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VIBRATIONS OF A FLOWING FLUID-FILLED VISCOUS-ELASTIC TRANSVERSELY STIFFENED CYLINDRICAL SHELL

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Abstract- In this paper, we study vibrations of flowing fluid-filled viscous-elastic cylindrical shell stiffened with rings. By means of the Ostrogradsky-Hamilton variation principle, the system of motion equations of the system under consideration was obtained, and based on this system a frequency equation was structured. The roots of the obtained frequency equation were found by the numerical method and the graph of their dependence on the motion speed of fluid was built for different characteristic parameters. It was shown that increase in the motion speed of fluid and of the viscosity of the shell material cases decrease of natural vibration frequencies of the system and their vanish at certain values of speed. This a frequency at which a shell loses its stability.

Keywords: Shell, Viscous-Elastic, Frequency, Orthotropic, Ring.

1. INTRODUCTION

At present the analysis of deformation and strength of ribbed shells made of composite materials is of particular relevance when calculating which it is necessary to take into account the anisotropy of the rigidity and strength of the shell and ribs, influence of flowing fluid.

Natural vibrations of a smooth isotropic cylindrical shell in an infinite elastic medium were studied in [1]. One of the dynamic rigidity characteristics, natural vibration frequencies of a system consisting of a soil-contacting viscous-elastic orthotropic cylindrical shell stiffened with discretely distributed rods was studied in [2] using the Hamilton-Ostrogradsky variational principle. A frequency equation for finding the rots of vibration frequencies of the system was constructed and influence on physical geometrical parameters characterizing the system on these roots were studied.

Ref [3] was devoted to one of the dynamical rididity characteristics, natural vibrations frequencies of a system consisting of a solid medium-filled viscous-elastic orthotropic cylindrical shell stiffened with discretely distributed ring-shaped ribs. Using the Hamilton-Ostrogradsky variational principle, a frequency equation for finding vibrations of the system under consideration was constructed and were studied depending on physical and geometrical parameters characterizing the system.

Free vibrations of an isotropic inhomogeneous, moving fluid-contacting cylindrical shell stiffened with cross system of ribs were studied in [4]. The vibrations of an anisotropic cylindrical shell stiffened with cross system of ribs were considered in [5]. Natural and forced axially-symmetric vibrations of a cylindrical shell fluid with fluid a fluid-filled isotropic loaded with axial compressive forces were considered in [6, 7]. Free vibrations of fluid-filled isotropic cylindrical shells stiffened with annular cross system of ribs under the axial compression and with regard to dislocation o ribs were researched in [8, 9].

In the paper [10] natural vibrations frequency of the system is studied that consisting of a solid medium-filled elastic-plastic orthotropic cylindrical shell strengthened with discretely distributed rings established on a plane perpendicular to its axis. Utilizing the Hamilton-Ostrogradsky principle, a frequency equation for determining vibration frequencies of the system following consideration was created; its roots were obtained by mathematical method.

In this paper [11] free vibrations of an orthotropic, laterally stiffened, ideal fluid-filled cylindrical shell inhomogeneous in thickness and in circumferential direction is studied. Using the Hamilton Ostrogrdasky variational principle, the systems of equations of the motion of an orthotropic, ideal fluid filled cylindrical shell stiffened in thickness and circumference, are constructed.

In order to calculate inhomogeneity of the shell material in thickness and circumference, it is accepted that the Young modulus and the density of the material of the shell are the functions of normal and circumferential coordinates. Frequency equations are constructed and free vibrations of an orthotropic, ideal fluid-filled, laterally stiffened cylindrical shell inhomogeneous in thickness and in circumference are numerically implemented. The characteristically dependence curves were constructed.

In the present paper, by means of the variational principle we solve a problem on natural vibrations of a transversely stiffened flowing fluid-filled orthotropic viscous-elastic cylindrical shell.

2. PROBLEM STATEMENT

Based on the Ostrogradsky-Hamilton variational principle we get differential equation of motion for an ideal flowing fluid-filled, transversely stiffened orthotropic cylindrical shell. To apply the Ostrogradsky-Hamilton principle, we preliminarily write the total energy of the system.

We can write orthotropic cylindrical in shell the form [13]:

$$J=\frac{1}{2}R^2\int_{x_1}^{x_2}\int_{y_1}^{y_2}\{N_{11}\varepsilon_{11}+N_{22}\varepsilon_{22}+N_{12}\varepsilon_{12}-$$
$$-M_{11}\chi_{11}-M_{22}\chi_{22}-M_{12}\chi_{12}\}dxdy+$$
$$+\frac{1}{2}\sum_{j=1}^{k_2}\int_{y_1}^{y_2}\left[E_jF_j\left(\frac{\partial\vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+E_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right.$$
$$\left.+E_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y^2}-\frac{\varphi_{kpj}}{R}\right)^2+G_jJ_{kpj}\left(\frac{\partial\varphi_{kpi}}{\partial y}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]dy+ \quad (1)$$
$$+\rho_0 h\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dxdy+$$
$$+\sum_{j=1}^{k_2}\rho_jF_j\int_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial\vartheta_j}{\partial t}\right)^2+\left(\frac{\partial w_j}{\partial t}\right)^2+\right.$$
$$\left.+\frac{J_{kpj}}{F_j}\left(\frac{\partial\phi_{kpj}}{\partial t}\right)^2\right]dy-\int_{x_1}^{x_2}\int_{y_2}^{y_2}\left(q_xu+q_y\vartheta+q_zw\right)dxdy$$

where,

$$b_{11}=\frac{E_1}{1-\nu_1\nu_2},\ b_{22}=\frac{E_2}{1-\nu_1\nu_2}$$
$$b_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2}=\frac{\nu_1E_2}{1-\nu_1\nu_2}$$

E_1, E_2 are the main module of elasticity of the orthotropic material, R denotes the radius of shell median surface of the shell, h is shell's thickness, u, v, w denotes the components of displacements of the points of the shell median surface, x_1, x_2, y_1, y_2 are the coordinates of curvilinear and rectilinear edges of the shell; $F_j, J_{zj}, J_{yj}, J_{kpj}$ are the area and inertia moments of the cross section of the jth cross rod with regard to the axis Oz and the axis parallel to the axis Oy and passing through the gravity center of the section and also its inertia moment at torsion; E_j, G_j are module of elasticity and shift of the material of the jth cross rod, respectively, t is a time coordinate, $t_1=\omega_0 t$, $\omega_0=\sqrt{\frac{E_1}{\left(1-\nu^2\right)\rho_0R^2}}$, ρ_0, ρ are densities of materials from which the shell was made, i is a longitudinal rod, respectively.

We represent the expressions for inner forces and moments as follows:

$$N_{ij}=\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)dz$$
$$M_{ij}=-\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)zdz \quad (2)$$
$$w_{11}=b_{11}\chi_{11}+b_{12}\chi_{22}$$
$$w_{22}=b_{12}\chi_{11}+b_{22}\chi_{22}$$
$$w_{21}=w_{12}=b_{66}\chi_{12}$$

The stresses σ_{ij} and strains ε_{ij} in the median surface in relations (2) are determined in the following way:

$$\sigma_{11}=b_{11}\varepsilon_{11}+b_{12}\varepsilon_{22}$$
$$\sigma_{22}=b_{12}\varepsilon_{11}+b_{22}\varepsilon_{22} \quad (3)$$
$$\sigma_{12}=b_{66}\varepsilon_{12}$$

$$\varepsilon_{ij}=\bar{\varepsilon}_{ij}+\int_{-\infty}^{t}\Gamma(t-\tau)\bar{\varepsilon}_{ij}d\tau$$
$$\bar{\varepsilon}_{11}=\frac{\partial u}{\partial x};\ \bar{\varepsilon}_{22}=\frac{\partial v}{\partial y}+w;\ \bar{\varepsilon}_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \quad (4)$$
$$\chi_{11}=\frac{\partial^2 w}{\partial x^2};\ \chi_{22}=\frac{\partial^2 w}{\partial y^2};\ \chi_{12}=-2\frac{\partial^2 w}{\partial x\partial y}$$
$$\Gamma(t)=Ae^{-\psi t}$$

The equations of motion of a ribbed orthotropic shell with flowing fluid, were obtained based on the Ostrogradsky-Hamilton stationarity of action principle:

$$\delta W=0 \quad (5)$$

where, $W=\int_{t'}^{t''}Ldt$ is Hamilton's action, $L=K-\Pi$ is a Lagrange function, t' and t'' are the given arbitrary moments of time.

Supposing that the main speed of flow equals U and deviations from this speed are negligible, we use a wave equation for the potential of perturbed speeds φ with respect to [14]:

$$\Delta\varphi-\frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2}+2U\frac{\partial^2\varphi}{R\partial\xi\partial t}+U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right)=0 \quad (6)$$

Continuity of radial speeds and pressures on the contact surface a shell-fluid is observed. The condition of impermeability or smooth flow at the shell wall is of the form [14]:

$$\vartheta_r\big|_{r=R}=\frac{\partial\varphi}{\partial r}\bigg|_{r=R}=-\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right)\bigg|_{r=R} \quad (7)$$

Equality of radial pressures as viewed from fluid on the shell

$$q_z=-p\big|_{r=R} \quad (8)$$

Complementing by the contact conditions (7), (8) the expression for the total energy equation motion shell (1), the of fluid (6) we arrive at a problem on natural vibrations of flowing fluid-filled orthotropic cylindrical shell stiffened with longitudinal ribs.

3. PROBLEM SOLUTION

We will look for shell displacements in the form:

$$u = u_0 \sin\chi\xi \cos n\theta \sin\omega_1 t_1$$
$$\vartheta = \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega_1 t_1 \quad (9)$$
$$w = w_0 \cos\chi\xi \cos n\theta \sin\omega_1 t_1$$

where, u_0, ϑ_0, w_0 are unknown constants; χ, n are wave numbers in longitudinal and peripheral directions, respectively.

Using (6)-(8), we can determine pressure as viewed from fluid on the shell:

$$p = \Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \quad (10)$$

where,

$$\Phi_{\alpha n} = \begin{cases} \dfrac{I_n(\beta r)}{I_n'(\beta R)}, M_1 < 1 \\ \dfrac{J_n(\beta r)}{J_n'(\beta R)}, M_1 > 1 \\ \dfrac{r^n}{nR^{n-1}}, M_1 = 1 \end{cases} \quad (11)$$

where,

$$M_1 = \frac{U + \omega_0 R\omega_1/\alpha}{a_0}$$
$$M_1 = \frac{U + \omega_0 R\omega_1/a}{a_0}$$
$$\beta^2 = R^{-2}(1 - M_1^2)\chi^2$$
$$\beta_1^2 = R^{-2}(M_1^2 - 1)\chi^2$$

I_n is a modified Bessel function of first kind, and J_n is nth order Bessel functions of the first kind.

After substitution of (10), (9) in (5), the problem is reduced to the homogeneous system of third order linear algebraic equations:

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0 \quad (i = 1,2,3) \quad (12)$$

where,

$$a_{11} = \left\{\frac{\pi Lh}{4R}\left(b_{11}\chi^2 + b_{66}n^2\right)\left(\frac{\pi}{2\omega} + F(\omega)\right) + \right.$$
$$+\frac{\pi}{2\omega}\left[\rho_0 h\frac{\pi L}{2R}\omega^2 + \frac{1}{4R}\sum_{i=1}^{k_1}\cos^2 n\theta_i\frac{\tilde{E}_i F_i\chi^2}{R^2} + \right.$$
$$\left.\left. + \frac{1}{2R}\chi^2\psi_1\sum_{i=1}^{k_1}F_i\tilde{E}_i\cos^2 n\theta_i - \omega^2\psi_2\sum_{i=1}^{k_1}F_i\rho_i\cos^2 n\theta_i\right]\right\}$$

$$a_{22} = \left\{\frac{\pi Lh}{4R}\left(b_{22}n^2 + b_{66}\chi^2\right)\left(\frac{\pi}{2\omega} + F(\omega)\right) + \right.$$
$$+\frac{\pi}{2\omega}\left[\rho_0 h\frac{\pi L}{2R}\omega^2 + \frac{1}{4R}\sum_{i=1}^{k_1}\cos^2 n\theta_i\frac{\tilde{E}_i F_i\chi^4}{R^4} + \right.$$
$$\left.\left. + \frac{1}{2R}I_{kp.c}\chi^2\psi_2\sum_{i=1}^{k_1}\tilde{G}_i\sin^2 n\theta_i\right]\right\}$$

$$a_{33} = \frac{\pi Lh}{4R}\left[-\frac{h}{4}\chi^2 b_{12} - \frac{h}{4}b_{22}n^2 + \right.$$
$$\left. + \frac{h^2}{12}\left(n^2\chi b_{12} + n^4 b_{22}\right) + \frac{h^2}{3}\chi^2 n^2 b_{66} + b_{22}\right]\times$$
$$\times\left(\frac{\pi}{2\omega} + F(\omega)\right) + \frac{\pi}{2\omega}\left[\rho_0 h\frac{\pi L}{2R}\frac{\omega_1^2}{\omega_0^2} + \frac{1}{4R}\sum_{i=1}^{k_1}\left[\cos^2 n\theta_i E_i + \right.\right.$$
$$+ J_{yi}\frac{\chi^4}{R^4} + \left(n^2 + 1\right)G_i J_{kp.i}\frac{\chi^4}{R^4} -$$
$$\left.\left. - \frac{\pi\xi_1}{2}\tilde{\Phi}_{\alpha n}\rho_m\left(\omega_0^2\omega_1^2 - \frac{U^2\chi^2}{R^2}\right)\right]\right]$$

$$a_{12} = -2n\chi\frac{\pi Lh}{4R}\left(b_{12} + b_{66}\right)\left(\frac{\pi}{2\omega} + F(\omega)\right)$$

$$a_{13} = \left(-2\chi b_{12} + \frac{h}{4}\chi^3 b_{11} + \right.$$
$$\left. + \frac{h}{4}n^2\chi b_{12} - \frac{h}{2}b_{66}\chi n^2\right)\left(\frac{\pi}{2\omega} + F(\omega)\right)$$

$$a_{23} = \left[(2nb_{22} - \frac{h}{4}n\chi^2 b_{12} + \right.$$
$$+ \frac{h}{4}b_{22}n^3 + \frac{h}{2}b_{66}n\chi^2)\left(\frac{\pi}{2\omega} + F(\omega)\right) -$$
$$\left. - \frac{\pi}{2\omega}(2nG_i J_{kp.i}\frac{\chi^4}{R^4} + \frac{\tilde{G}_i}{\tilde{E}_i}J_{kp.i}n\chi^2\psi_3\sum_{i=1}^{k_1}\sin 2n\theta_i)\right]$$

$$F(\omega) = \int_0^{\pi/\omega}\gamma(t)\sin\omega t dt$$

The nontrivial solution of the system linear algebraic Equations (12) is possible only is the case when ω_1 is the root of of third order its determinant. The definition of ω_1 is reduced to a transcendental equation since ω_1 enters to the arguments of the Bessel function J_n.

4. CONCLUSIONS

Let us consider some results of calculations carried out proceeding from the above dependences. For geometrical and physical parameters that characterize the shell and medium materials we accept:

$$\rho_0/\rho_m = 0.105$$
$$\rho_0 = \rho_o = 0.26\times10^4 \text{ Nc}^2/\text{m}^4$$
$$A = 0.1615;\ \beta = 0.05$$
$$b_{11} = 18.3\text{QPa};\ b_{12} = 2.77 \text{ QPa};\ b_{22} = 25.2 \text{ QPa}$$
$$\frac{I_{kp.i}}{2\pi R^3 h} = 0.5305\times10^{-6}$$
$$h^* = \frac{h}{R} = 0.25\times10^{-2};\ \xi_1 = 1$$
$$E_1 = E = 6.67\times10^9 \text{N/m}^2$$
$$v = 0.3;\ h_j = 1.39 \text{ mm}$$
$$R = 160 \text{ mm};\ L_1 = 800 \text{ mm}$$
$$h = 0.45 \text{ mm};\ F_j = 5.75 \text{ mm}^2$$
$$I_{xj} = 19.9 \text{ mm}^4;\ I_{kpj} = 0.48 \text{ mm}^4$$

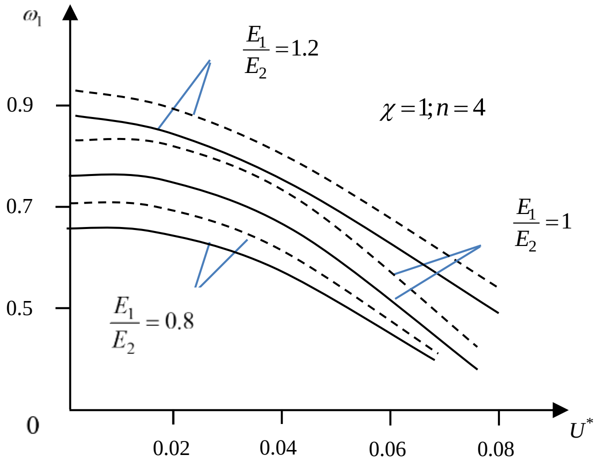

Figure 1. Dependence of the frequency parameter on the flow speed for a system of cylindrical shell with flowing fluid and stiffened with transverse systems of ribs

Dependence of the frequency parameter ω_1 on relative speed of flow $U^{\div} = U / c,\ c = \omega_0 R$ for different values of χ and n are given in Figure 1. In these graphs, the dotted lines correspond to vibrations of a flowing fluid-filled, transversely stiffened elastic cylindrical shell, the solid lines to vibrations of a flowing fluid-filled, transversely stiffened viscous-elastic cylindrical shell. It is seen that increase in the speed and account of viscosity of the shell material leads to frequency reduction. It is important to note the values of $U^{\div}$ under which frequency of vibrations vanishes. Obviously, here a loss of stability of the shell should occur.

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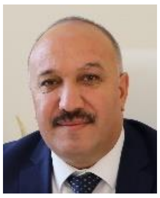

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