

## FORCED VIBRATIONS OF A LONGITUDINALLY STIFFENED INHOMOGENEOUS ORTHOTROPIC CYLINDRICAL SHELL IN FLUID

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**Abstract-** This paper forces vibrations of a longitudinally stiffened orthotropic cylindrical shell inhomogeneous in thickness under the action of internal radial pressure pulsating in time is studied. Based on the Ostrogradsky-Hamilton principle, a system of equations for determining displacements of the points of median surface of a longitudinally stiffened orthotropic cylindrical shell inhomogeneous in thickness under dynamic interaction with fluid, is constructed. The surface loads acting on a longitudinally stiffened cylindrical shell inhomogeneous in thickness as viewed from fluid, are determined from the solutions of the equation of motion of fluid, written in potentials. Analytic formulas for finding displacements of the points of the median surface of a longitudinally stiffened, fluid-contacting cylindrical shell inhomogeneous in thickness are obtained

**Keywords:** Forced Vibrations, Shell, Ideal Fluid, Stress, Stiffened, Variation Principle.

### 1. INTRODUCTION

For greater rigidity the shells are stiffened with ribs and this time slight increase in the mass of the structure significantly increases its strength. Such structures may be in contact with external medium and be subjected not only to statically but also to dynamical loads. Strength analysis, stability and vibrations of such constructions play an important role in designing modern machines and apparatus. Ref. [1] was devoted to the study of free vibrations of solid medium-fluid isotropic cylindrical shells longitudinally stiffened with cross system of ribs and loaded with axial compressive forces. Using the variational principle, a frequency equation of vibrations of a medium-contacting isotropic, stiffened cylindrical shell is constructed and numerically realized. Free vibrations of fluid-filled isotropic ridge cylindrical shells under axial compression were considered in [2].

The stiffening of shells was conducted with longitudinal, lateral and cross system of ribs. In [3], a problem of forced axially symmetric vibrations of a stiffened, fluid-filled cylindrical shell loaded with axial compressive forces, is studied. The vibration of laterally stiffened orthotropic cylindrical shells with flowing fluid in the soil was considered in [4]. Using the variational principle and the Pasternak model, parametric vibrations

of a nonlinear, visco-elastic, unstiffened cylindrical shell with a filler and inhomogeneous in thickness was studied in the paper [5].

Variational-parametric studies, modeling of cylindrical shells of step-variable thickness under dynamical loading were given in [6]. Note that the solutions described in references refer mainly to a stiffened isotropic, medium less cylindrical shell [7]. Vibrations of smooth cylindrical shells with a filler and fluid was completely studied in [8, 9]. The papers [10, 11] deal with vibrations of a longitudinally stiffened isotropic cylindrical shell with a filler under axial compression and with regard to friction between contact surfaces of the shell and filler. Analysis of these works shows that behavior of thin-shelled constructions made of orthotropic material and with discrete arrangements of ribs, under dynamical interaction with fluid was not studied sufficiently. Therefore, elaboration of mathematical models of behavior of stiffened shells, that completely takes into account their work under dynamical loads and carrying out investigations of fluid-contacting forced vibrations on their bases are urgent problems.

The present paper is devoted to the study of forced vibrations of a longitudinally stiffened orthotropic cylindrical shell in fluid under the action of time-pulsating internal radial pressure. The system of equations for determining the displacements of the points of the median surface of a longitudinally stiffened orthotropic cylindrical shell inhomogeneous in thickness, under dynamic interaction with fluid was constructed based on the Ostrogradsky-Hamilton variational principle. The surface loads acting on a longitudinally stiffened cylindrical shell inhomogeneous in thickness as viewed from fluid are determined from the solution of the equation of fluid motion written in potentials.

### 2. PROBLEM STATEMENT

The ridge shell is considered as a system consisting of a singular anisotropic shell and longitudinal ribs rigidly connected with it along contact lines (Figure 1). It is accepted that the stress-strain state of the shell may be completely determined within linear theory of elastic thin shells, based on the Kirchhoff-Liav hypothesis, while for calculating the ribs, theory of Kirchhoff-Liav curvilinear bars is applicable.

The system of coordinates is chosen so that the coordinate lines coincide with the lines of principle curvature of the median surface of the shell. It is assumed that the ribs were arranged along the coordinate lines, and their edges as the edges of the casing lie in the same coordinate plane.

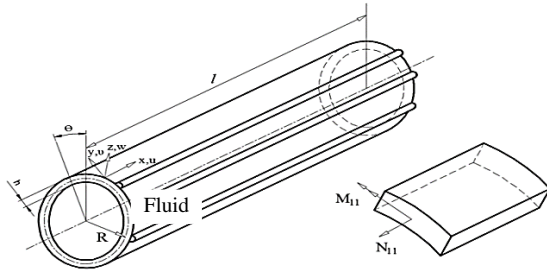


Figure 1. A longitudinally stiffened inhomogeneous cylindrical shell

Strain state of the casing may be determined by three components of displacements of its median surface  $u, \mathcal{G}$  and  $w$ . This time, the turning angles of normal elements  $\varphi_1, \varphi_2$  with respect to coordinate lines  $y$  and  $x$  are expressed by  $w$  and  $\mathcal{G}$  by means of dependences  $\varphi_1 = -\frac{\partial w}{\partial x}, \varphi_2 = -\left(\frac{\partial w}{\partial y} + \frac{\mathcal{G}}{R}\right)$ , where  $R$  is a radius of median surface of the shell.

To describe the strain state of ribs, in addition to components of displacements of center of gravity of their cross-sections ( $u_i, \mathcal{G}_i, w_i$  of the  $i$ th longitudinal bar), it is necessary to determine also the twisting angle  $\varphi_{kpi}$ .

Taking into account that according to accepted hypothesis we have constancy of radial deflections along the height of sections, and equality of appropriate twisting angles following from the conditions of rigid connection of ribs and a shell, we write the following relations:

$$\begin{aligned} u_j(y) &= u(x_j, y) + h_j \varphi_1(x_j, y); \\ \mathcal{G}_j(x) &= \mathcal{G}(x_j, y) + h_j \varphi_2(x_j, y); \\ w_j(x) &= w(x_j, y); \varphi_j = \varphi_2(x_j, y); \\ \varphi_{kpi}(x) &= \varphi_1(x_j, y); \end{aligned} \tag{1}$$

where,  $h_i = 0, 5h + H_i^1$ ,  $h$  is the thickness of the shell,  $H_i^1$  is the distance from the axis of the  $i$ th longitudinal bar to the shell surface,  $x_i$  and  $y_i$  are the coordinates of conjunction lines of ribs and the shell,  $\varphi_i, \varphi_{kpi}$  are angles of turning and twisting of cross-sections of longitudinal and lateral bars, respectively.

For external actions, it is assumed that the surface loads acting of the ridge shell as viewed from fluid, can be reduced to normal constituents  $q_z$ , applied to the median surface of the shell.

Differential equations of motion and natural boundary conditions for a longitudinally stiffened orthotropic cylindrical shell with fluid under axial compression are

obtained based on the Ostrogradsky-Hamilton variational principle. For that we preliminarily write potential and kinetic energies of the system.

There are various ways for calculating the inhomogeneity of the material of a shell. One of them is that the Young modulus and density of the shell material are accepted as functions of a normal coordinate [10]. It is assumed that the Poisson ratio is constant.

In this case, the functional of the total energy of elastic deformation of an orthotropic cylindrical shell is of the form:

$$\begin{aligned} \Pi_0 &= \frac{hR}{2} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h/2}^{h/2} \left\{ b_{11}(z) \left( \frac{\partial u}{\partial x} \right)^2 - \right. \\ &- 2(b_{11}(z) + b_{12}(z)) \frac{w}{R} \frac{\partial u}{\partial x} + \\ &+ \frac{w^2}{R^2} (b_{11}(z) + 2b_{12}(z) + b_{22}(z)) + b_{22}(z) \left( \frac{\partial \mathcal{G}}{\partial y} \right)^2 + \\ &2(b_{12}(z) + b_{22}(z)) \times \frac{w}{R} \frac{\partial \mathcal{G}}{\partial y} + 2b_{12}(z) \frac{1}{R} \frac{\partial u}{\partial x} \frac{\partial \mathcal{G}}{\partial y} + \\ &b_{66}(z) \left( \frac{\partial u}{\partial y} \right)^2 + b_{66}(z) \left( \frac{\partial \mathcal{G}}{\partial x} \right)^2 + 2b_{66}(z) \frac{\partial u}{\partial y} \frac{\partial \mathcal{G}}{\partial x} \left. \right\} dx dy dz \\ K_0 &= h \int_{-h/2}^{h/2} \rho(z) \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial \mathcal{G}}{\partial t} \right)^2 + \left( \frac{\partial w}{\partial t} \right)^2 \right] dx dy dz \end{aligned} \tag{2}$$

where,

$$\begin{aligned} b_{11}(z) &= \frac{E_1(z)}{1 - \nu_1 \nu_2}; b_{22}(z) = \frac{E_2(z)}{1 - \nu_1 \nu_2} \\ b_{12}(z) &= \frac{\nu_2 E_1(z)}{1 - \nu_1 \nu_2} = \frac{\nu_1 E_2(z)}{1 - \nu_1 \nu_2} \\ b_{66}(z) &= G_{12}(z) = G(z) \end{aligned} \tag{3}$$

where,  $R$  is the radius of the median surface of the shell,  $h$  is the shell thickness,  $u, \mathcal{G}, w$  are constituents of displacements of the points of the median surface of the shell.

Assume that

$$\begin{aligned} E_1(z) &= E_{10} f(z) \\ E_2(z) &= E_{20} f(z) \\ G(z) &= G_0 f(z) \\ \rho(z) &= \rho_0 f(z) \end{aligned} \tag{4}$$

where,  $E_{10}, E_{20}$  are the module of elasticity of the shell material in coordinate directions,  $G_0$  is the modulus of elasticity of the shell in shear,  $\rho_0$  is density of the material of a homogeneous shell.

Allowing for (3) and (4) the functional of the total energy of the cylindrical shell is of the form:

$$\begin{aligned} \Pi_0 &= \frac{hR}{2(1 - \nu_1 \nu_2)} \int_{-h/2}^{h/2} f(z) dz \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left\{ E_{10} \times \right. \\ &\times \left( \frac{\partial u}{\partial x} \right)^2 - 2(E_{10} + \nu_1 E_{20}) \frac{w}{R} \frac{\partial u}{\partial x} + \end{aligned}$$

$$\begin{aligned}
 & + \frac{w^2}{R^2} (E_{10} + 2\nu_1 E_{20} + E_{20}) + E_{20} \left( \frac{\partial \mathcal{G}}{\partial y} \right)^2 + \\
 & - 2(\nu_2 E_{10} + E_{20}) \frac{w}{R} \frac{\partial \mathcal{G}}{\partial y} + 2\nu_2 E_{10} \frac{1}{R} \frac{\partial u}{\partial x} \frac{\partial \mathcal{G}}{\partial y} + \\
 & + G_0 \left( \frac{\partial u}{\partial y} \right)^2 + G_0 \left( \frac{\partial \mathcal{G}}{\partial x} \right)^2 + 2G_0 \frac{\partial u}{\partial y} \frac{\partial \mathcal{G}}{\partial x} \Big\} dx dy \\
 & K_0 = h\rho_0 \int_{-h/2}^{h/2} f(z) dz \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial \mathcal{G}}{\partial t} \right)^2 + \right. \\
 & \left. + \left( \frac{\partial w}{\partial t} \right)^2 \right] dx dy
 \end{aligned} \tag{5}$$

The expressions for potential energy of elastic deformation of the  $i$  longitudinal rib are as follows [1]:

$$\begin{aligned}
 \Pi_i = \frac{1}{2} \int_{x_1}^{x_2} \left[ \tilde{E}_i F_i \left( \frac{\partial u_i}{\partial x} \right)^2 + \tilde{E}_i J_{yi} \left( \frac{\partial^2 w_i}{\partial x^2} \right)^2 + \right. \\
 \left. + \tilde{E}_i J_{zi} \left( \frac{\partial^2 \mathcal{G}_i}{\partial x^2} \right)^2 + \tilde{G}_i J_{kpi} \left( \frac{\partial \varphi_{kpi}}{\partial x} \right)^2 \right] dx
 \end{aligned} \tag{6}$$

where,  $F_i, J_{zi}, J_{yi}, J_{kpi}$  are area and moments of the inertia of the cross-section of the  $i$ th longitudinal bar with respect to the axis  $Oz$  and the axis parallel to the axis  $Oy$  and passing through the gravity center and its inertia moment at torsion;  $E_i, G_i$  are modulus of elasticity and shear of the material of the  $i$ th longitudinal bar.

Potential energy of external surface loads  $\bar{q}(q_x, q_y, q_z)$  and loads  $q_{zz}$ , acting as viewed from fluid, applied to the shell is determined as a work performed by these loads when taking the system from the deformed state to initial not deformed one and is represented as follows:

$$A_0 = - \int_{x_1}^{x_2} \int_{y_1}^{y_2} (q_x + q_y + q_z + q_{zz}) w dx dy \tag{7}$$

The total potential energy of the system is equal to the sum of potential energies of elastic deformations of the shell and longitudinal ribs, and also potential energies of all external loads acting as viewed from fluid and potential energy from the compressive stress:

$$\Pi = \Pi_0 + \sum_{i=1}^{k_1} \Pi_i + A_0 \tag{8}$$

Kinetic energies of the shell and longitudinal ribs are written in the form [6]:

$$\begin{aligned}
 K_i = \sum_{i=1}^{k_1} \rho_i F_i \int_{x_1}^{x_2} \left[ \left( \frac{\partial u_i}{\partial t} \right)^2 + \left( \frac{\partial \mathcal{G}_i}{\partial t} \right)^2 + \right. \\
 \left. + \left( \frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left( \frac{\partial \varphi_{kpi}}{\partial t} \right)^2 \right] dx
 \end{aligned} \tag{9}$$

where,  $t$  is time coordinate,  $\rho_i$  is density of materials from which the  $i$ th longitudinal bar was made.

Kinetic energy of the longitudinally stiffened shell

$$K = K_0 + \sum_{i=1}^{k_1} K_i \tag{10}$$

The equations of motion of the ridge shell are obtained based on the Ostrogradsky-Hamilton principle of stationarity of actions:

$$\delta W = 0 \tag{11}$$

where,  $W = \int_{t'}^{t''} \tilde{L} dt$  is Hamilton action,  $\tilde{L} = K - \Pi$  is a Lagrange function,  $t'$  and  $t''$  are the given arbitrary moments of time.

The surface load  $q_{zz}$ , acting as viewed from fluid on a longitudinally stiffened shell is determined from the solutions of the equation of motion of ideal fluid [7]:

$$\Delta \varphi - \frac{1}{a_0^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \tag{12}$$

where,  $\varphi$  is a potential of perturbed velocities,  $a_0$  is perturbation propagation velocity in fluid.

On the contact surface of shell-fluid we observe continuity of radial velocities and pressures. The condition of impermeability or smoothness of flow at the shell wall is of the form:

$$\mathcal{G}_r \Big|_{r=R} = \frac{\partial \varphi}{\partial r} \Big|_{r=R} = -\omega_0 \frac{\partial w}{\partial t_1} \tag{13}$$

Equality of radial pressures as viewed from fluid on shell  $q_{zz} = -p|_{r=R}$ ;  $q_x = q_y = 0$

By means of (12), (13) and (14), the pressure  $p$  as viewed from fluid on the shell may be represented as follows:

$$p = \omega_0^2 \Phi_{0n} \rho_m \frac{\partial^2 w}{\partial t_1^2} \tag{15}$$

$$\Phi_{an} = \begin{cases} K_n(\beta r) / K_n'(\beta R), & M_1 < 1 \\ N_n(\beta_1 r) / N_n'(\beta_1 R), & M_1 > 1 \\ \frac{R^n}{nr^{n-1}}, & M_1 = 1 \end{cases} \tag{16}$$

$$\text{In (16) } t_1 = \omega_0 t, M_1 = \frac{\omega / m}{a_0}, \beta^2 = R^{-2} (1 - M_1^2) \chi^2,$$

$$\omega_0 = \sqrt{\frac{b_{11}}{\rho_0 R^2}}, \beta_1^2 = R^{-2} (M_1^2 - 1) \chi^2, \xi = x / L,$$

$\omega_1 = \sqrt{\frac{\rho_0 R^2 \omega^2}{b_{11}}} = \frac{\omega}{\omega_0}$ ,  $K_n$ , is a  $n$ th order, modified Bessel function of second kind,  $N_n$  are  $n$ th order Bessel or Neumann functions of second kind.

We will suppose that a longitudinally stiffened orthotropic cylindrical shell in fluid is under action of time-pulsating internal radial pressure  $q_z$ :

$$q_z = q_0 \cos n\theta \sin \frac{m\pi}{\xi_1} \xi \sin \omega_1 t_1 \tag{17}$$

3. PROBLEM SOLUTION

We consider Hingely supported shells, i.e. for  $\xi = 0$  and  $\xi = \xi_1$  ( $\xi_1 = L/R$ ) the following boundary conditions are fulfilled  $\mathcal{G} = w = 0, T_1 = M_1 = 0$ .

We look for components of the vector of displacements of points of the median surface of the shell in the form

$$\begin{aligned}
 u &= u_0 \cos n\theta \cos \frac{m\pi}{\xi_1} \xi \sin \omega_1 t_1 \\
 \mathcal{G} &= \mathcal{G}_0 \sin n\theta \sin \frac{m\pi}{\xi_1} \xi \sin \omega_1 t_1 \\
 w &= w_0 \cos n\theta \sin \frac{m\pi}{\xi_1} \xi \sin \omega_1 t_1
 \end{aligned}
 \tag{18}$$

where,  $u_0, \mathcal{G}_0, w_0$  are unknown constants.

Using (9), (14), (15), (17), (18) and accepting  $f(z) = 1 + \gamma \frac{z}{h}$ ,  $\gamma$  is an inhomogeneity parameter, and  $0 \leq \gamma \leq 1$  the problem is reduced to the inhomogeneous system of linear algebraic equations of third order with respect to the constants  $u_0, \mathcal{G}_0, w_0$ :

$$a_{i1}u_0 + a_{i2}\mathcal{G}_0 + a_{i3}w_0 = q_i \quad (i = 1, 2, 3) \tag{19}$$

where,  $q_1 = q_2 = 0, q_3 = q_0$ . The elements  $a_{i1}, a_{i2}, a_{i3}$  ( $i = 1, 2, 3$ ) have a bulky form and we do not give them here.

As the system (17) is inhomogeneous from it for displacements amplitude we get:

$$u_0 = \frac{\Delta_1}{\Delta}, \mathcal{G}_0 = \frac{\Delta_2}{\Delta}, w_0 = \frac{\Delta_3}{\Delta} \tag{20}$$

where

$$\begin{aligned}
 \Delta_1 &= q_0 (a_{12}a_{23} - a_{22}a_{13}) \\
 \Delta_2 &= q_0 (a_{21}a_{13} - a_{11}a_{23}) \\
 \Delta_3 &= q_0 (a_{11}a_{22} - a_{21}a_{12}) \\
 \Delta &= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - \\
 &\quad - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{21}a_{12}a_{33}
 \end{aligned}$$

Note that for  $\Delta = 0$  the displacements amplitudes go to infinity and this corresponds to the resonance case.

4. NUMERICAL RESULTS

Let us consider some results of calculations carried out proceeding from the above dependences as (18) of displacements amplitude by means of ECM. The followings were accepted for geometrical and physical parameters characterizing the materials of shell, fluid and longitudinal bars:

$$E_i = 6.67 \times 10^9 \text{ N/m}^2$$

$$\rho_0 = \rho_i = 7800 \text{ Kg/m}^3$$

$$F_i = 3.4 \text{ mm}^2$$

$$J_{yi} = 5.1 \text{ mm}^4$$

$$\rho_m / \rho_0 = 0.105$$

$$\frac{J_{yi}}{2\pi R^3 h} = 0.8289 \times 10^{-6}$$

$$\frac{J_{zi}}{2\pi R^3 h} = 0.13 \times 10^{-6}$$

$$\frac{J_{kpi}}{2\pi R^3 h} = 0.5305 \times 10^{-6}$$

$$R = 0.16 \text{ m}$$

$$h = 0.00045 \text{ m}$$

$$v_1 = 0.11 \tag{18}$$

$$v_2 = 0.19$$

$$L = 0.8 \text{ m}$$

$$h_i = 1.39 \text{ mm}$$

$$a_0 = 1350 \text{ m/sec}$$

$$E_{10} = 18.3 \text{ QPa}$$

$$E_{20} = 25.2 \text{ QPa}$$

$$G_0 = 3.5 \text{ QPa}$$

The dependences of  $w_0/q$  on the frequency  $\omega_1$  for various ratio of  $E_{10}/E_{20}$  were represented in Figure 2 and  $E_{10}/E_{20}=1.25$  corresponds to solid lines,  $E_{10}/E_{20}=0.75$  to the dotted lines. From Figure 2 it can be seen that under certain frequencies the peaks of curves go to infinity. These frequencies are resonant and are determined from the equation  $\Delta = 0$ . Furthermore, strengthening of orthotropic properties of the shell material reduces to decrease in the value of the shell deflection.

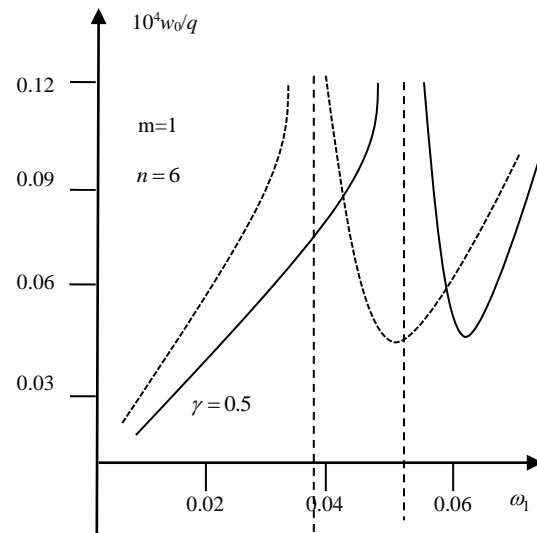


Figure 2. Dependence of the shell deflection on vibration frequency

Dependence of  $w_0/q$  on inhomogeneity parameter  $\gamma$  for different ratios of  $E_{10}/E_{20}$  was given in Figure 3, and  $E_{10}/E_{20}=0.25$  corresponds to solid lines,  $E_{10}/E_{20}=0.75$  to dotted ones. From the figure it can be seen that with increasing the inhomogeneity parameter  $\gamma$ , the shell deflection decreases.

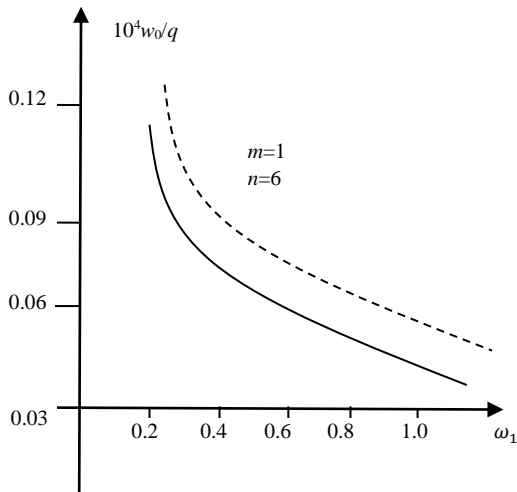


Figure 3. Dependence of shell deflection on inhomogeneity parameter

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