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STUDY OF OPERATING MODES OF SMALL HYDROELECTRIC UNITS EQUIPPED WITH SYNCHRONOUS GENERATOR

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Abstract- A universal analytical expression is obtained for the power of hydraulic turbines of small hydropower plants such as Francis, Pelton and Kaplan with fixed blades as a function of water flow. This expression allows us to predict the generation of active power of each of the turbines, depending on the adjusted water flow rate, which varies in the seasonal, monthly and daily periods of the year. It also allows you to solve the inverse problem, when it is necessary, depending on the load schedule dictated by the electrical system to which these small hydroelectric power stations are connected, to regulate the amount of water flow by changing the opening angles of the guiding devices of the hydraulic turbines. Studies on the proposed mathematical model containing a synchronous generator with electromagnetic excitation coupled to a Francis hydraulic turbine with a 60% change in water flow from q = 1 (p.u) to q=0.4 (p.u). The value of the required active power of a small hydroelectric power station, i.e. power output of the generator will jam (either by the dispatcher, or by the value of the load schedule), i.e. it is an input value, and as the output value of the controller is the opening angle of the guiding apparatus of the turbine.

Keywords: Small Hydropower Plants, Francis Turbine, Pelton Turbine, Axial Turbines, Synchronous Generator with Electromagnetic Excitation, Electrical Grid.

1. INTRODUCTION

In recent years small HPSs (hydroelectric power stations) are successfully adopted as the electric power producers at regional level in many world countries. One of the main advantages of this power type is its environmental purity, and a disadvantage is the demand for significant investments in their construction.

In small HPSs, whose power varies from several kW to 25-30 MW, in most cases as the hydraulic turbines the Francis and Pelton turbines are used, and also a series of hydraulic units with axial flow turbines.

The purpose of this paper is the modeling of behavior of stated turbines in general analytic form and the solution of some issues of their combined action with electromechanical converter connected to electric power network of power system.

2. MATERIALS AND METHODS

To deduce the analytical relations, describing the processes in above turbines, as a basis the approach presented in [1] is assumed.

It is known, that the active power of hydraulic turbines is described by the expression of [2] form:

$$P = g\cdot\eta\cdot Q\cdot H \quad (1)$$

where, P is power in [kW]; η is efficiency, expressed in relative units; Q is water discharge [m^3/s], H is head [m], and g=9.81 is acceleration of gravity [m/s^2].

It is clear that the rated power of hydraulic turbine will be:

$$P_{rat} = 9.81\cdot\eta_{rat}\cdot Q_{rat}\cdot H_{rat} \quad (2)$$

If to write down the current power in relative units, taking the rated values as the basis, we will obtain:

$$p^* = \eta^*\cdot q^*\cdot h^* \quad (3)$$

where, $p^* = \frac{P}{P_{rat}}$; $\eta^* = \frac{\eta}{\eta_{rat}}$; $q^* = \frac{Q}{Q_{rat}}$; $h^* = \frac{H}{H_{rat}}$.

The expression for efficiency of hydraulic turbines is presented in [1] in the form of:

$$\eta^* = a\cdot q^{*2} + b\cdot q^* + c \quad (4)$$

In (4) equation the a, b and c indexes depend on turbine's type. For example for the Francis hydraulic turbines they are equal to a=–0.537; b=1.047; c=0.49 with η_{rat}=0.9 for Pelton turbines they are equal to a=–0.224; b=0.483; c=0.741 also with; and finally for axial flow turbines a=–0.219; b=0.476; c=0.743 with η_{rat}=0.9 [1].

It is also known, that in general case the rated head is described by the expression of following form:

$$H_{rat} = H_0\cdot(1-\lambda) \quad (5)$$

where, H_0 is head corresponded to theoretical fall, i.e. without taking into account the friction loss, λ is index, taking into account the water friction when passing from top level to hydraulic turbine.

It is natural that a value of this λ index depends on a value of water flow q^*, this dependence is imaged in the expression for current value of head, which is presented in the form of:

$$H = H_0(1-\lambda\cdot q^{*2}) \quad (6)$$

Thus, with taking into account (5) and (6) the expression of current head in relative units has the form:

$$h^* = \frac{H}{H_{rat}} = \frac{1-\lambda \cdot q^{*2}}{1-\lambda} \quad (7)$$

Inserting the (4) and (7) expressions into (3) expression and making the simple conversions we finally obtain the expression for relative power of hydraulic turbines of small HPSs in the form of:

$$p^* = -A \cdot q^{*5} - B \cdot q^{*4} + C \cdot q^{*3} + D \cdot q^{*2} + E \cdot q^* \quad (8)$$

where, $A = \frac{\lambda \cdot a}{1-\lambda}$; $B = \frac{\lambda \cdot b}{1-\lambda}$; $C = \frac{a - c \cdot \lambda}{1-\lambda}$; $D = \frac{b}{1-\lambda}$; $E = \frac{c}{1-\lambda}$.

Thus, the expression (8) for power of hydraulic turbines is the universal analytic expression, linking a power value of three types of turbines Francis, Pelton and axial flow with water discharge q, in this process only change the values of a, b, c indexes and friction factor λ.

Table 1. Calculated relationship $p^*=f(q^*)$ for Francis turbine

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.33	0.47	0.59	0.7	0.816	0.915	1	1.11	1.12

Table 2. Calculated relationship $p^*=f(q^*)$ for Pelton turbine

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.39	0.502	0.61	0.718	0.819	0.916	1	1.14	1.22

Table 3. Calculated relationship $p^*=f(q^*)$ for Kaplan turbine water rate

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.39	0.5	0.61	0.716	0.818	0.914	1	1.1	1.2

If to assume the friction factor λ is equal to λ=0.1, then the calculated dependences $p^*=f(q^*)$ for Francis turbines will be presented in Table 1, for Pelton turbines in Table 2 and for axial flow turbines in Table 3.

Analyzing the results, given in 1÷3 Tables, the single-valued conclusions can be drawn, that the power factors of Pelton turbine and axial flow turbine in turndown of flow rate q^* from 0.4 to 1.4 are fully coincide in practice. For Francis hydraulic turbines in the range of water discharge from 0.4 to 0.7 the power value p^* as a function of flow rate q^* is a little bit less than for above turbines, which indicates that the efficiency of this turbine for small flow rates is lower than the one of Pelton's and axial flow turbines (the less flow rate the less output power value). The curves of $p^*=f(q^*)$ change for above hydraulic turbines of small HPSs are shown in Figure 1 (further "*" indexes are removed).

It needs to point to the circumstance, that all above mentioned correlations were obtained for constant (rated) rotational frequency of hydraulic turbines. Thus, if the hydraulic turbines were jointed with uncontrolled electric generators – synchronous or asynchronous, whose rotational frequency is constant in steady-state mode, then (8) equation becomes automatically the one of driving torque, developed by hydraulic turbine as:

$$m_{ht} = \frac{p^*}{n^*} = p_{ht} \quad (9)$$

where, n^*=1.

3. RESULTS AND DISCUSSION

The issues of operating modes study of hydraulic units with adjustable rotational frequency are presented in [3, 4].

Let's consider the equations of state of hydraulic unit, composed of Francis turbine and classical synchronous generator with electromagnetic excitation, operating to electric network.

The equations of synchronous generator with electromagnetic excitation are presented in [5]. Thus, with consideration for the equations of hydraulic turbine the general equations will be presented in the form of:

$$\left.\begin{aligned}
&p\psi_{ds} = U_{ds} - \omega_r \cdot \psi_{qs} - r_s \cdot i_{ds} \\
&p\psi_{qs} = U_{qs} + \omega_r \cdot \psi_{ds} - r_s \cdot i_{qs} \\
&p\psi_{dr} = -\frac{r_{dr}}{x_{dr}} \cdot \psi_{dr} + \frac{r_{dr} \cdot x_{ad}}{x_{dr}} \cdot i_{ds} + \frac{r_{dr} \cdot x_{ad}}{x_{dr}} \cdot i_{df} \\
&p\psi_{qr} = \frac{r_{qr}}{x_{qr}} \cdot \psi_{qr} + \frac{r_{qr} \cdot x_{aq}}{x_{qr}} \cdot i_{qs} \\
&p\psi_{df} = \frac{r_{df}}{x_{ad}} \cdot U_{df}^* - r_{df} \cdot i_{df} \\
&p\omega_r = \frac{1}{T_j} \cdot m_{ht} - \frac{1}{T_j} \cdot m_{em} \\
&i_{ds} = \frac{x_{dr}}{\Delta d} \cdot \psi_{ds} - \frac{\Delta d_1}{\Delta d} \cdot i_{df} - \frac{x_{ad}}{\Delta d} \cdot \psi_{dr} \\
&i_{qs} = \frac{x_{qr}}{\Delta q} \cdot \psi_{qs} - \frac{x_{aq}}{\Delta q} \cdot \psi_{qr} \\
&i_{df} = \frac{x_{dr}}{\Delta d_2} \cdot \psi_{df} - \frac{\Delta d_1}{\Delta d_2} \cdot i_{ds} - \frac{x_{ad}}{\Delta d_2} \cdot \psi_{dr} \\
&m_{em} = \psi_{ds} \cdot i_{qs} - \psi_{qs} \cdot i_{ds} \\
&\omega_r = p\theta - 1 \\
&m_{ht} = k_{tran} \cdot \left(-Aq^5 - Bq^4 + Cq^3 + Dq^2 + Eq\right)
\end{aligned}\right\} \quad (10)$$

where, $U_{ds} = -U_s \cdot \sin\theta$; $U_{qs} = U_{qs} \cdot \cos\theta$

$\Delta d = x_{ds} \cdot x_{dr} - x_{ad}^2$; $\Delta q = x_{qs} \cdot x_{qr} - x_{aq}^2$

$\Delta d_1 = x_{dr} \cdot x_{ad} - x_{ad}^2$; $\Delta d_2 = x_{dr} \cdot x_{df} - x_{ad}^2$

where, U_{df}^* is excitation voltage as a fraction of excitation voltage of no-load operation [6]; m_{ht} is driving torque of hydraulic turbine; k_{tran} is index of transfer of hydraulic turbine's basic units to the basic units of synchronous generator; and q is water discharge in relative units.

If to automatize the system, then it becomes necessary to link the water discharge q, which determines identically a value of turbine's driving torque m_{ht}, with the output power of generator p_{get}, i.e. besides the generator's equations it needs to take into account the equation of controller. For Francis hydraulic turbines and propeller (axial flow) ones a water discharge is controlled with the help of opening of wicket gates, which in one's turn are controlled by servomotors. A time constant of control tract is rather significant and reaches 1-2 seconds.

If to take for simplicity of analysis the inertia governor as a controller (link of first order), then its equation will be in the form of:

$$U_{out} = \frac{K}{T_p + 1} U_{in} \qquad (11)$$

As it was noted, as the output power of controller U_{out} the angle of opening of wicket gates is used, which determine a water discharge q flowing through turbine, and the input power is the active power at the output of generator, which is set either by dispatcher or by a value of load diagram p_{gset}.

Thus, the equations of controller and expressions for current value of active power on the terminals of generator will add to the equations of generator and hydraulic turbine (10):

$$\left.\begin{aligned} pq &= \frac{k_{12}}{T_p}(p_{gset}) - \frac{q}{T_p} \\ p_{gen} &= U_{ds} \cdot i_{ds} + U_{qs} \cdot i_{qs} \end{aligned}\right\} \qquad (12)$$

where, p is symbol of differentiation with respect to synchronous time τ=314·t, T_p is time constant of controller in [rad], k_{12} is amplification factor (of transfer) of controller, and p_{gset} is set value of power at generator's output.

In accordance with Figure 1, k_{12} can be determined by approximation of curve q=f(p), in this process at least two approximations are needed: up to q≤1, k_{12}=k_1 (for example for axial flow turbines k_1≈1), and for q≥1, k_{12}=k_2, (rarely carried out mode).

$$k_{12} = \begin{vmatrix} k_1 & npu & q \le 1 \\ k_2 & npu & q \ge 1 \end{vmatrix} \qquad (13)$$

Let's perform the approximate calculation of small hydraulic unit with Francis turbine, which parameters are equal to: rated head H_{rat} =50 m, rated water discharge for diameter of hydraulic turbine D=2.45 m is equal to Q =56 m^3/s, then the power of small HPS in nominal conditions will be equal to:

$$P_{rat.ht} = 9.81 \times 0.9 \times 56 \times 50 = 24.72\,\text{MW}$$

Rotational frequency of hydraulic turbine with reduced rotational frequency equal to $n^1_{rat} = 130\,\text{rpm}$ [2] is determined by the formula [3].

$$n_{ht} = \frac{n^1_{rat} \cdot \sqrt{H}}{D} = \frac{130\sqrt{50}}{2.45} \approx 375\,\text{rpm}$$

The generator is chosen with the power of P_{rat}=25 MW, total power of generator S, which is taken as a basis one, is equal to S_{rat}=31.25 MW with rated revolutions equal to n_{rat}=375 rpm. In this case an index of transfer is k_{trat}=0.8.

The parameters of steady-state mode of the system are given in Table 4.

Table 4. Parameters of steady-state mode of the system

q	rel.unit	0.4	0.6	0.8	1
p	rel.unit	0.33	0.59	0.816	1
m_{ht}	0.8·m_{ht}	0.264	0.47	0.65	0.8
p_{gset}	rel.unit	0.258	0.46	0.642	0.787
$q_{st\text{-}state}$	rel.unit	0.328	0.585	0.816	1

Two first rows of the table were determined in accordance with Figure 1 for Francis hydraulic turbine (i.e. data of Table 1). Third row displays a value of driving torque of hydraulic turbine m_{ht} with taking into account the index of transfer k_{trat}=0.8 according to (9) expression. The data of set active power at the output of generator p_{gset} (in relative units) were placed in the fourth row. Finally, in the fifth row the data of controller output were placed, i.e. the values $q_{st\text{-}state}$ in steady-state mode (Equation (12)).

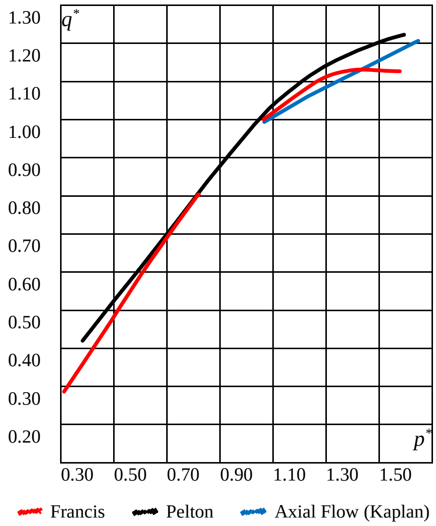
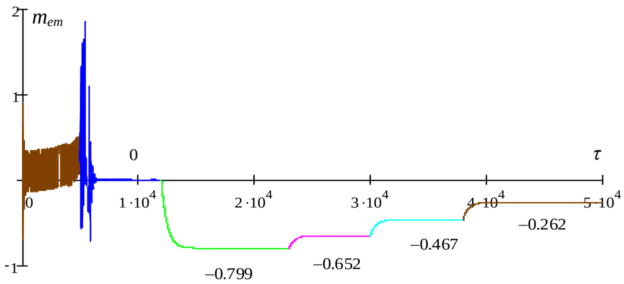


Figure 1. Relationship between turbine output power and water flow rate

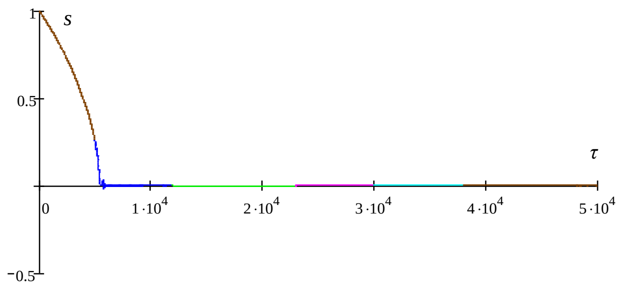
In this section, we design a tracking controller for the chosen parameters of controller are the following: time constant of controller T_{gen}=1.5 (471.22 rad), amplification factor (of transfer) k_{12}=k_1=1.27.

The fluktogrammas of mode parameters' change of small HPS's generator with automatic control system of water discharge (and therefore of the hydraulic turbine's power) are presented in Figure 2. The data of generator and controller and algorithm of modeling are given in Appendix. The fluktogramma of change of generator's electromagnetic torque m_{em}=f(τ) is given in Figure 2(a). According to the modeling algorithm at the first stage the asynchronous start is carried out without load and with short-circuited excitation winding within the period up to τ=5000 radian. A slip s changes in accordance with Figure 2(b). At 5000 radian the open-circuit voltage U^*_f =1 is supplied to the excitation winding, and the machine locks in synchronism, the rotational frequency of generator sets at the mark ω=1 (Figure 2(c)).

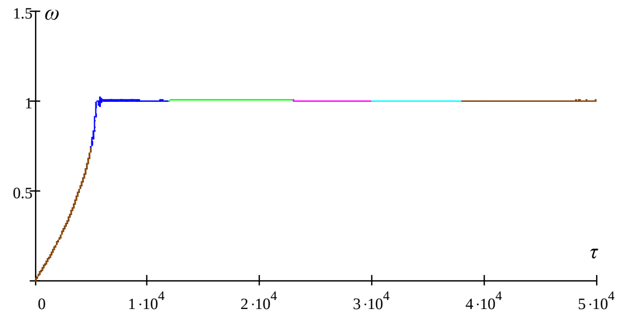
At 15000 radian the set value of active power p_{set1}=0.787 inputs to the equation of controller, a torque of hydraulic turbine in this process is formed according to expression m_{ht}=k_{tran}·q_1=–0.8q_1, herewith a value of water discharge q sets at a level of q_1=1 (Figure 2(d)) and the relevant to it value of driving torque of turbine is equal to m_{ht}=–0.8 (minus sign indicates generator mode) (Figure 2(a)). Further in accordance with set values of output power of generator p_{set2}=0.642, p_{set3}=0.46, p_{set4}=0.258, the q values, accordingly equal to q_2=0.815, q_3=0.584, q_4=0.327 (Figure 2(d)), are set at controller output.



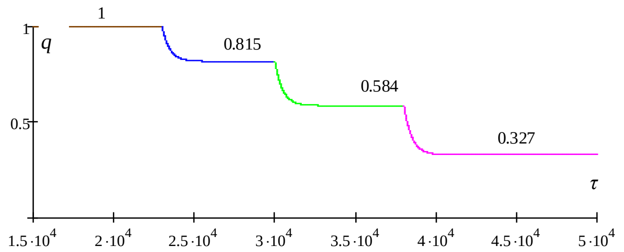
(a)



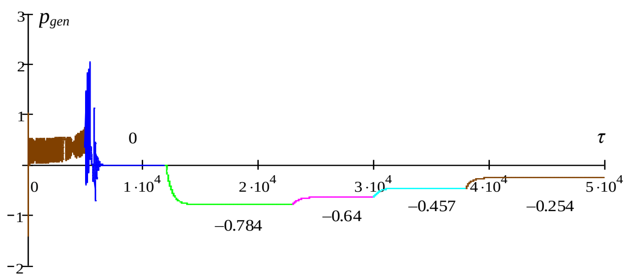
(b)



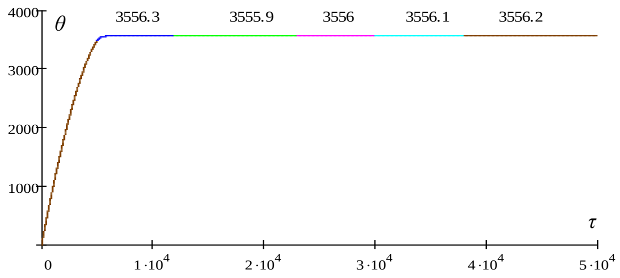
(c)



(d)



(e)



(f)

Figure 2. Fluctograms of operating parameters variations of small hydroelectric power generator

In accordance with these q values the torque values m_{ht} and m_{em} automatically form, the values of last ones are accordingly equal to m_{em1}=–0.799, m_{em2}=–0.652, m_{em3}=–0.467 and m_{em4}=–0.262. These values of electromagnetic torque are corresponded to the values of active powers at the generator's output p_{gen1}=–0.784, p_{gen2}=–0.64, p_{gen3}=–0.45 and p_{gen4}=–0.254 (Figure 2(e)). Comparing the values of current powers of generator $p_{gen1\div}p_{gen4}$ with the set values it can be stated, that the error amount because of approximation and offset of chosen type of controller doesn't exceed 3%.

The curve of change of synchronous machine's interior angle θ is given in Figure 2(f); for almost rated active load (p_{gen1}=–0.784) this angle is equal to θ_{rat}=0.4 radian (~23 deg), and for minimum active load (p_{gen4}=–0.254) is equal to θ_4=0.1 radian (~5.7 deg).

In conclusion it needs to note again, that the controller in these researches is chosen as the relaxation circuit of first order, which is a simplification, needs for more visual demonstration of system's principle of operation. The choice of more complicated controller will not influence on the idea and algorithm of construction of general modeling system.

4. CONCLUSIONS

The mathematical model is developed for study of operating modes of small HPSs' hydroelectric units, equipped with Francis, Pelton and axial flow turbines (Kaplan turbines with antideflection mounting of blades). For three above mentioned types of turbines the analytic expression was obtained in a form of multinomial, which connects the power values of turbines, expressed in relative units, with a value of flow rate of energy carrier – water; the structure of multinomial is invariable and only the values of its factors change according to turbines' types.

Collaborate research of hydraulic turbines with synchronous generators on full equations and also of the controller has demonstrated the working capacity, accuracy and effectiveness of developed mathematical model, which allows it is using for oriented calculations of operating modes of small HPSs both at the designing stage and in operating conditions.

APPENDICES

Appendix 1. Parameters of Synchronous generator

x_{ds}=0.986	x_{ad}=0.787
x_{qs}=0.63	x_{dr}=1
x_{aq}=0.435	r_s=0.02
x_{qr}=0.7	r_{qr}=0.019
x_{df}=1.1	r_{dr}=0.028
T_j=1000 rad (~1.5 s)	U_s=1

Calculated value Δd=0.367; Δd_1=0.167; Δd_2=0.48; Δq=0.255

Controller parameters: gain ratio k_{12}=k_1=1.27; time constant T_{gen}=471.22 rad

Appendix 2. Algorithm of the mathematical model

$Y_0=\Psi_{ds}$; $Y_1=\Psi_{qs}$; $Y_2=\Psi_{dr}$; $Y_3=\Psi_{qr}$; $Y_4=\Psi_{df}$; $Y_5=s$; $Y_6=\theta$; $Y_7=q$.

$\omega = p\theta - 1 = s - 1$

$i_{ds} = 3.23 \cdot Y_0 - 1.12 \cdot Y_4 - 1.65 \cdot Y_2$

$i_{qs} = 2.745 \cdot Y_1 - 1.7 \cdot Y_3$

$i_{df} = 2.47 \cdot Y_4 - 1.12 \cdot Y_0 - 1.07 \cdot Y_2$

p_{set} = active power preset value in generator output

$$D(t,Y) = \begin{bmatrix} -\sin(Y_6) + Y_1 - Y_1 \cdot Y_5 - 0.02 \cdot i_{sd} \\ \cos(Y_6) - Y_0 + Y_0 \cdot Y_5 - 0.02 \cdot i_{sq} \\ -0.028 \cdot Y_2 - 0.022(3.23 \cdot Y_0 - 1.12 \cdot Y_4 - 1.65 \cdot Y_2) + 0.002 \cdot i_{df} \\ -0.027 \cdot Y_3 - 0.0117 \cdot i_{sq} \\ 0.057 \cdot U^*_{df} - 0.045 \cdot i_{df} \\ 0.001 \cdot (-k_{tran} \cdot Y_7) - 0.001\left[Y_0 \cdot (i_{sq}) - Y_1 \cdot (\cdot i_{sd})\right] \\ Y_5 \\ 0.0027 \cdot (p_{set}) - 0.002126 \cdot Y_7 \end{bmatrix}, \quad D(t,Y) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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