Free Vibration, Heterogeneity.

1. INTRODCTION

Polymeric, carbon, metal and organic-based composites and porous aluminum are widely used in different fields of engineering. To create heterogeneity in the bearing structures, by the diffusion method or using other technologies, another material with higher strength characteristics is introduced into its surface layers and as a result, there appears technological heterogeneity in the construction. There arises a need to develop methods for calculating such heterogeneous shells and study the influence of heterogeneity on the frequency of their natural vibrations. We need algorithms for determining the resonance frequencies leading to the destruction of heterogeneous shells. To give the greater rigidity, the thin-walled part of the shell is strengthened by the ribs and this significantly increases its strength at slight increase in the mass of construction even if the ribs have a small height.

Papers [1-2] were devoted to the study of parametric vibrations of a rectilinear bar nonlinear and heterogeneous in thickness in viscoelastic medium using the Pasternack's contact model. The influence of the main factors such as elasticity of foundation, damageability of the material of the bar and shell, dependence of the shear coefficient on the frequency of vibrations on the characteristics of longitudinal vibrations of a bar in a viscoelastic medium was studied. In all the cases under consideration, the dependences of dynamical stability zone of bar vibrations in a viscoelastic medium on the construction parameters were constructed in the plane the load-frequency.

In [3], free vibrations of a moving fluid-contacting, longitudinally strengthened, orthotropic cylindrical shell heterogeneous in thickness, were studied. Using the Hamilton-Ostrogradsky variational principle, the systems of equations of motion were constructed. Heterogeneity of the shell material in thickness was taken into account accepting that the Young modulus and shell material's density are the functions of normal coordinate. Frequency equations were obtained and numerically implemented. During the calculation process, linear and parabolic laws were accepted for the heterogeneity function. Characteristically curves of dependence were constructed.

In the presence of geometrical and physical nonlinearities of the shell, the equations describing its stress-strain state are complex linear partial differential equations and in the paper [4] the method of successive loadings was used for solving them. Derivation of these equations was given in [5, 6]. An effective two-step method of sequential perturbation of parameters has been developed to reduce error in linearization of the equation and to reduce the time of counting [7]. Influence of support condition along the contour on the stability of the polymer concrete shells was studied in [8-10].

2. PROBLEM STATEMENT

For applying the Hamilton-Ostrogradsky variational principle we use total energy of the studied construction that consists of a cylindrical form heterogeneous shell and strengthening annular elements hose number varies. Furthermore, the studied construction contacts with solid medium (Figure 1(a)). To take into account the heterogeneity in thickness of a cylindrical shell, we will proceed from the three-dimensional functional. In this case the functional of total energy of cylindrical shell will be:

$$V=\frac{1}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\Big(\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}++\sigma_{12}\varepsilon_{12}+ \\ +\rho\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\Big)dxdydz \quad (1)$$

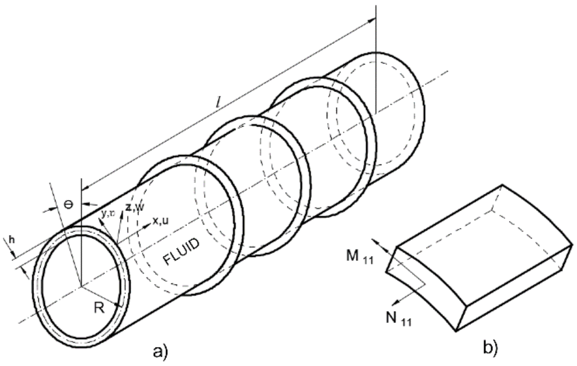

Figure 1. Strengthened heterogeneous cylindrical shell

There exist various methods to account for the heterogeneity of the shell material. One of them is that the Young modulus and density of the shell material are accepted as the functions of normal, lateral and longitudinal coordinate $[10]$. It is assumed that the Poisson ratio is constant. In this case, the strain-stress dependences are of the form:

$$\sigma_{11}=\frac{E(x,\theta,z)}{1-\nu^2}(\varepsilon_{11}+\nu\varepsilon_{22}) \\ \sigma_{22}=\frac{E(x,\theta,z)}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11}) \\ \sigma_{12}=G(x,\theta,z)\varepsilon_{12} \quad (2)$$

$$\varepsilon_{11}=\frac{\partial u}{\partial x};\ \varepsilon_{22}=\frac{\partial \vartheta}{\partial y}+\frac{w}{R};\ \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x} \quad (3)$$

Assume that

$$E(x,\theta,z)=E_0 f_1(z) f_2(x) f_3(\theta) \\ \rho(z,x)=\rho_0 f_1(z) f_2(x) f_3(\theta) \quad (4)$$

Taking into account (4) in (2), we get:

$$\sigma_{11}=\frac{E_0}{1-\nu^2}(\varepsilon_{11}+\nu\varepsilon_{22}) f_1(z) f_2(x) f_3(\theta) \\ \sigma_{22}=\frac{E_0}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11}) f_1(z) f_2(x) f_3(\theta) \\ \sigma_{12}=G\varepsilon_{12}=\frac{E_0}{2(1+\nu)}\varepsilon_{12} f_1(z) f_2(x) f_3(\theta) \quad (5)$$

where, E_0 is elasticity modulus, ρ_0 is density of the material of a homogeneous shell.

Allowing for (5), the functional of total energy of the cylindrical shell has the form:

$$V==\frac{RE_0}{2(1-\nu^2)}\int_{-h/2}^{h/2} f_1(z)dz\iint\left\{\varepsilon_{11}^2+2(1-\nu)\varepsilon_{11}\varepsilon_{22}+ \\ +\varepsilon_{22}^2+\varepsilon_{12}^2\right\}\times f_2(x) f_3(\theta)dxd\theta+ \\ +\int_{-\frac{h}{2}}^{\frac{h}{2}} f_1(z)dz\iint\left(\rho_0\left(\left(\frac{\partial u}{\partial t}\right)^2++\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\times \\ \times f_2(x) f_3(\theta)Rdxd\theta \quad (6)$$

The expression for the potential energy of elastic deformation of the jth lateral rib is as follows:

$$\Pi_j=\frac{1}{2}\int_0^{2\pi}\left[E_jF_j\left(\frac{\partial \vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+E_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+ \\ ++E_jJ_{xj}\left(\frac{\partial^2 u_i}{\partial y^2}-\frac{\varphi_{kpj}}{R}\right)^2+G_jJ_{kpj}\left(\frac{\partial \varphi_{kpi}}{\partial y}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]Rd\theta \quad (7)$$

Kinetic energies of the ribs are written in the form:

$$K_j=\rho_jF_j\int_0^{2\pi}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial \vartheta_j}{\partial t}\right)^2+ \\ +\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{kpj}}{F_j}\left(\frac{\partial \varphi_{kpj}}{\partial t}\right)^2\right]Rd\theta \quad (8)$$

In expressions (7) and (8) $F_j, J_{zj}, J_{yj}, J_{kpj}$ are area and inertia moments of the cross section of the jth lateral bar, respectively, with respect to the axis Oz and the axis parallel to the axis Oy and passing through the gravity center of the section and also its torsional inertia moment; E_j, G_j are elasticity and shear modules and also of the jth lateral bar, respectively, ρ_j is density of materials from which the jth lateral bar was made.

Potential energy of external surface loads acting on the part of the fluid on the shell, is determined as a work performed by these loads when transferring the system from the deformed state to the initial underformed state and is represented in the form:

$$A_0=-R\int_0^l\int_0^{2\pi} q_r w dxd\theta \quad (9)$$

The total energy of the system equals the sum of energy of elastic deformations of the shell and all lateral ribs and also potential energy of external loads acting on the part of the fluid:

$$J=V+\sum_{j=1}^{k_2}(\Pi_j+K_j)+A_0 \quad (10)$$

where, k_2 is the amount of lateral ribs.

The surface load q_r, acting on the part of the fluid on a longitudinally strengthened shell, is determined from the solution of the equation of motion of ideal fluid $[11]$:

$$\Delta\varphi - \frac{1}{a_0^2}\frac{\partial^2\varphi}{\partial t^2} = 0 \qquad (11)$$

where, φ is a potential of perturbed velocities, a_0 is perturbation propagation velocity in fluid.

On contact surface a shell-fluid we observe continuity of radial velocities and pressures. Condition of impermeability or smoothness of flow at the shell wall is:

$$\vartheta_r\big|_{r=R} = \frac{\partial\varphi}{\partial r}\bigg|_{r=R} = -\omega_0\frac{\partial w}{\partial t_1} \qquad (12)$$

Equality of radial pressures on the part of fluid on the shell:

$$q_r = -p_{|r=R} \qquad (13)$$

By means of (11), (12) and (13) we can represent pressure p on the part of fluid on the shell in the form:

$$p = \omega_0^2\Phi_{\alpha n}\rho_m\frac{\partial^2 w}{\partial t_1^2} \qquad (14)$$

where,

$$\Phi_{\alpha n} = \begin{cases} K_n(\beta r)/K_n'(\beta R), & M_1 < 1 \\ N_n(\beta_1 r)/N_n'(\beta_1 R), & M_1 > 1 \\ \dfrac{R^n}{nr^{n-1}}, & M_1 = 1 \end{cases} \qquad (15)$$

In (15)

$$M_1 = \frac{\omega/\text{m}}{a_0},\ \beta^2 = R^{-2}\left(1-M_1^2\right)\chi^2,\ \beta_1^2 = R^{-2}\left(M_1^2-1\right)\chi^2,$$

$$t_1 = \omega_0 t,\ \omega_0 = \sqrt{\frac{b_{11}}{\rho_0 R^2}},\ \omega_1 = \sqrt{\frac{\rho_0 R^2\omega^2}{b_{11}}} = \frac{\omega}{\omega_0},\ \xi = x/L,$$

where, K_n is a second kind *n*th order modified Bessel function, N_n are second kind Bessel or Neumann functions of *n*th order.

It is assumed that the conditions of hard contact between the shell and bars were satisfied:

$$u_j(y) = u(x_j, y) + h_j\varphi_1(x_j, y);$$
$$\vartheta_j(y) = \vartheta(x_j, y) + h_j\varphi_2(x_j, y);$$
$$w_j(y) = w(x_j, y);$$
$$\varphi_j(y) = \varphi_2(x_j, y);$$
$$\varphi_{kpj}(y) = \varphi_1(x_j, y);$$
$$h_j = 0.5h + H_j^1.$$

where H_j^1 is the distance from the axis of the *j*th bar to the surface of the cylindrical shell, φ_j, φ_{kpj} are turning and torsion angles of the cross section of the *j*th bar and are determined by shell displacements in the following way:

$$\varphi_j(y) = \varphi_2(x_j, y) = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{r}\right)\bigg|_{x=x_j};$$

$$\varphi_{kpj}(y) = \varphi_1(x_j, y) = -\frac{\partial w}{\partial x}\bigg|_{x=x_j}.$$

It is considered that on the lines $x = 0$ and $x = l$ the Navier boundary conditions are fulfilled:

$$\vartheta = 0,\ w = 0,\ N_{11} = 0,\ M_{11} = 0 \qquad (16)$$

where, l is the shell length, T_{11}, M_{11} are forces and moments acting on cross sections of the cylindrical shell (Figure 1(b)).

The frequency equation of an ribbed heterogeneous shell with flowing fluid was obtained based on the Ostrogradsky-Hamilton principle of stationarity of action:

$$\delta W = 0 \qquad (17)$$

where, $W = \int_{t'}^{t''} J dt$ is Hamilton's action, t' and t'' are the given arbitrary moments of time.

Completing the total energy of the system (10), the equation of motion of fluid (11) by contact conditions (12) and (13) we get a problem of natural vibrations of a fluid-contacting heterogeneous cylindrical shell strengthened with annular ribs in the main coordinate directions. In other words, a problem of natural vibrations of a fluid-filled heterogeneous cylindrical shell strengthened with annular ribs in the main coordinate directions is reduced to joint integration of the expression for total energy of the system (10), equation of motion of fluid (11) subject to condition (12) and (13) on their contact surface and boundary conditions.

3. PROBLEM SOLUTION

In expression (10) u, ϑ, w are varying variables. We approximate these unknown variables as follows:

$$u = u_0\cos\chi\xi\cos n\theta\sin\omega_1 t_1$$
$$\vartheta = \vartheta_0\sin\chi\xi\sin n\theta\sin\omega_1 t_1 \qquad (18)$$
$$w = w_0\sin\chi\xi\cos n\theta\sin\omega_1 t_1$$

where, u_0, ϑ_0, w_0 are unknown constants; χ, n are wave numbers in longitudinal and peripheral directions, respectively, $\xi = x/R$, $\chi = kR = \frac{m\pi R}{l}$, $t_1 = \omega_0 t$, ω is the desired frequency.

To calculate the work (9), by means of (14) we find contact surface forces q_r. When simplifying (10) we adopt the following dependences:

$$f_1(z) = 1 + \alpha\frac{z}{h},\ f_2(x) = 1 + \beta\frac{x}{l},\ f_3(x) = 1 + \gamma\frac{\theta}{2\pi R} \qquad (19)$$

where α, β, γ are constant parameters of heterogeneity in the direction along the normal, along the generatrix of the shell and in the peripheral direction, respectively, and $\alpha, \beta, \gamma \in [0,1]$.

Substituting the solution (19) in (10), taking into account expression (18) for the total energy (10), we get a second order polynomial with respect to the constants u_0, ϑ_0, w_0:

$$J_i = \varphi_{11}u_0^2 + \varphi_{22}\vartheta_0^2 + \varphi_{33}w_0^2 +$$
$$+\varphi_{44}u_0\vartheta_0 + \varphi_{55}u_0 w_0 + \varphi_{66}\vartheta_0 w_0$$

Since the expression for the coefficients $\varphi_{11}, \varphi_{22}, \varphi_{33}, \varphi_{44}, \varphi_{55}, \varphi_{66}$ have a bulky form, we do not cite them. If we vary the expression of Π with respect to the constants u_0, ϑ_0, w_0 and equate the coefficients of independent variations to zero, we get the following system of homogeneous algebraic equations

$$\begin{cases} 2\varphi_{11}u_0 + \varphi_{44}\vartheta_0 + \varphi_{55}w_0 = 0 \\ \varphi_{44}u_0 + 2\varphi_{22}\vartheta_0 + \varphi_{66}w_0 = 0 \\ \varphi_{55}u_0 + \varphi_{66}\vartheta_0 + 2\varphi_{33}w_0 = 0 \end{cases} \tag{20}$$

As the system (20) is a homogeneous system of linear algebraic equations, equality of its principal determinant to zero is the necessary and sufficient condition of the existence of its nonzero solution. As a result, we get the following frequency equation

$$\begin{vmatrix} 2\varphi_{11} & \varphi_{44} & \varphi_{55} \\ \varphi_{44} & 2\varphi_{22} & \varphi_{66} \\ \varphi_{55} & \varphi_{66} & 2\varphi_{33} \end{vmatrix} = 0 \tag{21}$$

We write Equation (21) in the form:

$$\begin{aligned} &4\varphi_{11}\varphi_{22}\varphi_{33} + \varphi_{44}\varphi_{55}\varphi_{66} - \varphi_{55}^2\varphi_{22} - \\ &-\varphi_{66}^2\varphi_{11} - \varphi_{44}^2\varphi_{33} = 0 \end{aligned} \tag{22}$$

Equation (22) was solved by numerical method. The following parameters were used in solution of problem:

$\rho_0 = \rho_j = 1850$ kg/m^3, $\tilde{E}_j = 6.67 \times 10^9$ H/m^2

$m = 1;\ n = 8;\ h_j = 1.39;\ R = 160$ cm

$I_{kpj} = 0.48$ mm^4; $I_{xj} = 19.9$ mm^4;

$F_j = 0.45$ mm^2; $h_i = 0.45$ mm, $\nu = 0.35$;

$\frac{l}{R} = 3,\ \frac{h}{R} = \frac{1}{6},\ \alpha = 0.4,\ \rho_m = 1\ \frac{\Gamma}{\text{cm}^3}$.

4. CONCLUSION

The results of calculations were given in Figure 2 in the form of dependence of the frequency parameter on the amount of the strengthening bars k_2 on the shell surface, in Figure 3 in the form of dependence of frequency parameters on the heterogeneity parameter in the direction of shell generatrix β. As we see from Figure 2, with increasing the amount of lateral ribs, the value of the frequency parameter increases

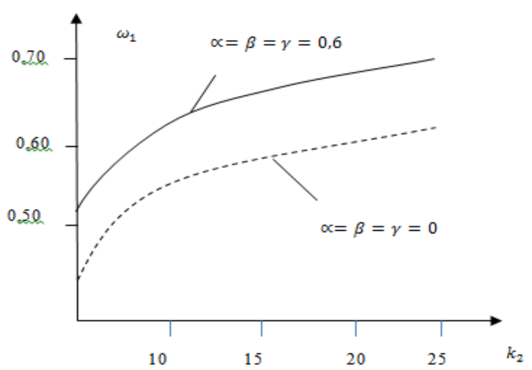

Figure 2. Dependence of the frequency parameter on k_2

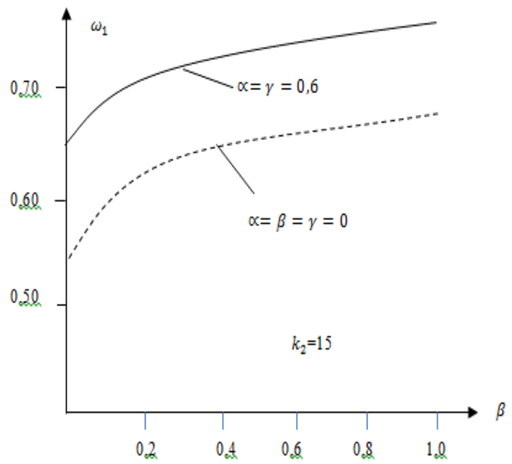

Figure 3. Dependence of the frequency parameters on β

REFERENCES

[1] I.T. Pirmamedov, "Parametric Oscillations of Nonlinear and No Uniform Thickness of Viscoelastic Cylindrical Shell under Dynamic Interaction with the Environment, Taking into Account the Friction", Series of Physical and Mathematical Sciences, Bulletin of Baku University, Baku, Azerbaijan, No. 1, pp. 82-89, 2005.

[2] I.T. Pirmamedov, "Calculation of Parametric Oscillations of a Homogeneous Thickness Viscoelastic rod in Viscoelastic Ground", International Scientific and Technical Journal, Joint Institute of Mechanical Engineering of the National Academy of Sciences of Belarus, Vol. 3, No. 8, pp. 52-56, 2009.

[3] F.S. Latifov, R.N. Aghayev, "Oscillations of Longitudinally Reinforced Heterogeneous Orthotropic Cylindrical Shell with Flowing Liquid", 13th International Conference on Technical and Physical Problems of Electrical Engineering (ICTEPE-2017), Van, Turkey, pp. 301-305, 21-23 September 2017.

[4] V.V. Petrov, "The Method of Successive Loading in the Nonlinear Theory of Plates and Shells", SGU, Publishing House, Saratov, Russia, 1975.

[5] V.V. Petrov, I.V. Krivoshein, "Methods for Calculating Structures from Nonlinearly Deformable Material", ASB Publishing House, Moscow, Russia, 2009.

[6] V.V. Petrov, I.V. Krivoshein, "Strength and Stability of Nonlinearly Deformable Shallow Shells", Academia, No. 3, pp. 83-87, 2009.

[7] V.V. Petrov, "Two-Step Method of Successive Perturbation of Parameters and its Application to Solving Non-Linear Problems of Solid Mechanics of a Deformable Body", Problems of Strength of Structural Elements under the Action of Loads and Working Environments: Intercollegiate Scientific Collection, Saratov, Russia, pp. 6-12, 2001.

[8] V.V. Petrov, I.V. Krivoshein, "Influence of Conditions of Support along the Contour on the Stability of Polymer-Concrete Shells", Herald VRO RAASN, Issue 13, Nizhny Novgorod, pp. 175-182, 2010.

[9] V.A. Lomakin, "The Theory of Inhomogeneous Bodies", MGU Publishing House, Moscow, Russia, 1975.

[10] F.S. Latifov, "Fluctuations of Shells with Elastic and Liquid Medium", Elm, Baku, Azerbaijan, 1999.

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