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FREE VIBERATIONS OF INHOMOGENEOUS MEDIUM-CONTACTING INHOMOGENEOUS CYLINDRICAL SHELL

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Abstract- In modern age, annular cross-section cylindrical shells are widely used in construction of engineering complexes, in many fields of power engineering. When operating they are contacting with different feature media. When they are in contact, they are dependent on natural conditions, on proper analysis, on the choice of mathematical models and solution algorithms. When these are taken into account, mathematical solution of the problems is complicated, but if they are ignored, some mistakes are allowed. The last years, along with constructions made of traditional materials, continuous inhomogeneous, orthotropic cylindrical shells made of artificial materials occupy an important place. Depending on technology of production and other reasons, the parameters characterizing elastic properties of a cylindrical shell continuously change along the generatrix of density of its material. Therefore, study of parameters characterizing elastic properties, changing continuously the density of its material along its generatrix, free vibrations of a viscous-elastic medium-contacting cylindrical shell is very important. Note that unlike the papers [4-10] not only lateral vibration of a cylindrical shell but also its vibrations on a tangential plane are studied.

Keywords: Cylindrical Shell, Orthotropic, Viscous-Elastic Medium, Inhomogeneous, Free Vibrations, Vibration Frequency.

1. INTRODUCTION

In [1], natural vibrations of a cylindrical shell-solid medium system stiffened with inhomogeneous rings in thickness and subjected to the action of compressive force are studied. In the paper, the inhomogeneity was taken into account by accepting as a function of coordinate changing the Young modulus and material`s density in thickness**.** The paper [2] was devoted to the study of one dynamical strength characteristics, the frequency of natural vibrations of a cylindrical shell inhomogeneous in thickness and along the generatrix, made of a fiberglass and stiffened with annular ribs and subjected to axial compression under the Navier boundary conditions in solid medium.

The motion of the medium is described by the Lame equations in displacements. Using the Hamilton-Ostrogradsky principle, frequency equations for calculating natural vibrations of the system under investigation were constructed in [2].

The paper [3] studies free vibrations of a moving fluid-contacting, orthotropic laterally stiffened cylindrical shell inhomogeneous in thickness. Using the Hamilton-Ostrogradsky variational principle, the system of equations of motion of a moving fluid-contacting, orthotropic, longitudinally stiffened cylindrical shell inhomogeneous in thickness. Inhomogeneity of the shell material was taken into account accepting that the Young modulus and shell`s material are the functions of normal coordinate. In the papers [4, 5, 6], vibrations of a variable thickness rectangular plate on a viscoelastic foundation were considered. The papers [7, 8, 9, 10] were devoted to vibrations of an inhomogeneous orthotropic, circular plate lying on an inhomogeneous elastic foundation.

2. PROBLEM STATEMENT

In the solution of the problem we will use the following system of equations [11]:

$$\begin{cases} \rho hR\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial N_x}{\partial \xi}+\dfrac{\partial T}{\partial \theta} \\ \rho hR\dfrac{\partial^2 \vartheta}{\partial t^2}=\dfrac{\partial N_y}{\partial \theta}+\dfrac{\partial T}{\partial \xi} \\ \rho hR\dfrac{\partial^2 w}{\partial t^2}=\dfrac{\partial^2 M_x}{R\partial \xi^2}+\dfrac{\partial^2 M_y}{R\partial \theta^2}+2\dfrac{\partial^2 H}{R\partial \xi\,\partial \theta}+N_yR+R^2P_z \end{cases} \quad (1)$$

where, u,ϑ,w are the displacements of the points of the shell, R,h are radius and thickness of the shell, respectively, ρ is the density of the shell material, t is time, $\xi=\dfrac{x}{l}$, P_z is a radial force affecting on the shell as viewed from the elastic medium.

The quantities contained in expression (1), the forces N_x,N_y,T and moments in M_x,M_y,H are determined from the following equalities:

$$\begin{aligned}
N_x &= \int_{-h/2}^{h/2} \left(\sigma_{11} + z\,w_{11}\right) dz \\
N_y &= \int_{-h/2}^{h/2} \left(\sigma_{22} + z\,w_{22}\right) dz \\
T &= \int_{-h/2}^{h/2} \left(\sigma_{12} + z\,w_{12}\right) dz \\
M_x &= -\int_{-h/2}^{h/2} \left(\sigma_{11} + z\,w_{11}\right) z dz \\
M_y &= \int_{-h/2}^{h/2} \left(\sigma_{22} + z\,w_{22}\right) z dz \\
H &= -\int_{-h/2}^{h/2} \left(\sigma_{12} + z\,w_{12}\right) z dz
\end{aligned} \tag{2}$$

where,

$$\begin{aligned}
&\sigma_{11} = b_{11}\varepsilon_{11} + b_{12}\varepsilon_{22} \\
&\sigma_{22} = b_{12}\varepsilon_{11} + b_{22}\varepsilon_{22},\ \sigma_{12} = b_{66}\varepsilon_{12} \\
&w_{11} = b_{11}\chi_{11} + b_{12}\chi_{22} \\
&w_{22} = b_{12}\chi_{11} + b_{22}\chi_{22},\ w_{12} = w_{21} = b_{66}\chi_{12}
\end{aligned} \tag{3}$$

By means of the displacements of the shell points the deformation components are expressed as follows:

$$\begin{aligned}
\varepsilon_{11} &= \frac{1}{R}\frac{\partial u}{\partial \xi} \\
\varepsilon_{22} &= \frac{1}{R}\left(\frac{\partial \vartheta}{\partial \theta} + w\right) \\
\varepsilon_{12} &= \frac{1}{R}\left(\frac{\partial u}{\partial \theta} + \frac{\partial \vartheta}{\partial \xi}\right) \\
\chi_{11} &= -\frac{1}{R^2}\frac{\partial^2 w}{\partial \xi^2} \\
\chi_{22} &= -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} \\
\chi_{12} &= -\frac{2}{R^2}\frac{\partial^2 w}{\partial \theta\,\partial \xi}
\end{aligned} \tag{4}$$

For taking inhomogeneity into account, it is assumed the main elasticity module of the orthotropic material $b_{11}, b_{22}, b_{12}, b_{66}$ and the density ρ exchange by the following law [12]:

$$\begin{aligned}
b_{11} &= \tilde{b}_{11}\, f(\xi) \\
b_{22} &= \tilde{b}_{22}\, f(\xi) \\
b_{12} &= \tilde{b}_{12}\, f(\xi) \\
b_{66} &= \tilde{b}_{66}\, f(\xi) \\
\rho &= \tilde{\rho}\, g(\xi)
\end{aligned} \tag{5}$$

where, $\tilde{b}_{11}, \tilde{b}_{22}, \tilde{b}_{12}, \tilde{b}_{66}, \tilde{\rho}$ correspond to homogeneous orthotropic material, $g(x)$ is a continuous function, $f(x)$ is a continuous function together with its second order derivative.

Allowing for expressions (3)-(5) in equalities (2), we can write:

$$\begin{aligned}
N_x &= \frac{h}{R} f(\xi)\left[\tilde{b}_{11}\frac{\partial u}{\partial \xi} + \tilde{b}_{12}\left(\frac{\partial \vartheta}{\partial \theta} + w\right)\right] \\
N_y &= \frac{h}{R} f(\xi)\left[\tilde{b}_{12}\frac{\partial u}{\partial \xi} + \tilde{b}_{22}\left(\frac{\partial \vartheta}{\partial \theta} + w\right)\right] \\
T &= \frac{h}{R} f(\xi)\tilde{b}_{66}\left(\frac{\partial u}{\partial \theta} + \frac{\partial \vartheta}{\partial \xi}\right) \\
M_x &= \frac{h^3 f(\xi)}{12R^2}\left(\tilde{b}_{11}\frac{\partial^2 w}{\partial \xi^2} + \tilde{b}_{12}\frac{\partial^2 w}{\partial \theta^2}\right) \\
M_y &= \frac{h^3 f(\xi)}{12R^2}\left(\tilde{b}_{12}\frac{\partial^2 w}{\partial \xi^2} + \tilde{b}_{22}\frac{\partial^2 w}{\partial \theta^2}\right) \\
H &= \frac{h^3 f(\xi)}{6R^2}\,\tilde{b}_{66}\frac{\partial^2 w}{\partial \xi \partial \theta}
\end{aligned} \tag{6}$$

Taking into account (6) in (1), we get a system of equations of motion in displacements of a viscous-elastic medium- contacting cylindrical shell with parameters characterizing its elastic features changing continuously the density of its material along the generatrix:

$$\begin{aligned}
&R^2\tilde{\rho} g(x)\frac{\partial^2 u}{\partial t^2} = \tilde{b}_{11}\, f(\xi)\frac{\partial^2 u}{\partial \xi^2} + \tilde{b}_{66}\, f(\xi)\frac{\partial^2 u}{\partial \theta^2} + \\
&+ f(\xi)\left(\tilde{b}_{12} + \tilde{b}_{66}\right)\frac{\partial^2 \vartheta}{\partial \xi\,\partial \theta} + f'(\xi)\tilde{b}_{11}\frac{\partial u}{\partial \xi} + \\
&+ f(\xi)\tilde{b}_{12}\frac{\partial w}{\partial \xi} + f'(\xi)\tilde{b}_{12}\frac{\partial \vartheta}{\partial \theta} + f'(\xi)\tilde{b}_{12}\, w \\
&R^2\tilde{\rho} g(x)\frac{\partial^2 \vartheta}{\partial t^2} = f(\xi)\left(\tilde{b}_{12} + \tilde{b}_{66}\right)\frac{\partial^2 u}{\partial \xi\,\partial \theta} + \\
&+ \tilde{b}_{66}\, f(\xi)\frac{\partial^2 \vartheta}{\partial \xi^2} + f(\xi)\tilde{b}_{22}\frac{\partial^2 \vartheta}{\partial \theta^2} + \tilde{b}_{66} f'(\xi)\frac{\partial u}{\partial \theta} + \\
&+ \tilde{b}_{66}\, f'(\xi)\frac{\partial \vartheta}{\partial \xi} + f(\xi)\tilde{b}_{22}\frac{\partial w}{\partial \theta} \\
&R^2\tilde{\rho} g(x)\frac{\partial^2 w}{\partial t^2} = h\, f(\xi)\tilde{b}_{12}\frac{\partial u}{\partial \xi} + h\, f(\xi)\tilde{b}_{22}\frac{\partial \vartheta}{\partial \theta} + \\
&+ \frac{h^3 f(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^4 w}{\partial \xi^4} + \frac{h^3 f(\xi)}{R^2}\tilde{b}_{22}\frac{\partial^4 w}{\partial \theta^4} + \\
&+ \frac{2h^3 f(\xi)}{R^2}\left(\tilde{b}_{12} + \tilde{b}_{66}\right)\frac{\partial^4 w}{\partial \xi^2\,\partial \theta^2} + \\
&+ \frac{2h^3 f'(\xi)}{R^2}\tilde{b}_{66}\frac{\partial^3 w}{\partial \xi\,\partial \theta^2} + \frac{h^3 f'(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^2 w}{\partial \xi^2} + \\
&+ \frac{h^3 f'(\xi)}{R^2}\tilde{b}_{12}\frac{\partial^2 w}{\partial \theta^2} + h\, f(\xi)\tilde{b}_{22}\, w + R^2 P_z
\end{aligned} \tag{7}$$

In the problem under consideration, it is assumed that the cylindrical shell is in contact with inhomogeneous medium and during vibrations its reaction is characterized by the curvature P_z in the following way [13]:

$$P_z = k_1(\xi)\, w + k_2(\xi)\frac{\partial^2 w}{\partial t^2} \tag{8}$$

where, $k_1(\xi)$ and $k_2(\xi)$ are the characteristics of the foundation and are continuous functions, w is a curvature, t is time. Thus, the solution of the problem is reduced to the integration of system (7) taking into account (8).

It is considered that the displacements u, ϑ, w satisfy the hinge connection conditions at the ends of the cylindrical shell:

$$u=\vartheta=N_x=M_x=0$$

3. PROBLEM SOLUTION

The system (7) of equations of motion of a cylindrical shell consists of variable coefficient complex equations. As it is difficult to find its solution, there arises necessity to find its exact solution by approximate analytic methods. Taking this into account, we will solve system (7) using the method of separation of variables and the Bubnov-Galerkin method. In the first step we will look for the functions u, ϑ, w in the following form:

$$\begin{aligned} u&=U(\xi,\theta)e^{i\omega t} \\ \vartheta&=V(\xi,\theta)e^{i\omega t} \\ w&=W(\xi,\theta)e^{i\omega t} \end{aligned} \tag{9}$$

Taking into account equations (8) and (9) in (7) we can write:

$$\begin{aligned} &-\omega^2R^2\tilde{\rho}g(x)U=\tilde{b}_{11}\,f(\xi)\frac{\partial^2 u}{\partial\xi^2}+\tilde{b}_{66}\,f(\xi)\frac{\partial^2 u}{\partial\theta^2}+ \\ &+f(\xi)\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^2 V}{\partial\xi\,\partial\theta}+f'(\xi)\tilde{b}_{11}\frac{\partial U}{\partial\xi}+ \\ &+f(\xi)\tilde{b}_{12}\frac{\partial W}{\partial\xi}+f'(\xi)\tilde{b}_{12}\frac{\partial V}{\partial\theta}+f'(\xi)\tilde{b}_{12}W- \\ &-\omega^2R^2\tilde{\rho}g(x)V=f(\xi)\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^2 U}{\partial\xi\,\partial\theta}+ \\ &+\tilde{b}_{66}\,f(\xi)\frac{\partial^2 V}{\partial\xi^2}+f(\xi)\tilde{b}_{22}\frac{\partial^2 V}{\partial\theta^2}+\tilde{b}_{66}f'(\xi)\frac{\partial U}{\partial\theta}+ \\ &+\tilde{b}_{66}\,f'(\xi)\frac{\partial V}{\partial\xi}+f(\xi)\tilde{b}_{22}\frac{\partial W}{\partial\theta} \\ &\left(k_2(\xi)-\omega^2R^2\tilde{\rho}g(x)\right)W= \\ &=h\,f(\xi)\tilde{b}_{12}\frac{\partial U}{\partial\xi}+h\,f(\xi)\tilde{b}_{22}\frac{\partial V}{\partial\theta}+ \\ &+\frac{h^3f(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^4 W}{\partial\xi^4}+\frac{h^3f(\xi)}{R^2}\tilde{b}_{22}\frac{\partial^4 W}{\partial\theta^4}+ \\ &+\frac{2h^3f(\xi)}{R^2}\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^4 w}{\partial\xi^2\,\partial\theta^2}+ \\ &+\frac{2h^3f'(\xi)}{R^2}\tilde{b}_{66}\frac{\partial^3 W}{\partial\xi\,\partial\theta^2}+\frac{h^3f'(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^2 W}{\partial\xi^2}+ \\ &+\frac{h^3f'(\xi)}{R^2}\tilde{b}_{12}\frac{\partial^2 W}{\partial\theta^2}+\left(h\,f(\xi)\tilde{b}_{22}+k_1(\xi)\right)W \end{aligned} \tag{10}$$

In the second stage the solution of the system (10) is constructed by the Bubnov-Galerkin method. The function $U(\xi,\theta), V(\xi,\theta), W(\xi,\theta)$ is sought in the following way:

$$\begin{aligned} U(\xi,\theta)&=u_0\cos m\pi\xi\cos n\theta \\ V(\xi,\theta)&=\vartheta_0\sin m\pi\xi\sin n\theta \\ W(\xi,\theta)&=w_0\sin m\pi\xi\cos n\theta \end{aligned} \tag{11}$$

where, u_0, ϑ_0, w_0 are unknown constants, m, n are the number of half-waves in the direction of the generatrix of the cylindrical shell and in the circular direction, respectively.

Using the Bubnov-Galerkin method, we can write:

$$\begin{aligned} &\int_0^1\int_0^{2\pi}\Bigg[\omega^2R^2\tilde{\rho}g(\xi)U+\left(\tilde{b}_{11}\,f(\xi)\frac{\partial^2 U}{\partial\xi^2}+\tilde{b}_{66}\,f(\xi)\frac{\partial^2 U}{\partial\theta^2}\right)+ \\ &+f(\xi)\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^2 V}{\partial\xi\,\partial\theta}+f'(\xi)\tilde{b}_{11}\frac{\partial U}{\partial\xi}+ \\ &+f(\xi)\tilde{b}_{12}\frac{\partial W}{\partial\xi}+f'(\xi)\tilde{b}_{12}\frac{\partial V}{\partial\theta}+f'(\xi)\tilde{b}_{12}W\Bigg]\times \\ &\times\sin m\pi\xi\cos n\theta\,d\xi\,d\theta=0 \end{aligned}$$

$$\begin{aligned} &\int_0^1\int_0^{2\pi}\Bigg[\omega^2R^2\tilde{\rho}gV+f(\xi)\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^2 U}{\partial\xi\,\partial\theta}+ \\ &+\tilde{b}_{66}\,f(\xi)\frac{\partial^2 V}{\partial\xi^2}+f(\xi)\tilde{b}_{22}\frac{\partial^2 V}{\partial\theta^2}+\tilde{b}_{66}f'(\xi)\frac{\partial U}{\partial\theta}+ \\ &+\tilde{b}_{66}\,f'(\xi)\frac{\partial V}{\partial\xi}+f(\xi)\tilde{b}_{22}\frac{\partial W}{\partial\theta}\Bigg]\times \\ &\times\sin m\pi\xi\cos n\theta\,d\xi\,d\theta=0 \end{aligned} \tag{12}$$

$$\begin{aligned} &\int_0^1\int_0^{2\pi}\Bigg[\left(\omega^2R^2\tilde{\rho}g(\xi)-k_2(\xi)\right)W+h\,f(\xi)\tilde{b}_{12}\frac{\partial U}{\partial\xi}+ \\ &+h\,f(\xi)\tilde{b}_{22}\frac{\partial V}{\partial\theta}+\frac{h^3f(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^4 W}{\partial\xi^4}+ \\ &+\frac{h^3f(\xi)}{R^2}\tilde{b}_{22}\frac{\partial^4 W}{\partial\theta^4}+\frac{2h^3f(\xi)}{R^2}\left(\tilde{b}_{12}+\tilde{b}_{66}\right)\frac{\partial^4 W}{\partial\xi^2\,\partial\theta^2}+ \\ &+\frac{2h^3f'(\xi)}{R^2}\tilde{b}_{66}\frac{\partial^3 W}{\partial\xi\,\partial\theta^2}+\frac{h^3f'(\xi)}{R^2}\tilde{b}_{11}\frac{\partial^2 W}{\partial\xi^2}+ \\ &+\frac{h^3f'(\xi)}{R^2}\tilde{b}_{12}\frac{\partial^2 W}{\partial\theta^2}+\left(h\,f(\xi)\tilde{b}_{22}+k_1(\xi)\right)W\Bigg]\times \\ &\times\sin m\pi\xi\cos n\theta\,d\xi\,d\theta=0 \end{aligned}$$

Substituting expressions (11) in the system (12), we get the following system of algebraic equations with respect to the unknown constants, u_0, ϑ_0, w_0:

$$\left(\omega^2 R^2 \tilde{\rho} I_0 - m^2\pi^2 \tilde{b}_{11} I_4 - n^2 \tilde{b}_{66} I_4 + m\pi \tilde{b}_{11} I_2\right) u_0 +$$
$$+\left[\left(\tilde{b}_{12}+\tilde{b}_{66}\right) I_4 nm\pi + \tilde{b}_{12} n I_3\right]\vartheta_0 + \left(m\pi I_4 + I_3\right)\tilde{b}_{12} w_0 = 0$$
$$\left(\omega^2 R^2 \tilde{\rho} I_0 - \tilde{b}_{66} m^2\pi^2 I_2 - \tilde{b}_{22} n^2 I_2 + \tilde{b}_{66} m\pi I_5\right)\vartheta_0 = 0$$
$$h\tilde{b}_{12} m\pi I_2 u_0 + h\tilde{b}_{22} n I_2 \vartheta_0 + \qquad (13)$$
$$+\left[\omega^2 R^2 \tilde{\rho} I_0 - I_6 + \frac{h^3}{R^2}\left(m^4\pi^4 \tilde{b}_{11} I_2 + \tilde{b}_{22} n^4 I_2 +\right.\right.$$
$$+2\left(\tilde{b}_{12}+\tilde{b}_{66}\right) m^2\pi^2 n^2 I_2 - 2\tilde{b}_{66} m\pi n^2 I_5 -$$
$$\left.-\tilde{b}_{11} m^2\pi^2 I_3 - \tilde{b}_{12} n^2 I_3 + h\tilde{b}_{22} I_2 + I_7\right] w_0 = 0$$

where,

$$I_0 = \frac{1}{2}\int_0^1 g(\xi)\sin 2m\pi\xi\, d\xi; \quad I_1 = \int_0^1 g(\xi)\sin^2 m\pi\xi\, d\xi$$
$$I_2 = \int_0^1 f(\xi)\sin^2 m\pi\xi\, d\xi; \quad I_3 = \int_0^1 f'(\xi)\sin^2 m\pi\xi\, d\xi \qquad (14)$$
$$I_4 = \frac{1}{2}\int_0^1 f(\xi)\sin 2m\pi\xi\, d\xi; \quad I_5 = \frac{1}{2}\int_0^1 f'(\xi)\sin 2m\pi\xi\, d\xi$$
$$I_6 = \int_0^1 k_2(\xi)\sin^2 m\pi\xi\, d\xi; \quad I_7 = \int_0^1 \sin^2 m\pi\xi\, d\xi$$

As the system (13) is a system of linear homogeneous equations, the necessary and sufficient condition for the existence of its non-zero solution is the equality of its principal determinant to zero. As a result, we get a frequency equation:

$$\begin{vmatrix} \omega^2 R^2 \tilde{\rho} I_0 + \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 0 & \omega^2 R^2 \tilde{\rho} I_0 + \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \omega^2 R^2 \tilde{\rho} I_0 + \alpha_{33} \end{vmatrix} = 0 \qquad (15)$$

where,

$$\alpha_{11} = -m^2\pi^2 \tilde{b}_{11} I_4 - n^2 \tilde{b}_{66} I_4 + m\pi \tilde{b}_{11} I_2;$$
$$\alpha_{12} = \left(\tilde{b}_{12}+\tilde{b}_{66}\right) I_4 nm\pi + \tilde{b}_{12} n I_3;$$
$$\alpha_{13} = \left(m\pi I_4 + I_3\right)\tilde{b}_{12}; \ \alpha_{22} = -\tilde{b}_{66} m^2\pi^2 I_2 - \tilde{b}_{22} n^2 I_2 +$$
$$+\tilde{b}_{66} m\pi I_5; \ \alpha_{31} = h\tilde{b}_{12} m\pi I_2; \ \alpha_{32} = h\tilde{b}_{22} n I_2;$$
$$\alpha_{33} = -I_6 + \frac{h^3}{R^2}\left(m^4\pi^4 \tilde{b}_{11} I_2 + \tilde{b}_{22} n^4 I_2\right) +$$
$$+2\left(\tilde{b}_{12}+\tilde{b}_{66}\right) m^2\pi^2 n^2 I_2 - 2\tilde{b}_{66} m\pi n^2 I_5 -$$
$$-\tilde{b}_{11} m^2\pi^2 I_3 - \tilde{b}_{12} n^2 I_3 + h\tilde{b}_{22} I_2 + I_7$$

and,

$$\left(\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{22}\right)\left[\left(\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{11}\right)\times\right.$$
$$\left.\times\left(\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{33}\right) - \alpha_{13}\alpha_{31}\right] = 0 \qquad (16)$$

$$\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{22} = 0$$
$$\left(\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{11}\right)\left(\omega^2 R^2 \tilde{\rho} I_0 + \alpha_{33}\right) - \alpha_{13}\alpha_{31} = 0 \qquad (17)$$

From the first one of equations (17) we get:

$$\omega^2 = -\frac{\alpha_{22}}{R^2 \tilde{\rho} I_0} \qquad (18)$$

The second one of equations (17) is a biquadratic equation with respect to ω:

$$R^2 \tilde{\rho}^2 I_0^4 \omega^4 + \left(\alpha_{11}+\alpha_{33}\right) R^2 \tilde{\rho} I_0 \omega^2 +$$
$$+\alpha_{33}\alpha_{11} - \alpha_{13}\alpha_{31} = 0 \qquad (19)$$

We can easily find the roots of equations (18) and (19):

$$\omega_{1,2} = \pm\sqrt{-\frac{\alpha_{22}}{R^2 \tilde{\rho} I_0}}$$
$$\omega_{3,4} = \pm\sqrt{\frac{-\left(\alpha_{11}+\alpha_{33}\right)+\sqrt{\left(\alpha_{11}+\alpha_{33}\right)^2 + 4\alpha_{13}\alpha_{31}}}{R^2 \tilde{\rho} I_0}} \qquad (20)$$
$$\omega_{5,6} = \pm\sqrt{\frac{-\left(\alpha_{11}+\alpha_{33}\right)-\sqrt{\left(\alpha_{11}-\alpha_{33}\right)^2 + 4\alpha_{13}\alpha_{31}}}{R^2 \tilde{\rho} I_0}}$$

4. CONCLUSIONS

Let us consider a particular case. Assume that the function, $f(\xi)$, $k_1(\xi)$, $k_2(\xi)$ and $g(\xi)$ change by the linear law:

$$f(\xi) = 1+\varepsilon\xi, \ g(\xi) = 1+\mu\xi, \ k_1(\xi) = k_2(\xi) = 1+\alpha\xi,$$
$$\varepsilon, \mu, \alpha \in [0;1]$$

By means of these functions the integrals (14) were calculated and the roots of (20) were found by the numerical method. For the parameters participating in the calculations, the following values were taken:

$$\tilde{b}_{11} = 18.3 \text{ QPa}, \tilde{b}_{12} = 2.77 \text{ QPa}, \tilde{b}_{22} = 25.2 \text{ QPa}, \ \tilde{b}_{66} = 3.5 \text{ QPa}$$
$$h = 0.8 \text{ mm}, R = 160 \text{ mm}, \tilde{\rho} = 7.8 \text{ q/cm}^3$$

The results of calculations were given in Figure 1 in the form of dependence of a frequency parameter on inhomogeneity parameter of the shell, in Figure 2 on the parameter characterizing inhomogeneity of the density of the shell material, in Figure 3 on the inhomogeneity parameter of medium.

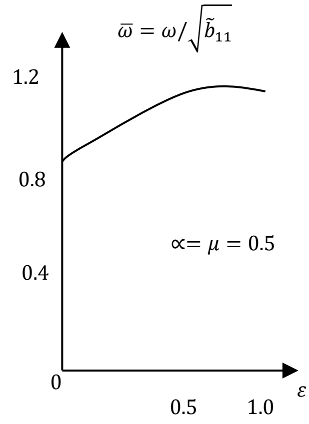

Figure 1. Dependence of frequency parameter on inhomogeneity parameter of the shell material

As can be seen from Figure 1, as the value of inhomogeneity parameter of the shell material increases, the value of the frequency parameter increases. This is explained by the fact that the rigidity of the cylindrical shell increases due to the increase in the value of inhomogeneity parameter. Increase in the value of the parameter characterizing the density of the cylindrical shell material, as can be seen from Figure 2 causes decrease in the value of the frequency parameter. This is explained by the fact that increase in parameter of the density of the shell material increases inertial effect on vibration frequency of the system. As can be seen from Figure 3, increase in the value of inhomogeneity parameter of the medium contacting with the shell causes increase in the value of frequency parameter.

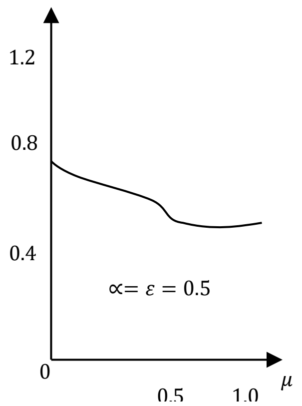

Figure 2. Dependence of frequency parameter on inhomogeneity parameter of the density of the shell material

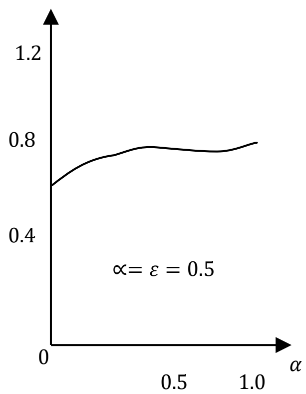

Figure 3. Dependence of frequency parameter on inhomogenety parameter of the medium

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