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FREE VIBRATIONS OF A CONICAL SHELL WITH SPRING ASSOCIATED MASS AND STIFFENED WITH A CROSS SYSTEM OF RIBS IN MEDIUM

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Abstract- We consider a circular, closed, truncated, medium-contacting, constant thickness, medium-contacting conical shell stiffened with longitudinal and lateral ribs supported with associated mass connected with the shell by means of two identical springs in points which are diametrically opposite. The problem of definition of natural frequencies of vibrations of such a shell is solved in leaner statement by the energy method. Discrete allocation of stiffening longitudinal and lateral ribs was taken into account.

Keywords: Conical Shell, Vibrations, Medium, Energy Method, Winkler Model.

1. INTRODUCTION

Conical shells are widely used in aeronautical engineering and machine building. One of the first works on studying stability of conical shells was the paper of Kh.M. Mushtari [1], and free vibrations of a conical cone with spring associated mass [11]. The paper [2] considers free vibrations of a longitudinally stiffened, the paper [3, 12] a laterally stiffened conical shell with spring associated mass in medium. Mathematical model of deformation of stiffened orthotropic shells of general type was represented in [4]. The paper [5] was devoted to constructing mathematical model of deformation of conical shell constructions based on total energy functional, deformation with regard to orthotropy of the material, geometrical nonlinearity and lateral shifts.

For the considered shell, a solution of the oscillation problem is studied taking into account the energy expressions in the Lagrange equations of type II.

2. PROBLEM STATEMENT

The coordinate system (r, φ) consisting of a variable radius r and a dihedral angle φ is chosen.

Potential energy of the shell is calculated by means of the following expression [10]:

$$\Pi_1=\frac{Eh}{12\left(1-\nu^2\right)}\int_{r_2}^{r_1}\int_0^{2\pi}\left(\varepsilon_1^2+\varepsilon_2^2+2\nu\varepsilon_1\varepsilon_2+\frac{1-\nu}{2}\psi^2\right)\frac{rdrd\varphi}{sin\gamma}+\frac{D}{2}\int_{r_2}^{r_1}\int_0^{2\pi}\left(\chi_1^2+\chi_2^2+2\nu\chi_1\chi_2+2(1-\nu)\tau^2\right)\frac{rdrd\varphi}{sin\gamma} \quad (1)$$

when in accordance to [9] it is accepted

$$\varepsilon_1=\frac{\partial u}{\partial r}sin\gamma$$

$$\varepsilon_2=\frac{u}{r}sin\gamma+\frac{\partial \upsilon}{r\partial\varphi}+\frac{w}{r}cos\gamma$$

$$\psi=\frac{\partial u}{r\partial\varphi}+\frac{\partial\upsilon}{\partial r}sin\gamma-\frac{\upsilon}{r}sin\gamma$$

$$\chi_1=-\frac{\partial^2 w}{\partial r^2}\sin^2\gamma$$

$$\chi_2=-\frac{u}{r^2}sin\gamma cos\gamma-\frac{w}{r^2}\cos^2\gamma-\frac{\partial^2 w}{r^2\partial\varphi^2}-\frac{\partial w}{r\partial r}\sin^2\gamma$$

$$\tau=-\frac{cos\gamma}{r}\left(\frac{\partial u}{r\partial\varphi}-\frac{\partial\upsilon}{\partial r}sin\gamma+\frac{\upsilon}{r}sin\gamma\right)-\frac{2sin\gamma}{r}\left(\frac{\partial^2 w}{\partial r\partial\varphi}-\frac{\partial w}{r\partial\varphi}\right)$$

The notation used in the last equations represents the following mechanical, physical, and geometric quantities, respectively,

- E and ν are mechanical constants (Young's modulus and Poisson's ratio) of the conical shell;
- u, v and w are displacements of the median surface of conical shell in three directions;
- h, r_1, r_2 and γ are the geometric dimensions of conical shell and the angle of inclination from vertical direction.

$$D=\frac{Eh^3}{12\left(1-\nu^2\right)}$$

Potential energy Π_2 of deformation of longitudinal and lateral ribs equal [6]:

$$\Pi_2 = \frac{1}{2}\sum_{i=1}^{k_1}\int_{r_2}^{r_1}\left[\tilde{E}_i F_1\left(\frac{\partial u}{\partial r}\sin\gamma\right)^2 + \tilde{E}_i I_1\left(\frac{\partial^2 w}{\partial r^2}\sin^2\gamma\right)^2 + \right.$$
$$\left. +\tilde{E}_i \tilde{I}_1\left(\frac{\partial^2 \vartheta}{r^2\partial\varphi^2}\right)^2 + +\tilde{G}_i I_{1kp}\left(\frac{\partial^2 w}{r\partial r\partial\varphi}\sin\gamma\right)^2\right]_{\varphi=\varphi_i}\frac{dr}{\sin\gamma} + \quad (2)$$
$$+G_j I_{2kp}\left(\frac{\partial^2 w}{r\partial r\partial\varphi}sin\gamma\right)^2\Bigg|_{r=r_j} +$$
$$+\frac{1}{2}\sum_{j=1}^{k_2}\int_0^{2\pi}\left[E_j F_2\left(\frac{\partial\vartheta}{r\partial\varphi}+\frac{w}{r}\right)+E_j I_2\left(\frac{\partial^2 w}{r^2\partial\varphi^2}+\frac{w}{r^2}\right)^2\right.$$

where, F_1, I_1, $\tilde{I}_1$, I_{1kpl} denote the area and inertia moment of the cross-section of the longitudinal bar with respect to tangential and radial axis, respectively, and also inertia moment at torsion; F_2, I_2, I_{2kp} are the cross-sectional area of transverse rod and the moments of inertia. $\tilde{E}_1, E_j$ denote the elasticity module of the *i*th longitudinal and the *j*th lateral bars; $\tilde{G}_i, G_j$ are shear modulus of the materials of the *i*th longitudinal and the *j*th lateral bars, respectively; k_1, k_2 are the amounts of longitudinal and lateral bars, respectively; φ_i, r_j are the allocation coordinates of longitudinal and lateral bars, respectively.

Kinetic energy T_1 of the shell, longitudinal and lateral ribs is calculated by means of the expression

$$T_1 = \frac{\gamma_1 h}{2g}\int_{r_2}^{r_1}\int_0^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]\frac{rdrd\varphi}{\sin\gamma}+$$
$$+\frac{\gamma_1 F_1}{2g}\sum_{i=1}^{k_1}\int_0^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]_{\varphi=\varphi_i}\frac{dr}{\sin\gamma}+ \quad (3)$$
$$+\frac{\gamma_1 F_2}{2g}\sum_{j=1}^{k_2}\int_0^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]_{r=r_j} rd\varphi$$

where, γ_1 and g=9.81 m/s^2 are the physical constants of the shell material.

Influence of medium on the shell is determined as a work performed by these loads when taking the system from deformed state to initial undeformed state and can be represented in the form:

$$A = \int_{r_2}^{r_1}\int_0^{2\pi} q_r rdrd\varphi \quad (4)$$

Assuming that the influence of medium on a shell is subjected to the Winkler model, i.e. $q_r = kw,$ we can represent (4) in the form:

$$A = \int_{r_2}^{r_1}\int_0^{2\pi} kw^2 rdrd\varphi \quad (5)$$

where k is a constant.

Since the considered structure is a ballast mechanism, it is assumed that the mass moves in the direction of the *z* axis, remaining on the plane of the cross section of the shell, and the deformation in the direction of *u* component of the displacement vector is not taken into account (Figure 1).

The corresponding energies of the spring Π_3 and the mass T_2 are as follows:

$$\Pi_3 = \left(z - w_0 cos\gamma\right)^2 c$$
$$T_2 = \frac{1}{2}M\dot{z}^2 \quad (6)$$

where, w_0 is the displacement of the springs attachment points located in diametrical coordinate plane $\varphi = 0.$

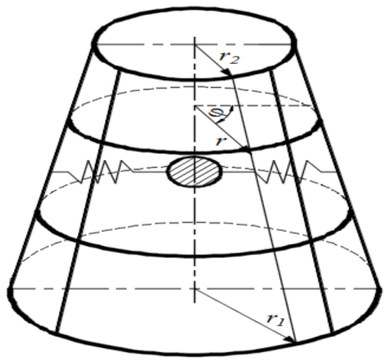

Figure 1. A conical shell with medium-contacting mass and stiffened with cross system of ribs

3. PROBLEM SOLUTION

We represent displacements of the shell in the form [7]:

$$w = \frac{r^2}{r_1^2}\sin\frac{m\pi(r-r_2)}{r_1-r_2}\sum_{n=1}^{\infty}A_n(t)cosn\varphi$$
$$\vartheta = \frac{r^2}{r_1^2}\sin\frac{m\pi(r-r_2)}{r_1-r_2}\sum_{n=1}^{\infty}B_n(t)\sin\sin n\varphi \quad (7)$$
$$u = \frac{r^2}{r_1^2}\cos\frac{m\pi(r-r_2)}{r_1-r_2}\sum_{n=1}^{\infty}D_n(t)cosn\varphi$$

where, *n* and *m* are the number of half-waves generated in the orthogonal direction in the shell body. Expressions for the displacement of the shell points must satisfy the elastic binding conditions at the points of the external structure, i.e. w=0, υ=0. Owing to the multiplier r^2, expressions (7) allow the energy integrals to be squared, and it simplifies the solution.

In addition, expressions (6) indicate that the half-waves along the generatrix of conical shell will move towards the larger base of the shell. It should also be noted that depending on whether the number of n-half-waves is odd or even, either the spring or the mass oscillates. Finally, generalizing expressions (1)-(6) and taking into account all the expressions for energy and work in the Lagrange equations of type II, we will obtain an infinite number of ordinary differential equations.

$$M\ddot{z}+2c(z-\alpha_0\sum_{n=1,3,\ldots}^{\infty}A_n)=0$$
$$2\ddot{A}_1 a+2A_1\left(l_1+\alpha_0^2 c+\beta_0\right)+B_1 f_1+D_1 q_1-2z\alpha_0 c+$$
$$+2c\alpha_0^2\left(A_3+A_5+A_7+\ldots\right)=0$$
$$\ddot{B}_1 a+2B_1 d_1+A_1 f_1+D_1 s_1=0 \qquad (8)$$
$$2\ddot{D}_1 b+2D_1 e_1+A_1 q_1+B_1 s_1=0$$
$$2\ddot{A}_3 a+2A_3\left(l_3+\alpha_0^2 c+\beta_0\right)+B_3 f_3+D_3 q_3-2z\alpha_0 c+$$
$$+2c\alpha_0^2\left(A_1+A_3+A_7+\ldots\right)=0$$

where,

$$l_n=\Omega_1=\left\langle \begin{array}{l} R_1\cos^2\gamma+\frac{h^3}{3}\left\{\sin^4\gamma\left(\frac{117}{8}R_2+\frac{31}{32}\alpha^2R_4+\frac{1}{192}\alpha^4R_6\right)+\right.\\ +\frac{1}{4}R_2\left[\left(1-n^2\right)^2-\sin^4\gamma\right]+\frac{1}{8}\left(\frac{1}{4}\alpha^2R_4+3R_2\right)\sin^4\gamma+\\ +v\left[\frac{1}{2}\alpha^2R_4\sin^4\gamma-R_2\sin^4\gamma-R_2\left(1-n^2+\sin^2\gamma\right)\sin^2\gamma-\right.\\ \left.\left.-2\left(1-n^2+\sin^2\gamma\right)\frac{1}{32}\alpha R_4-\frac{3}{2\alpha}R_2\right]-(1-v)\left(7R_2-\frac{1}{4}R_4\alpha\right)n^2\sin^2\gamma\right\} \end{array}\right\rangle+$$
$$+\frac{k_1\sin\gamma}{4r_1^4}\left[EI_1\left(10R_1+4\alpha\ R_3+0{,}01\alpha^4R_5-\right.\right.$$
$$\left.-\frac{1}{6}\alpha^2R_3+\frac{R_1}{\alpha}+\frac{3}{8}\alpha^3\Omega\right)\sin^2\gamma+$$
$$+\frac{\pi}{2r_1^4}\sum_{j=1}^{k_2}\left[EF_2r_j^4+EI_2(n^2-1)+GI_{kp}n^2\left(2\sin\theta_j-\frac{1}{2}\alpha r_j\cos\theta_j\right)^2\right]\times$$
$$\left.\times r_j\sin\theta_j+GI_{kp}\left(\frac{9}{4}R_2+\frac{\alpha^2}{64}R_3+R_1\right)\right];$$
$$e_n=\frac{1}{2}\Omega_1^2\sin^2\gamma\left\{\frac{1}{\alpha^2}R_2\left[9-12v+\frac{3(1-v)n^2}{4}\right]+\right.$$
$$\left.+R_4\left[v+\frac{7}{8}+\frac{1}{16}(1-v)n^2\right]+\frac{1}{48}\alpha^2R_6\right\}+$$
$$+\frac{h^3}{96}\Omega_1\left[\frac{1}{4}\sin^2 2\gamma+2(1-v)n^2\cos^2\gamma\right]R_2+$$
$$+\frac{Ek_1}{4r_1^2\sin\gamma}\left[F_1\left(\frac{1}{2}\alpha\Omega+\frac{1}{4}R_3+\frac{4R_1}{\alpha^2}+\frac{\alpha^2}{40}R_5\right)\sin\gamma+\frac{1}{2}I_1R_1n^4\right];$$
$$d_m=\Omega_1\left\{\frac{1}{2}n^2\left(\frac{1}{8}R_4-\frac{3}{2\alpha^2}R_2\right)+\frac{1-v}{2}\left(\frac{1}{4}R_4-\frac{3}{\alpha^2}R_2+\frac{\alpha^2}{48}R_6\right)\right]+$$
$$\left.+\frac{h^2(1-v)\sin 2\gamma}{12}\frac{1}{128}R_4-\frac{11}{32}R_2\right\}+\frac{EF_2n^2}{2r_1^4}\sum_{j=1}^{k_2}\sin^2\theta_j;$$
$$f_n=-\Omega\left[\left(\frac{1}{8}R_4-\frac{3}{2\alpha^2}R_2\right)n\cos\gamma+\frac{1}{12}h^2n\sin 2\gamma(1-v)\times\right.$$
$$\left.\times\left(\frac{9}{4}R_2+\frac{1}{16}\alpha^2R_4\right)\right]+\frac{\pi nEF_2}{r_1^4}\sum_{j=1}^{k_2}r_j^3\sin^3\theta_j;$$

$$q_n=\Omega_1\left\langle \begin{array}{l} \frac{1}{4}\sin 2\gamma\left(\Omega+\frac{v\alpha R_5}{10}\right)-\frac{h^2}{24}\left\{\left(1-n^2+\sin\gamma\right)\frac{1}{\alpha}R_2+\right.\\ +\left[\frac{\alpha}{4}R_3\left(v+\frac{1}{4}\right)+\frac{1}{\alpha}R_4(1+5v)+\Omega v\alpha^2\right]\times\\ \left.\times\frac{1}{2}\sin^2\gamma+\frac{\alpha}{8}R_3(1-v)n^2\right\}\sin 2\gamma \end{array}\right\rangle;$$
$$a=\beta_1-\beta_2;b=\beta_1+\beta_2;$$
$$\beta_1=\frac{\gamma_1}{2gr_1^4}\left(\frac{\pi hR_6}{12\sin\gamma}+\frac{F_1k_1R_5}{20\sin\gamma}+\pi F_2\sum_{j=1}^{k_2}r_j^5\sin^2\theta_j\right);$$
$$\beta_2=\frac{\gamma_1}{2gr_1^4}\left(\frac{\pi h}{\sin\gamma}\left(\frac{5}{2\alpha^2}R_4+\frac{30}{\alpha^4}R_2\right)+\frac{F_1k_1\Omega}{\alpha\sin\gamma}\right);$$
$$\Omega=\frac{r_1^3}{\alpha}-\frac{6r_1}{\alpha^3}-\left(\frac{r_2^3}{\alpha}-\frac{6r_2}{\alpha^3}\right);\Omega_1=\frac{Eh\pi}{\left(1-v^2\right)r_1^4\sin\gamma};\alpha=\frac{2\pi m}{r_1-r_2};$$
$$R_p=r_1^p-r_2^p\ (p=1,\ldots 6);$$
$$\beta_0=\frac{k\pi}{r_1^2}\left\{\frac{1}{12}\left(r_2^6-r_1^6\right)+\frac{5(r_2-r_1)\left(r_1^2-r_2^2\right)}{4m\pi}r_1^2+r_2^2-12\right\}$$

Looking for a solution of the resulting system of infinite equations as the product of the function sinωt (where ω is the oscillation frequency) and the unknowns z^*, A_n^*, B_n^*, D_n^*, we obtain homogeneous algebraic equations for these unknowns and set the determinant of this system to be zero.

$$\begin{vmatrix} \lambda_1+R-kof & R & R\ldots \\ R & \lambda_3+R-kof & R\ldots \\ R & R & \lambda_5+R-kof\ldots \end{vmatrix}=0 \qquad (9)$$

where, $kof=\omega^2+\beta_0/2\alpha$,

$$\lambda_n = \frac{l_n}{a} + \frac{f_n}{2a\Delta}\left[\frac{q_n s_n}{4ab} - \frac{f_n}{2a}\left(\frac{e_n}{b} - \omega^2\right)\right] +$$

$$+\frac{q_n}{2a\Delta}\left[\frac{f_n s_n}{4ab} - \frac{q_n}{2ab}\left(\frac{d_n}{a} - \omega^2\right)\right]$$

$$\Delta = \left(\frac{d_n}{a} - \omega^2\right)\left(\frac{e_n}{b} - \omega^2\right) - \frac{s_n^2}{4ab}$$

$$R = -\frac{\alpha_0^2\omega^2}{a}\left(\frac{c}{\frac{c}{M} - \omega^2}\right).$$

The effect of mass on the shell is calculated by the formula denoted by R, when the spring stiffness is c = 0 and the effect of mass is R = 0 the following frequency equation is obtained.

$$\lambda_n + R - \omega^2 + \frac{\beta_0}{2a} = 0$$

Also, if the mass is M=0, when c=∞ there is a rigid connection $R = -\frac{\alpha_0^2 M\omega^2}{a}$, and if M=∞ and $R = -\frac{\alpha_0^2 c}{a}$, the shell has internal elastic bonds.

4. NUMERICAL RESULTS

Natural frequencies of vibrations of the system were determined by means of numerical solution of equation (9). For the problem parameters were accepted: $r_1 = 160$ mm, $r_2 = 85$ mm a longitudinal bar of corner profile of sizes $5\times5\times1$ (in mm), $k_1 = 32$, $k_2 = 40$, $m = 1$. The associated mass whose quantity vary in the research process, was stiffened in the middle of the shell in diametrically opposite points.

The curve reflecting dependence of natural frequency $f^* = \omega/2\pi$ of vibrations of a shell without associated mass medium on the number of waves n in circumferential direction was given in Figure 2. It is seen that with increasing the amount of waves n in circumferential direction, frequency of vibrations of the shell without associated mass medium at first decreases and attaining minimum begin to increase.

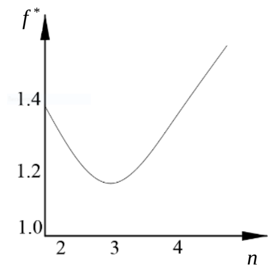

Figure 2. Dependence of natural frequency of shell's vibrations on the number of waves in peripheral direction

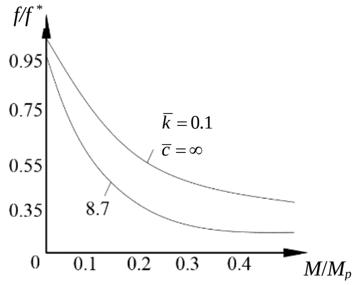

Figure 3. Dependence of natural frequency of shell's vibrations on the quantity of associated mass

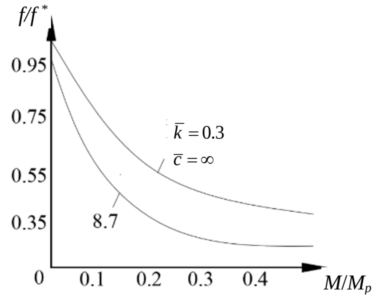

Figure 4. Dependence of natural frequency of shell's oscillations on the quantity of associated mass

The curves reflecting the dependence of minimum natural frequency of vibrations of the system on the quantity of associated mass, on relative rigidity of springs $\bar{c} = c/D$ with respect to the bed coefficient $\bar{k} = k/D = 0.1$ (in Figure 4 similar dependence for $\bar{k} = 0.3$) were given in Figure 3. Analysis of curves shows that influence of associated mass on minimal natural frequency of vibrations of the shell is very essential. With decreasing rigidities of bonds between the mass and shell, this influence increases. Comparison of curves in Figures 3 and 4 show that with increasing minimum natural frequency with respect to the bed coefficient, vibrations of the system increase. This is connected with the fact the Winkler model of inertial actions of the system on vibrations process of the system is not taken into account.

REFERENCES

[1] Kh.M. Mushtari, "On Stability of Thin-Walled Conical Shells of Circular Section in Torsion of a Pyramid", In the book: Collections of Scientific Works of KAI, Kazan, Russia, pp. 39-40, 1935.
[2] R.A. Iskanderov, H. Shafiei Matanagh, "Free Vibrations of Longitudinally Reinforced Conical Shell with Spring Associated Mass in Medium", Journal of Problems of Computational Mechanics and Strength of Structures, Oles Honchar Dnepropetrovsk National University, Dnipro, Ukraine, Vol. 26, pp. 175-185, 2017.
[3] R.A. Iskanderov, H. Shafiei Matanagh, "Free Vibrations of Lateral Reinforced Conical Shell with Spring Associated Mass in Medium", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 32, Vol. 9, No. 3, pp. 48-52, September 2017.

[4] V.V. Karpov, A.A. Semenov, "Mathematical Model of Deformation of Orthotropic Reinforced Shells of Revolution", Magazine of Civil Engineering, Saint Petersburg, Russia, No. 5, pp. 100-106, 2013.
[5] A.A. Semenov, A.A. Ovcharov, "Mathematical Model of Straining of Orthotropic Conical Shells", Engineering Journal of Don, No. 2, pp. 45-50, 2014.
[6] I.Ya. Amiro, V.A Zarutsky, P.S. Polyakov, "Ribbed Cylindrical Shells", Scientific Thought, Kiev, Ukraine, p. 245, 1973.
[7] L.M. Bunich O.M. Paliy, I.A. Piskovitina, "Stability of a Truncated Conical Shell under the Action of Uniform External Pressure", Engineering Collection, Issue 23, pp. 89-93, 1956.
[8] N.N. Buchholtz, "The Basic Course of Theoretical Mechanics", Science, Moscow, Russia, p. 467, 1972.
[9] V.Z. Vlasov, "Selected Works in 2 Volumes", AS of the USSR Publ. House, Vol. 1, Moscow, Russia, p. 528, 1962.
[10] S.P. Timoshenko, "Stability of Elastic Systems", Moscow, Russia, p. 567, 1955.
[11] V.G. Palamarchuk, A.M. Nosachenko, "Free Vibrations of a Ridge Conical Shell with String Associated Mass", Applied Mechanics, Vol. XVI, No. 4, pp. 40-46, 1978.
[12] H. Shafiei Matanagh, "Free Vibrations of Longitudinally Reinforced Conical Shell with a Mass on Springs Contacting with Inhomogeneous Medium", Problems of Computational Mechanics and Strength of Structures", Oles Honchar, Dnepropetrovsk National University, Dnipro, Ukraine, Vol. 29, pp. 221-234, 2019.

BIOGRAPHIES

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