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VIBRATIONS OF AN INHOMOGENEOUS CYLINDRICAL SHELL DYNAMICALLY INTERACTING WITH MOTIVE FLUID AND STIFFENED WITH RINGS

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Abstract- In connection with increase of motion speed, pressure, temperature and other factors, it is very important to study vibration processes occurring in machines and mechanisms used in modern engineering. In operational conditions such constructions are in contact with different media. Therefore, vibration processes occurring in a construction and structural elements should be studied with regard to influence of external factors. In the paper, natural vibrations of a cylindrical shell-fluid system stiffened with ribs in the direction of generatrix inhomogeneous along the thickness. The homogeneity the thickness of a cylindrical shed may be taken into account by means of two different methods: by introducing multilaminate and homogeneity function. In the paper, the homogeneity was taken into account by accepting the Young modulus and the material density as a function of the coordinate alternating along the thickness.

Keywords: Cylindrical Shell, Inhomogeneity, Stiffened, Fluid, Free Vibrations, Vibration Frequency.

1. INTRODUCTION

Natural vibrations of a cylindrical shell and fluid moving in an elastic medium stiffened with cross rings and ribs was considered in [2]. Natural vibrations of an isotropic cylindrical shell stiffened only with rings and fluid moving in infinite elastic medium were studied in [3]. Parametric vibrations of smooth cylindrical shells with regard to inhomogeneity along the thickness, were investigated in [4-6]. Using the variational principle when solving the problem, for finding vibrations frequencies of the considered system a frequency equation was constructed and depending on physical and geometrical parameters characterizing the system, in the studied force-frequency plane characteristic curves were constructed. Ref. [7] investigates the stability of cylindrical coatings subjected to the effect of changing forces over time.

In this paper natural vibrations of flowing fluid interacting, cylindrical shell inhomogeneous in thickness, are studied. Using the Hamilton-Ostrogradsky variation principle in the solution of the problem, for studying free vibrations of a flowing-fluid-contacting cylindrical shell inhomogeneous in thickness and stiffened with rings, a system of equations was constructed. Homogeneity of the thickness of the cylindrical shell was taking into account accepting the Young modulus and density of the material as a function of coordinate alternating along the thickness.

When studying vibrations of a cylindrical shell inhomogeneous along the thickness and stiffened with annular ribs and dynamically interacting with flowing fluid, we considered two cases: a) fluid is at rest inside the cylindrical shell b) fluid moves with constant velocity inside the cylindrical shell. In both cases, the frequency equation was structured and its roots were found. In the calculation process, linear and parabolic cases of alternation of inhomogeneity function with respect to the coordinate were considered.

2. PROBLEM STATEMENT

To take into account the inhomogeneity of a cylindrical shell along the thickness we will use three-dimensional functional. In this case, total energy of the cylindrical shell is in the following form:

$$U=\frac{1}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\Big(\sigma_\alpha e_\alpha+\sigma_\beta e_\beta+\tau_{\alpha\beta}e_{\alpha\beta}+ +\rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)d\alpha d\beta dz \qquad (1)$$

where

$$\sigma_\alpha=\frac{T_1}{h}+\frac{12M_1}{h^3}z$$
$$\sigma_\beta=\frac{T_2}{h}+\frac{12M_2}{h^3}z \qquad (2)$$
$$\tau_{\alpha\beta}=\frac{s}{h}+\frac{12H}{h^3}z$$

There is various way for taking inhomogeneity into account one of them is to accept the Young modulus and material density as a function of the coordinate alternating along the thickness [1]:

$$E=E(z),\ \ \rho=\rho(z)$$

We assume that the Poisson ratio of constant. In this case, the stress-strain relations are written as follows:

$$e_\alpha = \frac{1}{E(z)}\left(\sigma_\alpha - v\sigma_\beta\right)$$
$$e_\beta = \frac{1}{E(z)}\left(\sigma_\beta - v\sigma_\alpha\right) \quad (3)$$
$$e_{\alpha\beta} = \frac{2(1+v)}{E(z)}\sigma_{\alpha\beta}$$

Taking into account Equations (2)-(3) and the equality

$$\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\rho(z)\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)d\alpha d\beta dz =$$
$$=\iint\left(\rho_0\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)-\right.$$
$$\left.-2\rho_1\left(\frac{\partial^2 u}{\partial x\partial t}\cdot\frac{\partial u}{\partial t}+\frac{\partial^2 w}{\partial y\partial t}\cdot\frac{\partial \vartheta}{\partial t}\right)+\rho_2\left(\left(\frac{\partial^2 w}{\partial x\partial t}\right)^2+\left(\frac{\partial^2 w}{\partial y\partial t}\right)^2\right)\right)d\alpha d\beta$$

in Equation (1), we can write:

$$V = \frac{1}{h}\iint\left\{T_1\left[\frac{1}{E_0}\left(2T_2 - vT_1\right)+\frac{12}{E_1h^3}\left(M_2 - vM_1\right)\right]+\right.$$
$$+T_2\left[-\frac{vT_2}{E_0}+\frac{12}{E_1h^3}\left(M_1 - vM_2\right)\right]+$$
$$+2(1+v)\left(\frac{S}{E_0}+\frac{12H}{E_1h^3}\right)++\frac{72}{E_2h^6}\times \quad (4)$$
$$\left.\times\left(2M_1M_2 - vM_1^2 - vM_2^2 + 2(1+v)H^2\right)\right\}d\alpha d\beta$$

We write total energy of the system of rings:

$$V_1 = \frac{1}{2}\sum_{j=1}^{k_2}\int_{y_1}^{y_2}\left[E_jF_j\left(\frac{\partial \vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+E_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right.$$
$$+E_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y^2}-\frac{\varphi_{\kappa pj}}{R}\right)^2+\sum_{j=1}^{k_2}\rho_jF_j\int_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial \vartheta_j}{\partial t}\right)^2+\right. \quad (5)$$
$$\left.\left.+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{\kappa pj}}{F_j}\left(\frac{\partial \varphi_{\kappa pj}}{\partial t}\right)^2\right]dy$$

As fluid is ideal, for the forces on the cylindrical shells, the conditions $q_x = 0,\ q_y = 0$ are satisfied. The work performed by the forces acting on the cylindrical shell as viewed from fluid in displacements of the points of the shell will be

$$A_0 = -\int_0^{x_1}\int_0^{2\pi} q_z w dx dy$$

The total energy of the system under consideration will consist of the sum of

$$W = V + V_1 + A_0 \quad (6)$$

In Equations (1)-(6), u, ϑ, w are the displacements of the points of the cylindrical shell, u_j, ϑ_j, w_j are the displacements of the points of the ring, E, v are elasticity modulus and Poisson ratio of the cylindrical shell material, respectively, R, h are the radius and thickness of the cylindrical shell, respectively, E_j is the elasticity modulus of the ring, F_j is the area of the cross-section of the ring, $I_{zj}, I_{xj}, I_{kp.j}$ are inertia moments of the of the cross section of the ring, k_2 is the amount of rings, q_z is the component of pressure forces acting on the cylindrical shell as viewed from fluid and $\rho_i = \int_{-h}^{h.}\rho(z)z^i dz$, $\frac{1}{E_i} = \int_{-h}^{h.}\frac{z^i dz}{E(z)}$.

It is considered that the contact conditions between the cylindrical shell and rings are satisfied:

$$u_j(y) = u\left(x_j, y\right) + h_j\varphi_1\left(x_j, y\right)$$
$$\vartheta_j(y) = \vartheta\left(x_j, y\right) + h_j\varphi_2\left(x_j, y\right)$$
$$w_j(y) = w\left(x_j, y\right) \quad (7)$$
$$\varphi_j(y) = \varphi_2\left(x_j, y\right)$$
$$\varphi_{kpj}(y) = \varphi_1\left(x_j, y\right)$$
$$h_j = 0.5h + H_j^1$$

Motion of fluid moving with velocity U with respect to the potential φ is in the form [7].

$$\Delta\varphi - \frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2}+2U\frac{\partial^2\varphi}{R\partial\xi\partial t}+U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right)=0 \quad (8)$$

In shell-fluid contact, the equality of velocity and pressure in radial displacement is satisfied as:

$$\vartheta_r\big|_{r=R} = \frac{\partial\varphi}{\partial r}\bigg|_{r=R} = -\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right) \quad (9)$$

$$q_z = -p_{|r=R} \quad (10)$$

We look for the φ as potential of perturbations in the following form:

$$\varphi(\xi, r, \theta, t_1) = f(r)\cos n\varphi \sin kx \sin\omega t \quad (11)$$

Using (9) and (10) from (11) we get:

$$\varphi = -\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right)$$
$$p = \Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2}+2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1}+U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \quad (12)$$

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta r), & M_1 < 1 \\ J_n(\beta_1 r)/J_n'(\beta_1 r), & M_1 > 1 \\ \dfrac{R^n}{nR^{n-1}}, & M_1 = 1 \end{cases} \quad (13)$$

where, $M_1 = \frac{U + \omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, I_n is nth order modified first kind Bessel function, $U^* = U/c$, c is propagation velocity of sound in the cylindrical shell, and J_n is the nth order first kind Bessel function.

So, the solution of the stated problem is reduced to joint integration of total energy, of the system (12) of motion equation of fluid (12) to joint integration of the construction consisting of a cylindrical shell with flowing fluid in the inner domain within boundary conditions. Equation (6) shows the joint integration of the total energy, the system of equations of motion of the fluid (8)-(12) within the boundary conditions.

3. PROBLEM SOLUTION

In Equation (6) the variational values are $u, \vartheta, w, T_1, T_2, M_1, M_2, S, H$. We determine the stationary value of function (6). For that we use the Rits method. We will look for the unknown quantities in the form:

$$
\begin{aligned}
u &= \cos\frac{\pi x}{l}\sin(k\varphi)\left(u_0\cos\omega t + u_1\sin\omega t\right)\\
\vartheta &= \sin\frac{\pi x}{l}\cos(k\varphi)\left(\vartheta_0\cos\omega t + \vartheta_1\sin\omega t\right)\\
w &= \sin\frac{\pi x}{l}\sin(k\varphi)\left(w_0\cos\omega t + w_1\sin\omega t\right)\\
T_1 &= \sin\frac{\pi x}{l}\sin(k\varphi)\left(T_{10}\cos\omega t + T_{11}\sin\omega t\right)\\
T_2 &= \cos\frac{\pi x}{l}\cos(k\varphi)\left(T_{20}\cos\omega t + T_{21}\sin\omega t\right)\\
S &= \sin\frac{\pi x}{l}\sin(k\varphi)\left(S_{10}\cos\omega t + S_{11}\sin\omega t\right)\\
M_1 &= \cos\frac{\pi x}{l}\sin(k\varphi)\left(M_{10}\cos\omega t + M_{11}\sin\omega t\right)\\
M_2 &= \sin\frac{\pi x}{l}\sin(k\varphi)\left(M_{20}\cos\omega t + M_{21}\sin\omega t\right)\\
H &= \cos\frac{\pi x}{l}\cos(k\varphi)\left(H_{10}\cos\omega t + H_{11}\sin\omega t\right)
\end{aligned}
\tag{14}
$$

Substituting expressions (14) in functional (6) we get a function dependent on variables $u_0, u_1, \vartheta_0, \vartheta_1, w_0, w_1, T_{10}, T_{11}, T_{20}, T_{21},\ S_{10}, S_{11},\ M_{10}, M_{12},\ M_{20}, M_{22}, H_{10}, H_{11}$.

The stationarity condition of the obtained function is determined from the following system:

$$
\begin{aligned}
&\frac{\partial J}{\partial u_0}=0;\ \frac{\partial J}{\partial u_1}=0;\ \frac{\partial J}{\partial \vartheta_0}=0;\ \frac{\partial J}{\partial \vartheta_1}=0;\ \frac{\partial J}{\partial w_0}=0\\
&\frac{\partial J}{\partial w_1}=0;\ \frac{\partial J}{\partial T_{10}}=0;\ \frac{\partial J}{\partial T_{11}}=0;\ \frac{\partial J}{\partial T_{20}}=0;\ \frac{\partial J}{\partial T_{21}}=0\\
&\frac{\partial J}{\partial S_{10}}=0;\ \frac{\partial J}{\partial S_{11}}=0;\ \frac{\partial J}{\partial M_{10}}=0;\ \frac{\partial J}{\partial M_{11}}=0\\
&\frac{\partial J}{\partial M_{20}}=0;\ \frac{\partial J}{\partial M_{21}}=0;\ \frac{\partial J}{\partial H_{10}}=0;\ \frac{\partial J}{\partial H_{11}}=0
\end{aligned}
\tag{15}
$$

In (15) the system is inhomogeneous for existence of its non-zero solution, its principal determinant should be equal zero. Then, we get following frequency equation:

$$\det\left\|a_{ij}\right\| = 0\ ,\ i, j = 1,18 \tag{16}$$

4. CONCLUSIONS

Equation (16) was studied by numerical method. For medium and shell parameters the following value were taken:

$h^* = \dfrac{h}{R} = 0.25\times10^{-2}$

$\nu = 0.3$, $\alpha = 0.5$

$E_j = E = 6.67\times10^9\ \text{N/m}^2$

$h_j = 1.39\,\text{mm}$

$F_j = 5.75\ \text{mm}^2$

$\dfrac{J_{zj}}{2\pi R^3 h} = 0.23\times10^{-6}$

$J_{xj} = 19.9\ \text{mm}^4$

$\rho_j = 0.26\times10^4\ \text{Nsec}^2/\text{m}^2$

$E_0 = E$

$\rho_0 = \rho_j$

$J_{kp.j} = 0.48\ \text{mm}^4$

$U^* = 0.005$

Two cases of inhomogeneity functions were considered as linear $E(z) = E_0\left[1+\alpha\left(\dfrac{z}{h}\right)\right]$, $\rho(z) = \rho_0\left[1+\alpha\left(\dfrac{z}{h}\right)\right]$ and parabolic $E(z) = E_0\left[1+\alpha\left(\dfrac{z}{h}\right)^2\right]$, $\rho(z) = \rho_0\left[1+\alpha\left(\dfrac{z}{h}\right)^2\right]$ which are Young modulus, α is inhomogeneity parameter. It should be noted that in the case of linear function $|\alpha|<1$, in the case of parabolic law α is any number and $\omega_1 = \sqrt{\dfrac{\left(1-\nu^2\right)\rho_0 R^2\omega^2}{E}}$.

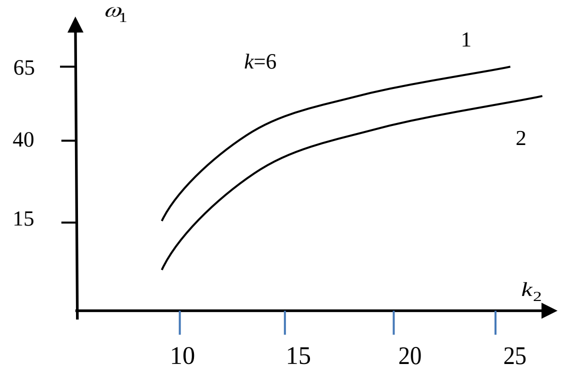

Figure 1. Dependence of frequency parameter on the number of rings

The results of calculations were given in Figure 1 in the form of dependence of frequency parameters on the number of rings, in Figure 2 in the form of dependence of the frequency parameter on the velocity of fluid motion.

The line of inhomogeneity laws corresponds to curves 1, parabolic change cases correspond to of inhomogeneity laws. Calculations show that vibrations frequencies corresponding to linear case of inhomogeneity laws are more than vibrations frequencies corresponding to the parabolic change case. Figure 1 shows that the first vibrations frequencies increase if the number of rings increase and the inertia effect of the ribs amplifies after same value decreases.

Figure 2 shows that with increasing the fluid motion velocity, the vibrations frequencies of the system decrease.

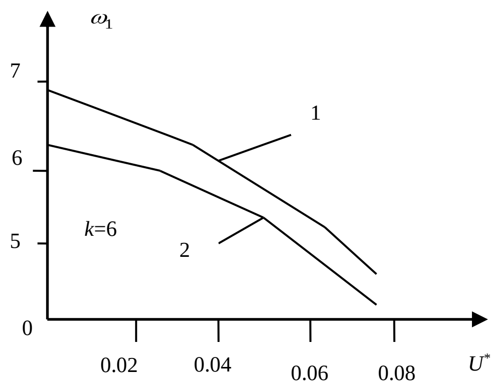

Figure 2. Dependence of frequency parameter on the velocity of fluid motion, 1 is linear law, 2 is a parabolic law

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