International Journal on
“Technical and Physical Problems of Engineering”
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

March 2020 | Issue 42 | Volume 12 | Number 1 | Pages 20-24

ALGORITHM AND MATLAB-BASED PROGRAM FOR MODELING THE NODAL ELECTRICITY PRICES

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Abstract- Through the last years of 20th century monopolist energy markets have become deregulated energy markets. Generation, transmission and distribution services are provided by various companies. Since power supply system reforms in various countries, including Azerbaijan, there is a possibility of using different energy sources in the power grids. It is known that electricity is traded in the wholesale market using such market procedures as the day-ahead market, the balancing market, and the market for regulated contracts. Price at different nodes represents the value of electricity and includes the cost of the electricity and the cost of power losses and system constraints. A mathematical model was developed based on Lagrange method for nodal prices calculation. The MATLAB program for calculating nodal pricing was developed. The equivalent scheme of the Azerbaijan power system was used for simulation. The possibility and effectiveness of the proposed model for the purpose of forecasting electricity prices was confirmed.

Keywords: Electricity Market, Nodal Price, Objective Function.

1. INTRODUCTION

It is known that electricity market historically was a monopoly industry. But during the 90s of last century monopolist power markets become deregulated power markets.

The first sign of the electricity market is that generation, transmission and distribution services are provided by various companies. According to principle of deregulated electricity market, all producers and consumers must have fair access to the network [1-2].

Note that two main market forms - the bilateral contract market and the pool market - are developed. Market participants - producers and consumers directly negotiate the price and the quantity of traded electrical energy in the bilateral market.

For the first time in the history an electricity market was established in Chile in 1982. The large consumers in Chile were allowed to choose their suppliers and negotiate the prices.

In the market of England and Wales the pool market mechanism was established in 1990. In 1991, the pool mechanism was adopted in Norway. This mechanism included Sweden in 1996, Finland in 1998 and Denmark in 1999. Nord Pool is the international electricity trading exchange. It allows users to choose their supplier. Thus, process of liberalization in electric power industry began.

In the electricity market, electricity is traded between buyers and sellers in the form of bilateral agreements. Two parties are involved here, exchanging electricity at negotiated prices and conditions. In wholesale electricity markets, generating companies can directly enter into contracts with customers.

When implementing bilateral agreements, the role of the system operator is minimized. Pool market is one of the types of electricity markets, which is established in some countries. However, there is no direct contract between producers and consumers in the pool market.

For example, in Scandinavian countries the Nord Pool is formed. In eastern countries the USA operates PJM. An installed capacity of the electricity market of PJM was more than 67000 MW [1].

In pool markets the electrical energy is traded through the market. It is the main characteristic of power pool market. Turkey’s electricity market, role and capacity of electricity generation companies is investigated in [2].

The methodology for the 2002 PJM market data has been tested and the results are presented in [3]. Note that pool market operates by Operator. Directly pool market can be operated by the operator named Independent System Operator, main task of which is to provide economic optimum. The electricity market operator obtains bids from suppliers and consumers for spot prices calculation. In Figure 1, supplier’s and consumer’s bid curves are presented.

This curve is classic supply-demand relationship. If the buyers and the sellers are both agree with the market results, the supply and demand curves intersect at the market equilibrium point. At the market equilibrium point marginal cost equals to linear inverse demand function. Since when the price becomes higher, the suppliers produce more power and the supply curve has a positive slope.

Price at node represents the value of EE. It includes the cost of the EE and the cost of power losses and system constraints. The price of transmission between any bus m and bus *k* is equal to the difference in the spot prices [1]:

$$TP_{mk} = TL_{mk} + TC_{mk} \quad (1)$$

where, TL_{mk} is the incremental change on the system losses and TC_{mk} is the incremental effect on the system congestion costs.

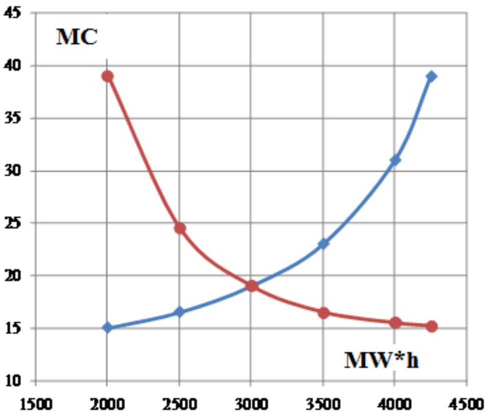


Figure 1. Supplier and consumer bid curves

When the bids are submitted, the market operator runs an optimal power flow program taking into account the network constraints. The objective of this optimal power flow program is to minimize the total costs. This objective also is known as a social welfare. The optimal power flow calculates spot prices for each bus of the power system and the quantity of supplying or buying that power by each of the market participant. Market participants are then billed to the spot price of their bus for the corresponding amount of electricity [4-7].

The scope of this work is the development of mathematical model and program for calculating nodal prices.

2. OPTIMIZATION MODEL OF POWER MARKET

Objective function of the electricity market has the form [4]:

- for unilateral auction:

$$\min \sum_{j=1}^{n} c_{g_j} P_{g_j} \quad (2)$$

or

$$\max \sum_{i=1}^{m} c_{d_i} P_{d_i} \quad (3)$$

- for bilateral auction:

$$\max \left\{ \sum_{i=1}^{m} c_{d_i} P_{d_i} - \sum_{j=1}^{n} c_{g_j} P_{g_j} \right\} \quad (4)$$

As shown from Equations (1) and (2), the aim of the operator is to minimize the cost of generating power. Constraints, which characterized the production, include demand-supply power balance, generator power constraints and transmission constraints.

That is, in minimizing the objective function it is necessary to account power balance and upper and lower bounds for powers, constraints to the generation and bus voltages [7]:

$$\sum_i P_{ij} + \sum_g P_g - \sum_c P_c = 0$$

$$\sum_i Q_{ij} + \sum_g Q_g - \sum_c Q_c = 0$$

$$P_s^{\min} \le \sum_{ij \in s} P_{ij} \le P_s^{\max} \quad (5)$$

$$0 \le P_g \le P_g^{\max},\ \ 0 \le Q_g \le Q_g^{\max}$$

$$0 \le P_c \le P_c^{\max},\ \ U_j^{\min} \le U_j \le U_j^{\max}$$

where, *d* is demand; *g* is generation; c_{d_i} is nodal price of active power at bus *i*; c_{g_j} is nodal price of active power generation at bus *i*; P_{di} is nodal active power demand; P_{gi} is nodal active power generation; *n*, *m* are bids of power generation and power demand; U_j is nodal voltage; P_{ij} and Q_{ij} are active and reactive power flows from bus *i* to bus *j* and *s* is controlled branches.

The demand-supply power balance constraints, the transmission constraints and generator constraints are the system constraints and generator constraints, respectively

3. MARGINAL COST CALCULATION USING LAGRANGE METHOD

Note that marginal cost is the cost of the next 1 MW. A function named Lagrange function is formulated for the above optimization problem. Lagrange function for mentioned optimization problem is given below [4, 6]:

$$L = \vec{C}(P_{gen}) - \lambda.\left(\sum_i P_{gen,i} + P_{\Sigma load} \right) - \quad (6)$$
$$-\vec{\mu}^T.\left([\alpha] \cdot \vec{P} - \vec{P}_L^{\max} \right)$$

where, P_{gen} is vector of active power generation, $P_{\Sigma load}$ is total active demand, $\vec{C}$ is vector of nodal generator's bids, $\vec{P}$ is vector of nodal active power, $\vec{P}_L$ is vector of constraints with respect to active power flow, and λ, μ are Lagrange multipliers with respect to set of equality and inequality constraints, respectively.

Thus, the nodal marginal cost of electricity is equal to the sum of the nodal price at the reference bus, cost of power losses, cost of system constraints and can be calculated by using the following expression [4, 6]:

$$\lambda_i = \lambda_{swing} + \lambda_{swing} \frac{\partial \Delta P}{\partial P_i} + \sum_s \mu_s \frac{\partial P_s}{\partial P_i} \quad (7)$$

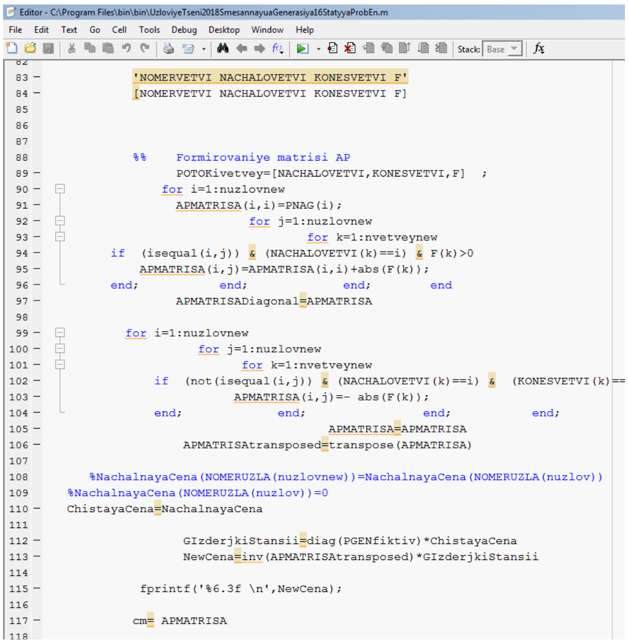
where, λ_i - marginal price at bus *i*, λ_{swing} is marginal price at reference bus, $\frac{\partial \Delta P}{\partial P_i} = L_i$ is marginal loss factor at bus *i*, P_i is active power injection at bus *i*, ΔP is total power losses, μ_s is price of *s*th constraint, and $\frac{\partial P_s}{\partial P_i}$ is coefficient of demand's change at bus *i* with respect to *s*th constraint.

If constraints exist, μ vector elements are not equal to zero, electricity price at demand nodes can be calculated using formula given below [7]:

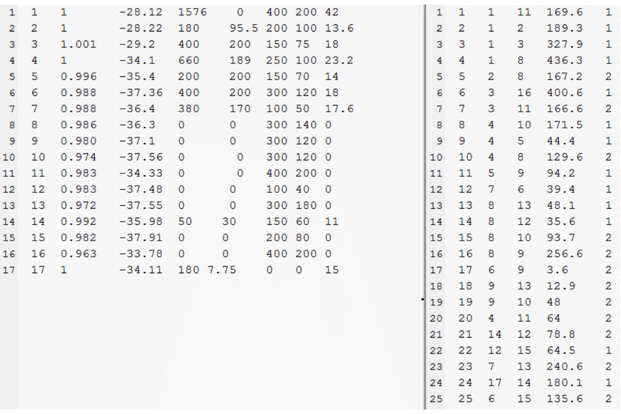
$$C = \lambda E + \mu^T [\alpha] \tag{8}$$

4. SIMULATION

The algorithm based on Lagrange multipliers and the MATLAB-based program are developed for modeling of the nodal prices [8-9]. Screen-shot of this program is shown in Figure 2. For simulation the 8-node test circuit was used (Figure 3).



```
'NOMERVETVI NACHALOVETVI KONESVETVI F'
[NOMERVETVI NACHALOVETVI KONESVETVI F]



%%    Formirovaniye matrisi AP
      POTOKivetvey=[NACHALOVETVI,KONESVETVI,F]   ;
    for i=1:nuzlovnew
      APMATRISA(i,i)=PNAG(i);
                   for j=1:nuzlovnew
                           for k=1:nvetveynew
if  (isequal(i,j)) & (NACHALOVETVI(k)==i) & F(k)>0
    APMATRISA(i,j)=APMATRISA(i,i)+abs(F(k));
end;            end;             end;        end
      APMATRISADiagonal=APMATRISA

  for i=1:nuzlovnew
            for j=1:nuzlovnew
                  for k=1:nvetveynew
      if  (not(isequal(i,j)) & (NACHALOVETVI(k)==i) &  (KONESVETVI(k)==
                 APMATRISA(i,j)=- abs(F(k));
      end;             end;                end;             end;
                              APMATRISA=APMATRISA
       APMATRISAtransposed=transpose(APMATRISA)

 %NachalnayaCena(NOMERUZLA(nuzlovnew))=NachalnayaCena(NOMERUZLA(nuzlov))
%NachalnayaCena(NOMERUZLA(nuzlov))=0
ChistayaCena=NachalnayaCena

       GIzderjkiStansii=diag(PGENfiktiv)*ChistayaCena
       NewCena=inv(APMATRISAtransposed)*GIzderjkiStansii

  fprintf('%6.3f \n',NewCena);

  cm= APMATRISA
```



```
 1  1   1       -28.12  1576     0   400 200 42
 2  2   1       -28.22  180     95.5 200 100 13.6
 3  3   1.001   -29.2   400     200  150 75  18
 4  4   1       -34.1   660     189  250 100 23.2
 5  5   0.996   -35.4   200     200  150 70  14
 6  6   0.988   -37.36  400     200  300 120 18
 7  7   0.988   -36.4   380     170  100 50  17.6
 8  8   0.986   -36.3   0       0    300 140 0
 9  9   0.980   -37.1   0       0    300 120 0
10  10  0.974   -37.56  0        0   300 120 0
11  11  0.983   -34.33  0        0   400 200 0
12  12  0.983   -37.48  0       0    100 40  0
13  13  0.972   -37.55  0       0    300 180 0
14  14  0.992   -35.98  50     30    150 60  11
15  15  0.982   -37.91  0      0     200 80  0
16  16  0.963   -33.78  0      0     400 200 0
17  17  1       -34.11  180 7.75     0   0   15
```

```
 1  1   1   11  169.6   1
 2  2   1   2   189.3   1
 3  3   1   3   327.9   1
 4  4   1   8   436.3   1
 5  5   2   8   167.2   2
 6  6   3   16  400.6   1
 7  7   3   11  166.6   2
 8  8   4   10  171.5   1
 9  9   4   5   44.4    1
10  10  4   8   129.6   2
11  11  5   9   94.2    1
12  12  7   6   39.4    1
13  13  8   13  48.1    1
14  14  8   12  35.6    1
15  15  8   10  93.7    2
16  16  8   9   256.6   2
17  17  6   9   3.6     2
18  18  9   13  12.9    2
19  19  9   10  48      2
20  20  4   11  64      2
21  21  14  12  78.8    2
22  22  12  15  64.5    1
23  23  7   13  240.6   2
24  24  17  14  180.1   1
25  25  6   15  135.6   2
```

Figure 2. Screen shot of the MATLAB-based program for nodal price simulation

The test scheme after addition of fictitious node and fictitious branches for simulation of nodal prices is shown in Figure 4.

An algorithm has been developed for modeling nodal prices. The algorithm consists of the following steps:

1. The calculation of the steady-state network;
2. Transfer of necessary information about nodes and branches to the program for nodal prices;
3. The formation of matrices of nodal injection of power, power flows;
4. Formation of additional nodes and branches
5. The formation of power flows matrix.
6. Determination of nodal prices.

Data on nodes and branches are shown in Tables 1 and 2. The steady-state mode's calculation of the circuit is presented in Table 3. Figure 4 illustrates fictitious nodes and fictitious branches added during simulation of nodal prices.

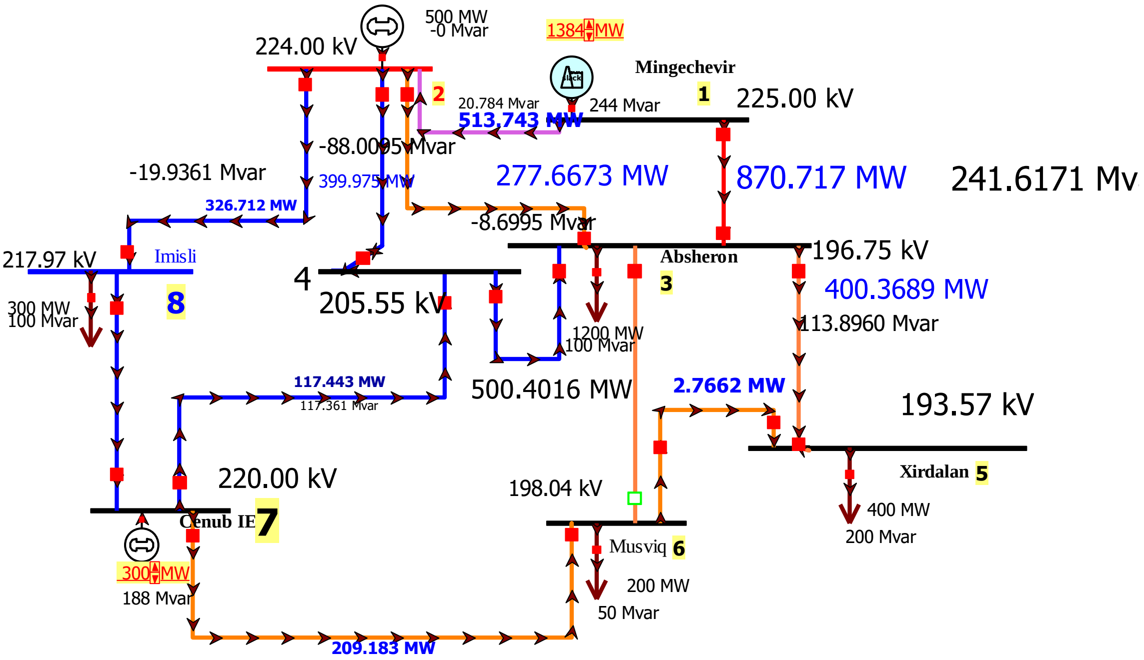


Figure 3. Test scheme for simulation of nodal prices

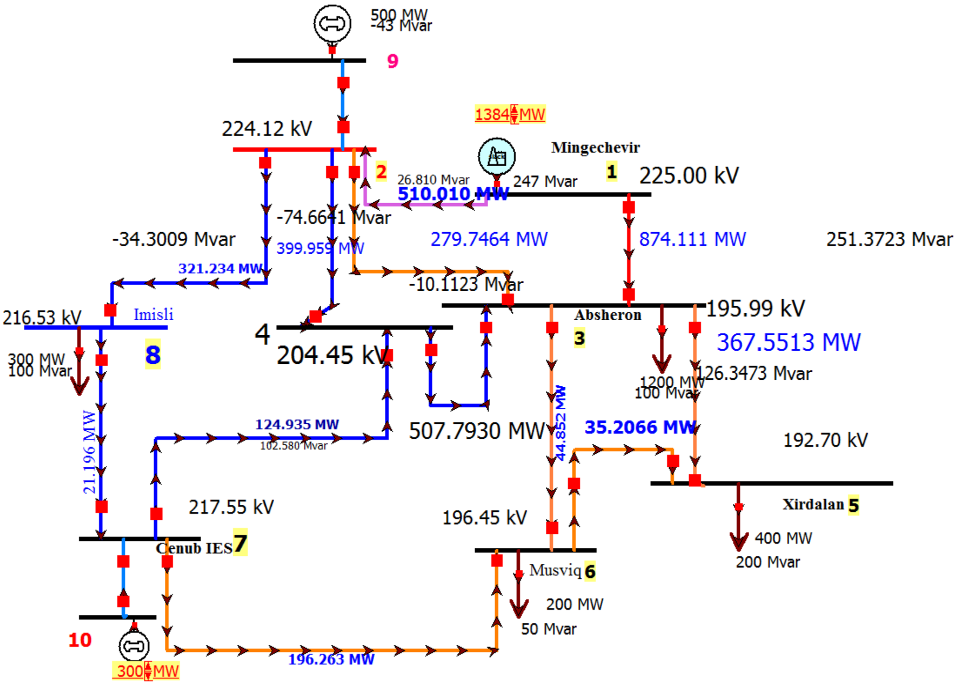


Figure 4. The test scheme after addition of fictitious node and fictitious branches for simulation of nodal prices

Initial data for buses and for branches are given in Tables 1 and 2. Power flow results for the circuit in Figure 1 are given in Table 3.

Note that bus 9 and bus 10 are the additional buses which are connected by buses 2 and 7 through additional branches 2-9 and 7-10. Formation of additional nodes and branches is illustrated in Figure 5. Results of nodal price simulation for the 8-node test circuit are presented in Table 4.

As shown from Table 4, the results of calculations of the nodal prices show that in different nodes of power system the different prices are formed. For example, on node 2 the nodal price is equal to 32.762, but on node 7 its value is 35.737. Note that nodal prices can be used in market conditions at price of bids.

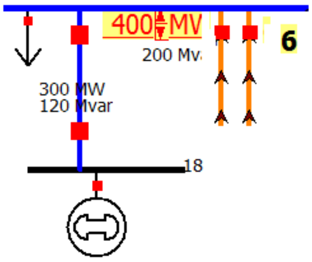


Figure 5. Formation of additional nodes and branches

Table 1. Initial data for buses

No	P_d MW	Q_d MVAR	P_G MW	Q_G MVAR
1	0	0	0	0
2	0	0	500	-13.79
3	1200	100	0	0
4	0	0	0	0
5	400	200	0	0
6	200	50	0	0
7	0	0	300	169.20
8	300	100	0	0.00

Table 2. Initial data for branches

From bus	To bus	*R*, ohm	*X*, ohm	*B*, S
1	3	1.66	21.29	-4800.02
1	2	0.08	4.35	0
2	3	10.66	61.30	-1187.81
2	8	4.03	26.17	-3313.43
2	4	4.36	28.95	3590.04
6	5	2.63	17.08	-2168.00
7	8	2.57	17.05	-2117.81
4	7	3.46	22.95	-244.98
7	6	5.6	45.91	-381.04
3	5	0.70	3.24	-190.50

Table 3. Power flow results for the 8-node test circuit

From Bus	To Bus	P_{ij}, MW	ΔP_{ij}, MW	Q_{ij}, MVAR	ΔQ_{ij}, MVAR
6	5	2.9	0.18	7	-85.79
1	3	870.7	29.05	241.6	163.84
1	2	513.7	0.42	1.9	22.68
7	6	209.2	6.24	91.3	34.28
7	4	117.4	2.08	117.4	2.64
7	8	-26.6	0.09	-20.9	-100.99
2	3	277.7	17.84	57.2	48.46
2	8	335.7	8.97	-88	-107.95
2	4	400	14.94	9.7	-73.01
3	5	400.4	3.14	113.9	6.68
3	4	-498.9	1.52	-127.4	70

Table 4. Results of nodal prices

Bus	Price of EE
1	30
2	32.7
3	32.82
4	34.6
5	33.1
6	36.8
7	35.7
8	32.7
9	35
10	36

5. DISCUSSION AND CONCLUSION

The transition to market relations in power systems leads to the need to use financial and technological models. Difference costs of electricity on different suppliers leads to the relevance of nodal prices.

The developed algorithm uses results of steady-state modes, formed the matrices of nodal injection, power flows, additional nodes with according branches then calculated nodal prices.

Note that the algorithm based on Lagrange multipliers can be used for modeling of the nodal prices. Simulation of the nodal prices was performed on example of 8 node scheme of electric network. Obtained results of the nodal prices can be used in market conditions.

NOMENCLATURES

EE: Electrical energy
PJM: Pennsylvania Jersey Maryland Interconnection Regional Transmission Organization
MW: Megawatt
MVAR: Megavolt-Amperes Reactive
S: Siemens

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BIOGRAPHIES

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