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FORCED VIBRATIONS OF VISCOUS-ELASTIC HETEROGENEOUS MEDIUM-CONTACTING VERTICAL RETAINING WALL CONSISTING OF THREE ORTHOTROPIC CYLINDRICAL PANELS

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Abstract- The supports formed by the combination of cylindrical panels are used in bridge construction. To save the material, the interior area of the support is filled with soil. Such supports are exposed to different nature forces. One of such forces is a force generated on the surface of cylindrical panels that form supports during flood flow. Under the action of these forces the support is exposed to forced vibration. Therefore, to study the supports formed from combination of cylindrical panels with regard to viscosity and heterogeneity of soil, orthotropic character of panels is of great practical importance. In the paper, based on the Hamilton-Ostrogradsky variational principle, we study forced vibrations of a vertical retaining wall consisting of three orthotropic cylindric panels contacting with viscous-elastic, heterogeneous soil, obtain analytic expressions to calculate the displacements of the points of cylindrical panels and structure characteristically curves. Account of heterogeneity of soil is performed by accepting its rigidity coefficients as a function of coordinate. It is assumed that the Poisson ratio is constant.

Keywords: Cylindrical Shell, Orthotropic, Viscous-Elastic Medium, Heterogeneous, Forced Vibration, Displacement.

1. INTRODUCTION

Ref [1] was devoted to the study of one dynamical strength characteristics, the frequency of natural vibrations of a vertical support consisting of three orthotropic soil-filled cylindrical panels reinforced with discretely distributed longitudinal rods. Using the Hamilton-Ostrogradsky principle for finding the frequencies of vibrations of a vertical support, a frequency equation is structured, its roots are found, influence of physical and geometrical parameters characterizing the system, are studied. Account of joint work on the contact line of three cylindrical panels is accepted as contact conditions. Ref. [2] was devoted to one of dynamical strength characteristics, the frequency of natural vibrations of the retaining wall consisting of two soil-contacting, orthotropic cylindrical shells reinforced with discretely distributed annular rods.

A problem of natural vibrations of a retaining wall consisting of two orthotropic, viscous-elastic soil-filled cylindrical shells reinforced with discretely distributed longitudinal rods was solved in [3]. The problems of connection of concave shells with contour constructions were solved in Kh.R. Seyfullayev's papers [4, 5]. Retaining walls consisting of three different isotropic materials in a plane strain state were analyzed in [6]. The problem was reduced to the solution of ordinary differential equations and analytic solution was obtained.

Ref. [7] was devoted to development of a technique for calculating cylindrical shells made of isotropic material with regard to compression and sliding in a contact surface. Calculations and studies were carried out based on moment theory of cylindrical shells. Analysis of the executed works shows that during the construction of retaining walls, stiffened cylindrical shells were not used and the soil reaction was not taken into account.

Paper [8] was devoted to vibrations of an orthotropic, cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linearly viscous-elastic foundation. Using the Hamilton-Ostrogradsky variational principle for finding vibrational frequencies of a cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linear elastic foundation, the frequency equation was constructed, its roots were found and the influences of physical and geometrical parameters characterizing the system, were studied.

2. PROBLEM STATEMENT

Assume that on the surface of each of three orthotropic panels contacting with viscous-elastic medium and composing vertical retaining wall is exposed to the following external forces in direction of normal:

$$P_i = P_{0i}\sin\chi\xi_i\cos n\theta_i\sin\omega_1 t_1 \qquad (1)$$

where, $\xi_i = \dfrac{x_i}{a}$, $t_1 = \omega_{01}t$, χ, n are wave numbers of the cylindrical panel in the direction of generatrix and circular direction $0 \le \theta_1 \le \tilde{\theta}_1$, $0 \le \theta_2 \le \tilde{\theta}_2$, $0 \le \theta_3 \le \tilde{\theta}$, $\omega_{01} = \sqrt{\dfrac{E_{11}}{\left(1-\nu_{11}^2\right)\rho_1 R_1^2}}$.

For studying forced vibrations of a viscous-elastic medium-contacting retaining wall consisting of three orthotropic panels we will use the Hamilton-Ostrogradsky variation principle. According to this principle, the total energy of the structure under consideration gets a stationary value for real stress-strain state. Since the structure studied consists of cylindrical shells, viscous-elastic soil, we will write the expressions of potential and kinetic energies of each element.

The potential energies of cylindrical shells:

$$G_i = \frac{h_i R_i}{2}\iint_{s_i}\left\{ b_{11i}\left(\frac{\partial u_i}{\partial x_i}\right)^2 - 2\left(b_{11i}+b_{12i}\right)\frac{w_i}{R_i}\frac{\partial u_i}{\partial x_i} + \right.$$
$$+\frac{w_i^2}{R_i^2}\left(b_{11i}+2b_{12i}+b_{22i}\right)+\frac{b_{22i}}{R_i^2}\left(\frac{\partial \vartheta_i}{\partial \theta_i}\right)^2 - \quad (2)$$
$$-2\left(b_{12i}+b_{22i}\right)\frac{w_i}{R_i^2}\frac{\partial \vartheta_i}{\partial \theta_i}+2b_{12i}\frac{1}{R_i^2}\frac{\partial u_i}{\partial x_i}\frac{\partial \vartheta_i}{\partial \theta_i}+$$
$$\left.+b_{66i}\frac{1}{R_i^2}\left(\frac{\partial u_i}{\partial \theta_i}\right)+b_{66i}\left(\frac{\partial \vartheta_i}{\partial x_i}\right)^2+b_{66i}\frac{1}{R}\frac{\partial \vartheta_i}{\partial x_i}\frac{\partial u_i}{\partial \theta_i}\right\}dx_i d\theta_i$$

The kinetic energies of cylindrical shells:

$$K_i = \frac{\rho_i h_i}{2R_i(1-\nu_i^2)}\iint_{s_i}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2\right]dx_i d\theta_i$$

The influence of soil on cylindrical shells is replaced by the external forces q_{xi}, q_{yi}, q_{zi}. The work done in displacements of the points of these forces is determined by means of the following expression:

$$A_i = \int_0^a \int_0^{3\pi/4}\left(q_{xi}u_i + q_{yi}\vartheta_i + q_{zi}w_i\right)dx d\theta_i \quad (3)$$

where, $i=1$ corresponds to the first cylindrical panel forming the support, $i=2$, to the second cylindrical panel, $i=3$ to the third cylindrical panel; u_i, ϑ_i, w_i are displacements of the points of cylindrical panels, R_i, h_i are curvature radii and thickness of cylindrical panels, $b_{11i}, b_{22i}, b_{12i}, b_{66i}$ are elasticity module of orthotropic cylindrical panels, E_{1i}, E_{2i} are elasticity module of orthotropic cylindrical panels in the direction of the coordinate axes x_i and θ_i, ν_{1i}, ν_{2i} are Poisson ratios, q_{xi}, q_{yi}, q_{zi} are the components of forces acting as viewed from soil on cylindrical panels, s_i are surfaces of cylindrical panels.

The elasticity module of orthotropic cylindrical panels are expressed by the constants $b_{11i}, b_{22i}, b_{12i}, b_{66i}$, $E_{1i}, E_{2i}, \nu_{1i}, \nu_{2i}$ in the following way:

$$b_{11i} = \frac{E_{1i}}{1-\nu_{1i}\nu_{2i}}, b_{22i} = \frac{E_{2i}}{1-\nu_{1i}\nu_{2i}}$$
$$b_{12i} = \frac{\nu_{2i}E_{1i}}{1-\nu_{1i}\nu_{2i}}, b_{66i} = \frac{\nu_{1i}E_{2i}}{1-\nu_{1i}\nu_{2i}}$$

The components q_{xi}, q_{yi}, q_{zi} of forces acting on cylindrical panels as viewed from soil are taken as follows:

$$q_{xi} = q_{yi} = 0$$
$$q_{z1} = p_1(x)w_1 + \frac{k_{s1}(x)}{R_1^2}\left(\frac{\partial^2 w_1}{\partial \xi_i^2}+\frac{\partial^2 w_1}{\partial \theta_1^2}\right) -$$
$$-\int_0^t \Gamma(t-\tau)w_1(\tau)d\tau \quad (4)$$
$$q_{z2} = p_2(x)w_2 + \frac{k_{s2}(x)}{R_2^2}\left(\frac{\partial^2 w_2}{\partial \xi_i^2}+\frac{\partial^2 w_2}{\partial \theta_2^2}\right) -$$
$$-\int_0^t \Gamma(t-\tau)w_2(\tau)d\tau$$
$$q_{z3} = p_3(x)w_3 + \frac{k_{s3}(x)}{R_2^3}\left(\frac{\partial^2 w_3}{\partial \xi_i^2}+\frac{\partial^2 w_3}{\partial \theta_3^2}\right) -$$
$$-\int_0^t \Gamma(t-\tau)w_3(\tau)d\tau$$

where, $p_i(x), k_{si}(x)$ are rigidity module of soils at compression and sliding and we will consider the cases when these quantities change by the linear law:

$$p_i(x) = p_{i0}\left(1+\alpha_i\xi_i\right),\ k_{si}(x) = k_{si0}\left(1+\gamma_i\xi_i\right) \quad (5)$$

where, $\alpha_i, \beta_i, \gamma_i \in [-1;1]$ are heterogeneity parameters p_{i0}, k_{sio} are rigidity module of homogeneous soil in compression and sliding, $\Gamma(t) = Ae^{-\psi t}$ is a relaxation core, A, ψ are empiric constants.

The work done by forces acting in normal direction of each three orthotropic cylindric panels forming an elastic-plastic medium-contacting vertical retaining wall is determined by means of the following expression:

$$B_i = -R\int_0^1\int_0^{\tilde{\theta}_i} P_i w_i d\xi_i d\theta_i \quad (6)$$

As a result, the total energy of the system is as follows:

$$\Pi_i = \sum_{i=1}^{3}\left(G_i + K_i + A_i + B_i\right) \quad (7)$$

To expression (7) we add contact and boundary conditions. We assume that hard contact conditions between the shell and bars are satisfied:

$$\begin{aligned}
&u_{ji}(y) = u_i(x_j, y) + h_j\varphi_1(x_j, y)\\
&\vartheta_{ji}(x) = \vartheta_i(x_j, y) + h_j\varphi_2(x_j, y)\\
&w_{ji}(x) = w_i(x_j, y)\\
&\varphi_{ji} = \varphi_2(x_j, y)\\
&\varphi_{kpji}(x) = \varphi_1(x_j, y)\\
&h_{ji} = 0.5h_i + H_{ji}^1 \qquad (8)\\
&u_{ki} = u_i(x, y_{ki}) + h_{1i}\varphi_1(x, y_{ki})\\
&\vartheta_{ki} = \vartheta_i(x, y_{ki}) + h_{1i}\varphi_2(x, y_{ki})\\
&\varphi_{kpki}(x) = \varphi_2(x, y_{ki})\\
&h_{ki} = 0.5h + H_{ki}^1\\
&w_{ki} = w(x, y_{ki})\\
&\varphi_{ki} = \varphi_1(x, y_{ki})
\end{aligned}$$

It is assumed that the cylindrical shells were elastically connected with each other. That is, in the contact the conditions

$$\begin{aligned}
&w_1(x)\Big|_{\theta_1=\tilde{\theta}_1}=w_2(x)\Big|_{\theta_2=0};\ \vartheta_1(x)\Big|_{\theta_1=\tilde{\theta}_1}=\vartheta_2(x)\Big|_{\theta_2=0};\\
&u_1(x)\Big|_{\theta_1=\tilde{\theta}_1}=u_2(x)\Big|_{\theta_2=0};\ \frac{\partial w_1(x)}{\partial x}\Big|_{\theta_1=\tilde{\theta}_1}=\frac{\partial w_2(x)}{\partial x}\Big|_{\theta_2=0};\\
&w_2(x)\Big|_{\theta_2=\tilde{\theta}_2}=w_3(x)\Big|_{\theta_3=0};\ \vartheta_2(x)\Big|_{\theta_2=\tilde{\theta}_2}=\vartheta_3(x)\Big|_{\theta_3=0};\\
&u_2(x)\Big|_{\theta_2=\tilde{\theta}_2}=u_3(x)\Big|_{\theta_3=0};\ \frac{\partial w_2(x)}{\partial x}\Big|_{\theta_2=\tilde{\theta}_2}=\frac{\partial w_3(x)}{\partial x}\Big|_{\theta_3=0}\\
&w_3(x)\Big|_{\theta_3=\tilde{\theta}_3}=w_1(x)\Big|_{\theta_1=0};\ \vartheta_3(x)\Big|_{\theta_3=\tilde{\theta}_3}=\vartheta_1(x)\Big|_{\theta_1=0};\\
&u_3(x)\Big|_{\theta_3=\tilde{\theta}_3}=u_1(x)\Big|_{\theta_1=0};\ \frac{\partial w_3(x)}{\partial x}\Big|_{\theta_3=\tilde{\theta}_3}=\frac{\partial w_1(x)}{\partial x}\Big|_{\theta_1=0}
\end{aligned} \tag{9}$$

are satisfied. It is accepted that cylindrical shells were highly supported on ideal diaphragms along the lines $x=0$ and $x=a$ in this case boundary conditions are expressed as follows:

$$\begin{aligned}
&u_i=0,\ w_i=0\\
&T_1=0,\ M_1=0
\end{aligned} \tag{10}$$

where, T_1, M_1 are force and moment acting on the cross-section of the cylindric shell.

Using the Ostrogradsky-Hamilton principle of stationarity of action one can obtain a frequency equation for determining natural vibrations frequency of retaining walls formed from connection of cylindrical shells:

$$\delta W=0 \tag{11}$$

where, $W=\int_{t_0}^{t_0^1}\Pi dt$ is Hamilton's action. If in the equality $\delta W=0$ we perform variation operation and take into account that the $\delta u_1, \delta\vartheta_1, \delta w_1$ variations are arbitrary, independent, we can get a system of equations for studying forced vibrations of retaining walls formed from connection of cylindrical shells dynamically contacting with soil.

Thus, the solution of vibrations of retaining walls formed from the connection of soil contacting cylindrical shells is reduced to joint integration of total energy (7) of the construction within contact conditions (8) and (9), boundary conditions (10).

3. PROBLEM SOLUTION

We look for displacements of the points of the cylindrical panel in the following form:

$$\begin{aligned}
&u_i=u_{0i}\cos\chi\xi_i(\cos n\theta_i+\sin n\theta_i)\sin\omega_1 t_1\\
&\vartheta_i=\vartheta_{0i}\sin\chi\xi_i(\cos n\theta_i+\sin n\theta_i)\sin\omega_1 t_1\\
&w_i=w_{0i}\sin\chi\xi_i(\cos n\theta_i+\sin n\theta_i)\sin\omega_1 t_1
\end{aligned} \tag{12}$$

where, $u_{0i}\vartheta_{0i}, w_{0i}$ are unknown constants. Using contact conditions (9) and solutions (12), we can express the constants $u_{02}, \vartheta_{02}, w_{02}$ and $u_{03}, \vartheta_{03}, w_{03}$ by the constants $u_{01}, \vartheta_{01}, w_{01}$.

$$\begin{aligned}
&u_{02}=u_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\\
&\vartheta_{02}=\vartheta_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\\
&w_{02}=w_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\\
&u_{03}=u_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2+\sin n\tilde{\theta}_2\right)\\
&\vartheta_{03}=\vartheta_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2+\sin n\tilde{\theta}_2\right)\\
&w_{03}=w_{03}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2+\sin n\tilde{\theta}_2\right).
\end{aligned} \tag{13}$$

In this case the following condition should be fulfilled.

$$\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2+\sin n\tilde{\theta}_2\right)\times$$
$$\times\left(\cos n\tilde{\theta}_3+\sin n\tilde{\theta}_3\right)=1$$

If we write solutions (12) in (7) and carry out the integration operation, for the total energy of the *i*th panel we get the following expression:

$$\begin{aligned}
\Pi_i=&\left\{\left[\frac{h_iR_ib_{11i}\chi^2q_0q_{2i}}{2}+\frac{\rho_ih_i\omega_1^2\omega_{01}^2q_1}{2R_i\left(1-v_i^2\right)}+b_{66i}\frac{n^2}{R_i^2}q_1q_{3i}\right]u_{0i}^2+\right.\\
&+\left[\frac{b_{22i}n^2q_0q_{3i}}{R_i^2}+\frac{\rho_ih_i\omega_1^2\omega_{01}^2q_0}{2R_i\left(1-v_i^2\right)}\right]\vartheta_{0i}^2+\\
&+\left[\frac{h_ia}{2R_i}(b_{11i}+2b_{12i}+b_{22i})q_0q_{2i}+\frac{\rho_ih_i\omega_1^2\omega_{01}^2q_0}{2R_i\left(1-v_i^2\right)}\right]w_{0i}^2+\\
&+\left[\frac{2b_{12i}}{aR_i^2}\chi nq_0q_{4i}+\frac{b_{66i}\chi n}{aR_i}q_1q_{3i}\right]u_{0i}\vartheta_{0i}+\\
&+h_iaR_i(b_{11i}+b_{12i})q_0q_{2i}u_{0i}w_{0i}+\\
&\left.+\left[-\frac{h_ia}{R_i}(b_{12i}+b_{22i})nq_0q_{4i}\right]\vartheta_{0i}w_{0i}\right\}\sin^2\omega_1t_1+\\
&+aq_0q_{2i}\left(p_i+\frac{\chi^2}{a^2}k_{si}+\frac{n^2}{R_i^2}k_{si}+A\frac{\psi e^{-\psi t}\sin\omega t+\psi\sin^2\omega t}{\psi^2+\omega^2}\right)w_{0i}^2+\\
&+P_{0i}\frac{1}{4}\left(\frac{1}{2}-\frac{\sin 2\chi}{2\chi}\right)\left(\tilde{\theta}_i+\frac{1}{2n}\sin 2n\tilde{\theta}_i\right)\sin^2\omega_1t_1w_{0i}
\end{aligned} \tag{14}$$

where,

$$q_{2i}=\tilde{\theta}_i+\frac{1}{2n}-\frac{\cos 2n\tilde{\theta}_i}{2n}$$

$$q_{5i}=1+\sin\frac{2R_i\tilde{\theta}_i}{k_i+1}$$

$$q_{3i}=\tilde{\theta}_i+\frac{1}{2n}+\frac{\cos 2n\tilde{\theta}_i}{2n}$$

$$q_{4i}=\frac{\sin 2n\tilde{\theta}_i}{2n},\ q_{6i}=1-\sin\frac{2R_i\tilde{\theta}_i}{k_i+1},\ q_{7i}=\cos\frac{2R_i\tilde{\theta}_i}{k_i+1}$$

As can be seen from expressions (14), the total energy of cylindrical shells composing retaining walls are two-degree polynomials with respect to the constants $u_{01}, \vartheta_{01}, w_{01}$.

We show them in the following way:

$$\begin{aligned}
\Pi_{is}=&\varphi_{11}u_{01}^2+\varphi_{22}\vartheta_{01}^2+\varphi_{33}w_{01}^2+\varphi_{44}u_{01}\vartheta_{01}+\\
&+\varphi_{55}u_{01}w_{01}+\varphi_{66}\vartheta_{01}w_{01}+\varphi_{77}P_{0i}w_{01}
\end{aligned} \tag{15}$$

Performing variation operations in the equality $\delta W = 0$ by using (15) and taking into account that the variations $\delta u_{01}, \delta\vartheta_{01}, \delta w_{01}$ are arbitrary, independent, we get the following system of heterogenous linear equations with respect to the constants $u_{01}, \vartheta_{01}, w_{01}$:

$$\begin{cases} 2\tilde{\varphi}_{11}u_{01} + \tilde{\varphi}_{44}\vartheta_{01} + \tilde{\varphi}_{55}w_{01} = 0 \\ \tilde{\varphi}_{44}u_{01} + 2\tilde{\varphi}_{22}\vartheta_{01} + \varphi_{66}w_{01} = 0 \\ \tilde{\varphi}_{55}u_{01} + \tilde{\varphi}_{66}\vartheta_{01} + 2\varphi_{33}w_{01} = \\ = -(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77}) \end{cases} \tag{16}$$

Since the system (16) is linear heterogeneous, we can find its solution by the Kramer rule

$$u_{01} = \frac{\Delta_1}{\Delta};\ \vartheta_{01} = \frac{\Delta_2}{\Delta};\ w_{01} = \frac{\Delta_2}{\Delta} \tag{17}$$

where,

$$\Delta = \begin{vmatrix} 2\tilde{\varphi}_{11} & \tilde{\varphi}_{44} & \tilde{\varphi}_{55} \\ \tilde{\varphi}_{44} & 2\tilde{\varphi}_{22} & \tilde{\varphi}_{66} \\ \tilde{\varphi}_{55} & 2\tilde{\varphi}_{66} & 2\tilde{\varphi}_{33} \end{vmatrix};$$

$$\Delta_1 = \begin{vmatrix} 0 & \tilde{\varphi}_{44} & \tilde{\varphi}_{55} \\ 0 & 2\tilde{\varphi}_{22} & \tilde{\varphi}_{66} \\ -(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77}) & \tilde{\varphi}_{66} & 2\tilde{\varphi}_{33} \end{vmatrix} =$$

$$= -(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(\tilde{\varphi}_{44}\tilde{\varphi}_{66} - 2\tilde{\varphi}_{22}\tilde{\varphi}_{55})$$

$$\Delta_2 = \begin{vmatrix} 2\tilde{\varphi}_{11} & 0 & \tilde{\varphi}_{55} \\ \tilde{\varphi}_{44} & 0 & \tilde{\varphi}_{66} \\ \tilde{\varphi}_{55} & -(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77}) & 2\tilde{\varphi}_{33} \end{vmatrix} =$$

$$= (\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(2\tilde{\varphi}_{11}\tilde{\varphi}_{66} - \tilde{\varphi}_{44}\tilde{\varphi}_{55})$$

$$\Delta_3 = \begin{vmatrix} 2\tilde{\varphi}_{11} & \tilde{\varphi}_{44} & 0 \\ \tilde{\varphi}_{44} & 2\tilde{\varphi}_{22} & 0 \\ \tilde{\varphi}_{55} & \tilde{\varphi}_{66} & -(\kappa_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77}) \end{vmatrix} =$$

$$= -(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})\left(4\tilde{\varphi}_{11}\tilde{\varphi}_{22} - \tilde{\varphi}_{44}^2\right)$$

As a result, for the amplitudes of displacements we get:

$$u_{01} = \frac{\Delta_1}{\Delta} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(\tilde{\varphi}_{11}\tilde{\varphi}_{66} - 2\tilde{\varphi}_{22}\tilde{\varphi}_{55})}{\Delta}$$

$$\vartheta_{01} = \frac{\Delta_2}{\Delta} = \frac{(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(2\tilde{\varphi}_{11}\tilde{\varphi}_{66} - \tilde{\varphi}_{44}\tilde{\varphi}_{55})}{\Delta} \tag{18}$$

$$w_{01} = \frac{\Delta_3}{\Delta} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})\left(4\tilde{\varphi}_{11}\tilde{\varphi}_{22} - \tilde{\varphi}_{44}^2\right)}{\Delta}$$

Taking into account the fisrt three equalities of (13), we can write:

$$u_{02} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(\tilde{\varphi}_{44}\tilde{\varphi}_{66} - 2\tilde{\varphi}_{22}\tilde{\varphi}_{55})}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right)$$

$$\vartheta_{02} = \frac{(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(2\tilde{\varphi}_{11}\tilde{\varphi}_{66} - \tilde{\varphi}_{44}\tilde{\varphi}_{55})}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right) \tag{19}$$

$$w_{02} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})\left(4\tilde{\varphi}_{11}\tilde{\varphi}_{22} - \tilde{\varphi}_{44}^2\right)}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right)$$

In the same way, taking into account the last three equalities of (13), we can write:

$$u_{03} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(\tilde{\varphi}_{44}\tilde{\varphi}_{66} - 2\tilde{\varphi}_{22}\tilde{\varphi}_{55})}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2 + \sin n\tilde{\theta}_2\right)$$

$$\vartheta_{03} = \frac{(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})(2\tilde{\varphi}_{11}\tilde{\varphi}_{66} - \tilde{\varphi}_{44}\tilde{\varphi}_{55})}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2 + \sin n\tilde{\theta}_2\right) \tag{20}$$

$$w_{03} = \frac{-(\varphi_{77}P_{01} + P_{02}\psi_{77} + P_{03}\chi_{77})\left(4\tilde{\varphi}_{11}\tilde{\varphi}_{22} - \tilde{\varphi}_{44}^2\right)}{\Delta} \times$$
$$\times\left(\cos n\tilde{\theta}_1 + \sin n\tilde{\theta}_1\right)\left(\cos n\tilde{\theta}_2 + \sin n\tilde{\theta}_2\right)$$

$$\Delta = 8\tilde{\varphi}_{11}\tilde{\varphi}_{22}\tilde{\varphi}_{33} + 2\tilde{\varphi}_{44}\tilde{\varphi}_{55}\tilde{\varphi}_{66} - 2\tilde{\varphi}_{22}\tilde{\varphi}_{55}^2 -$$
$$-2\tilde{\varphi}_{33}\tilde{\varphi}_{44}^2 - 2\tilde{\varphi}_{11}\tilde{\varphi}_{66}^2$$

4. CONCLUSIONS

The w_{01} component contained in (18) was calculated by a numerical method. For physical and mechanical parameters characterizing the panels and soil the following values were taken:

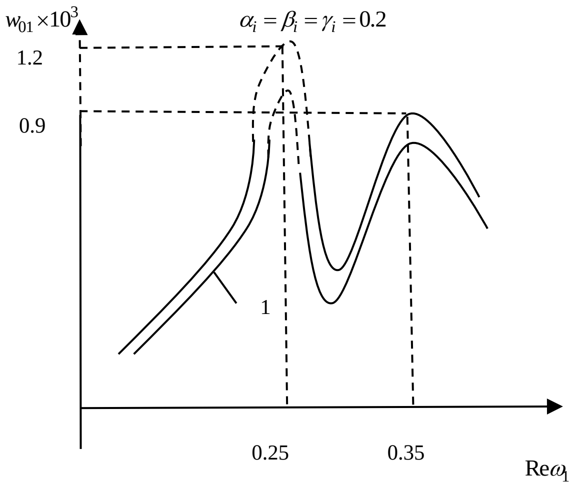

Figure 1. Dependence of the curve w_{01} versus $\mathrm{Re}\,\omega_1$

$$p_1 = p_2 = p_3 = 7\times10^8\ \text{N/m}^2,\ k_{si} = 11\times10^6\ \text{N/m}^2$$

$$\frac{a}{R_i} = 3,\ \nu_{1i} = \nu_{2i} = 0.35, R_i = 160\ \text{mm}, b_{11} = 18.3\ \text{QPa}$$

$$b_{12} = 2.77\,\text{QPa},\ b_{22} = 25.2\ \text{QPa}, b_{66} = 3.5\,\text{QPa}$$

$$\xi_i = \frac{a}{2},\ \rho_i = \rho_{ji} = 1850\,\text{kg/m}^3,\ h_i = 0.45\,\text{mm}$$

$$\frac{P_{02}}{P_{01}} = 0.8,\ \frac{P_{03}}{P_{01}} = 0.7, \Psi = 0.05, A = 0.1615$$

It can be seen from Figure 1 that, with an increase in the frequency of oscillations, the amplitude of displacements first increases, and then reaching a maximum decreases. The maximum values of displacements correspond to resonant frequencies. In Figure 1, the values of 0.25 and 0.35 are resonant frequencies.

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Dilgam Seyfeddin Qaniyev was born in Goyler, Shamakhi, Azerbaijan, in 1981. He graduated from Faculty of Transportation, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan in 2002. He received his M.Sc. degree from the same university in 2004. During 2004-2007, he studied the Ph.D. education at the same university and defended his Ph.D. thesis, earning his Ph.D. degree in Technical Sciences. Then, he started his career and worked as a leading engineer in a number of projects of Azerbaijan. He also worked as a bridge engineer in Akin Project company. He was awarded with "Tereqqi" medal in 2018. He was elected as a member of International Academy of Transport in 2017. He was awarded with the Jubilee Medal of Azerbaijan "100 Years of Azerbaijan Automobile Roads (1918-2018)" in 2018. Currently, he works as a Chief Engineer in Institute of Azerbaijan State Agency of Automobile Roads, Baku, Azerbaijan. He is also a lecturer in Azerbaijan University of Architecture and Construction.