International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

March 2020 | Issue 42 | Volume 12 | Number 1 | Pages 40-43

FREE VIBRATION OF INHOMOGENEOUS, ORTHOTROPIC AND MEDIUM-CONTACTING CULINDRICAL PANEL STIFFENED WITH ANNULAR RIBS

A.H. Movsumova

Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Baku, Azerbaijan
aytenmovsumova@mail.ru

Abstract- Cylindrical panels are widely used in modern technology, power engineering, and in various fields of construction and engineering. In many cases, depending on production technology and a number of various reasons, mechanical properties of the material of cylindrical panels become continuously inhomogeneous along the length of the panel. In operational conditions these panels are in contact with different nature medium and they are stiffened when it is necessary.

Keywords: Cylindrical Panel, Viscous-Elastic Medium, Inhomogeneous Cylindrical Panels, Ridge Panel, Free Vibrations.

1. INTRODUCTION

In paper [1], a problem of lateral vibrations of an annular cross-section inhomogeneous cylindrical shell lying on a viscous-elastic foundation, is considered. It is assumed that the modulus of elasticity and density are continuous functions of thickness coordinate. In the paper a problem of natural vibrations of an annular cross-section cylindrical shell inhomogeneous only along the length and lying on inhomogeneous viscous-elastic medium, is considered. The solution of the problem is reduced to the system of two linear differential equations with respect to the stress function and deflection.

The method of separation of variables and the Bubnov-Qalerkin method is used when solving the problem. The paper [5] was devoted to free vibrations of a flowing fluid-contacting, isotropic, inhomogeneous cylindrical shell stiffened with cross system of ribs. Using the Hamilton-Ostrogradsky variational principle, the system of equations of motion for a flowing fluid-contacting anisotropic cylindrical shell inhomogeneous in thickness and stiffened with cross systems of a ribs was solved. The paper [4] deals with natural vibrations of a soil-contacting cylindrical shell stiffened with annular ribs and subjected to compressive forces.

The present paper is devoted to vibrations of an orthotropic, cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linearly viscous-elastic foundation. Using the Hamilton-Ostrogradsky variational principle for finding vibrational frequencies of a cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linear elastic foundation, the frequency equation was constructed, its roots were found and the influences of physical and geometrical parameters characterizing the system, were studied.

2. PROBLEM STATEMENT

To apply the Hamilton-Ostrogradsky variational principle, we write the total energy of the structure under investigation, consisting of an orthotropic cylindrical panel inhomogeneous in thickness and stiffening elements whose number varies.

Furthermore, from the inside the construction is in contact with the medium (Figure 1). To take into account inhomogeneity of a cylindrical shell in thickness, we will proceed from three-dimensional functional. In this case, the functional of total energy of the cylindrical shell is of the form

$$V=\frac{R}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}+\sigma_{12}\varepsilon_{12}+\right.$$
$$\left.+\rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dxd\varphi dz \qquad (1)\right.$$

There are various ways for taking into account inhomogeneity of the shell material. One of them is that the Young modulus and density of the material are accepted as functions of the normal coordinate z [11]. It is supposed that the Poisson ratio is constant. In this case, the strain-stress ratio is of the form

$$\begin{aligned}\sigma_{11}&=b_{11}(z)\varepsilon_{11}+b_{12}(z)\varepsilon_{22}\\ \sigma_{22}&=b_{12}(z)\varepsilon_{11}+b_{22}(z)\varepsilon_{22}\\ \sigma_{12}&=b_{66}(z)\varepsilon_{12}\end{aligned} \qquad (2)$$

$$\begin{aligned}\varepsilon_{11}&=\frac{\partial u}{\partial x}\\ \varepsilon_{22}&=\frac{\partial \vartheta}{\partial y}+w\\ \varepsilon_{12}&=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x}\end{aligned} \qquad (3)$$

In (1) we can write:

$$V=\frac{R}{2}\iint\Big\{\tilde{b}_{11}\varepsilon_{11}^2+2\tilde{b}_{12}\varepsilon_{11}\varepsilon_{22}+2\tilde{b}_{26}\varepsilon_{12}\varepsilon_{22}+2\tilde{b}_{16}\varepsilon_{11}\varepsilon_{12}+$$
$$+\tilde{b}_{22}\varepsilon_{22}^2+\tilde{b}_{66}\varepsilon_{12}^2\Big\}dxd\varphi+ \tag{4}$$
$$+\iint\left(\tilde{\rho}\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\right)dxd\varphi$$

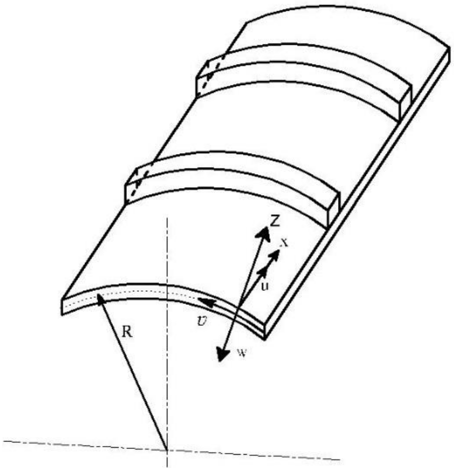

Figure 1. Inhomogeneous orthotropic, elastic medium-contacting cylindrical panel stiffened with annular ribs

where, $\tilde{b}_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{11}(z)dz;\ \tilde{b}_{12}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{12}(z)dz$;

$$\tilde{b}_{22}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{22}(z)dz;\ \tilde{b}_{66}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{66}(z)dz;$$

$$b_{11}(z)=\frac{E_1(z)}{1-\nu_1\nu_2};\quad b_{22}(z)=\frac{E_2(z)}{1-\nu_1\nu_2};$$

$$b_{66}(z)=G_{12}(z)=G(z);\ b_{12}(z)=\frac{\nu_2E_1(z)}{1-\nu_1\nu_2}=\frac{\nu_1E_2(z)}{1-\nu_1\nu_2}$$

are the main module of elasticity of the orthotropic material, $\tilde{\rho}=\int_{-h}^{h.}\rho(z)dz$, ν_1, ν_2 are Poisson ratios of the orthotropic material, h is shell thickens, u,ϑ,w are the components of displacements of the points of the median surface of the shell. It is assumed that $E_1(z)=\tilde{E}_1 f(z),\ E_2(z)=\tilde{E}_2 f(z),\ G(z)=\tilde{G}f(z)$.

The expressions for the potential energy of elastic deformation of jth lateral rib are as follows [12]:

$$\Pi_j=\frac{R}{2}\int_0^{\varphi_0}\left[\tilde{E}_jF_j\left(\frac{\partial\vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+\tilde{E}_jJ_{xj}\left(\frac{\partial^2w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right.$$
$$\left.+\tilde{E}_jJ_{zj}\left(\frac{\partial^2u_i}{\partial y^2}-\frac{\varphi_{\kappa pj}}{R}\right)^2+\tilde{G}_jJ_{\kappa pj}\left(\frac{\partial\varphi_{\kappa pi}}{\partial y}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]d\varphi \tag{5}$$

The kinetic energy of ribs are written in the form [12]:

$$K_j=\rho_jF_jR\int_0^{\varphi_0}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial\vartheta_j}{\partial t}\right)^2+\right.$$
$$\left.+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{kpj}}{F_j}\left(\frac{\partial\varphi_{kpj}}{\partial t}\right)^2\right]d\varphi \tag{6}$$

In expressions (4) and (6) $F_j, J_{zj}, J_{yj}, J_{kpj}$ are the area and moments of inertia of the cross-section of the jth bar with respect to the axis Oz and the axis parallel to the axis Oy and passing through the gravity center, and also its inertia moment at torsion; $\tilde{E}_j,\tilde{G}_j$ are elasticity and shear modulus of the material of the jth lateral bar, respectively; ρ_j is density of materials from which the jth lateral bars were made.

Potential energy of external surface loads acting as viewed from elastic medium, applied to the shell is determined as a work performed by these loads when taking the system from the deformed state to the initial not deformed one and is represented as follows:

$$A_0=-R\int_0^l\int_0^{\varphi_0}q_zwdxd\varphi \tag{7}$$

Suppose that the plate lies on two Pasternak-type two-constant foundation [7], where reaction q_z is connected with flexure w in the following relation:

$$q_z=k_\upsilon w-k_p\left(\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}\right) \tag{8}$$

where, k_v, k_p are Winkler's and Pasternak coefficients.

The total energy of the system equals the sum of energies of elastic deformations of the shell and lateral ribs, and also potential energies of all external loads acting as viewed from viscous-elastic medium:

$$J=V+\sum_{j=1}^{k_2}\left(\Pi_j+K_j\right)+A_0 \tag{9}$$

To describe the strain state of ribs, in addition to three components of displacements of gravity center of their cross-sections (u_j,ϑ_j,w_j of the jth lateral bar u_i,ϑ_i,w_i of the ith longitudinal bar) it is also necessary to determine the twist angles φ_{kpi} and φ_{kpj}.

Taking into account that according to the accepted assumptions we have constancy of radial flexures and also equality of corresponding twist angles following from the conditions of rigid connection of ribs with a shell, we write the following relations:

$$u_i(x)=u(x,y_i)+h_i\varphi_1(x,y_i);\ \vartheta_i(x)=\vartheta(x,y_i)+h_i\varphi_2(x,y_i)$$
$$w_i(x)=w(x,y_i);\ \varphi_i=\varphi_1(x,y_i);\ \varphi_{kpi}(x)=\varphi_2(x,y_i) \tag{10}$$
$$u_j(x)=u(x_j,y)+h_j\varphi_1(x_j,y);\ \vartheta_j(x)=\vartheta(x_j,y)+h_j\varphi_2(x_j,y)$$
$$w_j(x)=w(x_j,y);\ \varphi_j=\varphi_2(x_j,y)$$
$$\varphi_{kpj}(x)=\varphi_1(x_j,y)$$

where, $h_i = 0.5h + H_i^1$, $h_j = 0.5h + H_j^1$, h is shell thickness, H_i^1 and H_j^1 are distances from the axes of the ith longitudinal and jth lateral bar to the shell surface, x_i and y_i are the coordinates of lines of conjunction of ribs with the shell, φ_i, φ_{kpi} and φ_j, φ_{kpj} are turning and twisting angles of cross-section of longitudinal and lateral bars, respectively.

This time, the turning angles of normal elements φ_1, φ_2 with respect to coordinate lines y and x are expressed by w and ϑ by means of the dependences $\varphi_1 = -\frac{\partial w}{\partial x}, \varphi_2 = -\left(\frac{\partial w}{R\partial\varphi} + \frac{\vartheta}{R}\right)$, where R is the radius of curvature of the median surface of the plate.

Let the plate be comprehensively fixed with hinges. Then the following boundary conditions should be fulfilled:

$$u = v = w = M_x = 0 \quad \text{for} \quad x = 0; L \tag{10}$$

$$u = v = w = M_x = 0 \quad \text{for} \quad \varphi = 0; \varphi_0 \tag{11}$$

The frequency equation of a ridge, inhomogeneous, orthotropic, flowing-fluid contacting shell was obtained on the base of Ostrogradsky-Hamilton principle of stationarity of action:

$$\delta W = 0 \tag{12}$$

where $W = \int_{t'}^{t''} J dt$ is Hamilton action, t' and t'' are the given arbitrary moments of time.

Complementing the total energy of the system (9) with contact (10) and boundary conditions (11), we get a problem of natural vibrations of a viscous-elastic medium-contacting orthotropic cylindrical shell inhomogeneous in thickness and stiffened with lateral system of ribs.

In other words, the problem of natural vibrations of a viscous-elastic medium-contacting orthotropic, cylindrical shell inhomogeneous in thickness and stiffened with cross system of ribs is reduced to integration of expressions for the total energy of the system (9).

3. PROBLEM SOLUTION

In expression (9) u, ϑ, w_r are variable values. These unknown values are approximated in the following way:

$$u = u_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t$$

$$\vartheta = \vartheta_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t \tag{13}$$

$$w = w_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t$$

Substituting (13) in (9), after integration we get a function of variables u_0, ϑ_0, w_0. The stationary value of the obtained function is determined by the following system:

$$1)\frac{\partial J}{\partial u_0} = 0; \quad 2)\frac{\partial J}{\partial \vartheta_0} = 0; \quad 3)\frac{\partial J}{\partial w_0} = 0 \tag{14}$$

The non-trivial solution of the system of third order linear algebraic equations is possible only in the case when ω is the root of its determinant. The determination of ω is reduced to a transcendental equation as ω enters into the arguments of the Bessel function:

$$\det\left\|a_{ij}\right\| = 0, i, j = 1,3 \tag{15}$$

4. CONCLUSIONS

Equation (15) was calculated by the numerical method. The parameters contained in the solution of the problem were accepted as:

$\rho_0 = \rho_j = 1850$ kg/m^3;

$\tilde{E}_j = 6.67 \times 10^9$ N/m^2;

$m = 1$; $n = 8$; $h_j = 1.39$; $R = 160$ cm;

$I_{kpj} = 0.48$ mm^4; $I_{xj} = 19.9$ mm^4;

$F_j = 0.45$ mm^2; $\nu = 0.35$;

$\frac{l}{R} = 3$; $\frac{h}{R} = \frac{1}{6}$;

$\varepsilon \in [0;1]$; $f(z) = 1 + \varepsilon\left(\frac{z}{h}\right)^2$;

$k_\upsilon = 10^6$ N/m^3; $k_p = 10^4$ N/m

The result of calculations were given in Figure 2 in the form of dependence of the frequency ω on the amount of stiffening bars k_2 on the shell surface, in Figure 3 in the form of dependence of frequency ω on in homogeneity parameter ε.

It can be seen from Figure 2 that with increasing the amount of lateral ribs, the value of the vibrations frequency of the system increases. Figure 3 shows that with increasing the in- homogeneity parameter the vibrations frequencies of the system also increase.

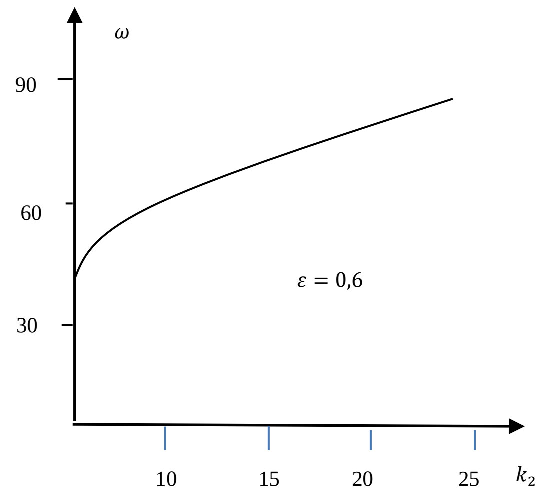

Figure 2. Dependence of vibrations frequen ω of the system on the amo bs k_2

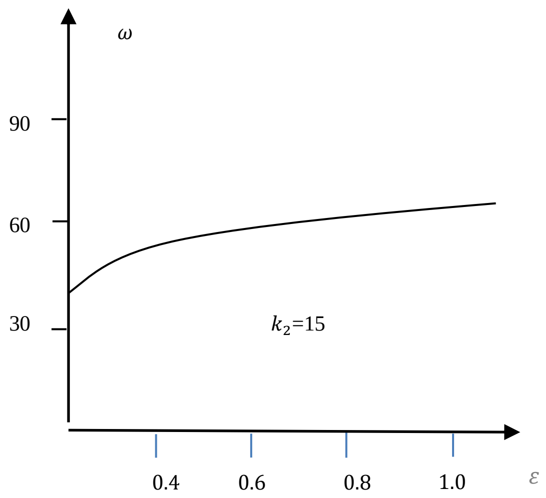

Figure 3. Dependence of vibrations frequency ω of the system on the inhomogeneity parameter ε

REFERENCES

[1] A.Kh. Movsumova, "Axially-Symmetric form Lateral Vibrations of Inhomogeneous Cylindrical Shell Lying on Viscous-Elastic Foundation", International Scientific Journal of Topical Science, Volgograd, Russia, No. 10, pp. 50-54, 2018.

[2] A.Kh. Movsumova, "On Free Vibrations of Exponentially Inhomogeneous Cylindrical Shell", Theoretical and Applied Mechanics, No. 3-4, pp. 117-120, 2018.

[3] V.I. Lomakin, "Theory of Elasticity of Inhomogeneous Bodies", Publication House, Moscow State University, Moscow, Russia, p. 376, 1976.

[4] F.S. Latifov, R.A. Iskanderov, K.A. Babaeva, "Vibrations of Nonhomogeneous Medium-Contacting Cylindrical Shell Stiffened with Rings and Subjected to Action of Compressive Force", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 31, Vol. 9, No. 2, pp. 1-5, June 2017.

[5] F.S. Latifov, R.N. Aghayev, "Oscillations of Longitudinally Reinforced Heterogeneous Orthotropic Cylindrical Shell with Flowing Liquid", 13th International Conference on Technical and Physical Problems of Electrical Engineering (ICTPE-2017), Van, Turkey, pp. 301-305, 21-23 September 2017.

[6] I.Y. Amiro, V.A. Zarutsky, "The Theory of Ribbed Shells: Methods for Calculating Shells", Scientific Thought, p. 367, 1980.

[7] P.L. Pasternak, "Principles of a New Method for Calculating Foundation of Elastic Base by Means of Two Bed Coefficients", State Prospect, Moscow, Russia, p. 89, 1959.

BIOGRAPHY

Ayten Hafiz Movsumova was born in Baku, Azerbaijan, in 1990. She studied in Physics, Mathematics and Computer Science in high school. She graduated from Faculty of Mechanics-Mathematics, Baku State University, Baku, Azerbaijan in 2012. She received her M.Sc. degree from Baku State University, Baku, Azerbaijan. She has been working at Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Baku, Azerbaijan since 2013. She is a post-graduate student of Institute of Mathematics and Mechanics. She has published 5 articles and 2 theses.