International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

September 2020 | Issue 44 | Volume 12 | Number 3 | Pages 56-61

FREE VIBRATIONS OF VISCOUS-ELASTIC HETEROGENEOUS MEDIUM-CONTACTING RETAINING WALL CONSISTING OF TWO ORTHOTROPIC CYLINDRICAL PLATES

D.S. Ganiyev

Azerbaijan University of Architecture and Construction, Baku, Azerbaijan, qanidilqem@gmail.com

Abstract- In the paper free vibrations of a viscous-elastic heterogeneous medium-contacting retaining wall consisting of two cylindrical plates were studied. Using the Hamilton-Ostrogradsky variational principle for finding vibrations frequencies of a retaining wall, a frequency equation was structured, its roots were found and the influences of physical and geometrical parameters characterizing the system were studied. Account of the joint work of two cylindrical panels on the contact line are accepted as contact conditions.

Keywords: Free Vibrations, Heterogeneous Medium, Orthotropic Plate, Viscous-Elastic Medium, Frequency Equation.

1. INTRODUCTION

The work [1] was devoted to one of the dynamical characteristics, the frequency of natural vibrations of a vertical support consisting of three orthotropic soil-filled cylindrical panels and reinforced with discretely distributed longitudinal rods. Using the Hamilton-Ostrogradsky variational principle for finding vibrations frequency of a vertical support a frequency equation was structured, its roots were found and influence of physical-geometrical parameters characterizing the system were studied. Account of joint work of three cylindrical panels on the contact line was accepted as contact conditions.

Allowing for viscosity and heterogeneity of soil, features of orthotropic property of plate, free vibrations of retaining wall consisting of three cylindrical plates were studied in [2], analytic expressions to calculate the displacements of the points of cylindrical plates were obtained, characteristically curves were structured. Account of heterogeneity of soil was performed by accepting its rigidity coefficients as a coordinate function. It was considered that the Poisson ratio is constant.

In the papers [3, 4] based on the Ostrogradskiy-Hamilton principle, we study natural vibrations of retaining walls composed of two orthotropic cylindrical shells under dynamical interaction with visco-elastic soil, find resonance frequencies of the wall under consideration and construct typical dependence curves.

The work [5] was devoted to the study of one of the dynamical strength characteristics, the frequency of natural vibrations of a retaining wall consisting of two soil-contacting orthotropic cylindrical shells stiffened with discretely distributed annular rods. Using the Hamilton-Ostrogradsky variational principle for finding vibrations frequencies of retaining walls, a frequency equation was structured, its roots were found and influence of physical and geometrical parameters characterizing the system was studied. Account of joint work of two cylindrical shells on contact line was accepted as contact conditions.

2. PROBLEM STATEMENT

To study free vibrations of a viscous-elastic medium-contacting retaining wall consisting of two orthotropic plates the Hamilton-Ostrogradsky variational principle was used. According to this principle, the total energy of the construction under investigation takes a stationary value for real stress-strain state. Since the construction under investigation consists of cylindrical shells, viscous-elastic heterogeneous soil, we write expressions for potential and kinetic energies of each element (Figure 1).

Potential energies of cylindrical shells:

$$G_i=\frac{h_iR_i}{2}\iint_{s_i}\left\{b_{11i}\left(\frac{\partial u_i}{\partial x_i}\right)^2-2\left(b_{11i}+b_{12i}\right)\times\frac{w_i}{R_i}\frac{\partial u_i}{\partial x_i}+\frac{w_i^2}{R_i^2}\left(b_{11i}+2b_{12i}+b_{22i}\right)+\frac{b_{22i}}{R_i^2}\left(\frac{\partial \vartheta_i}{\partial \theta_i}\right)^2-2\left(b_{12i}+b_{22i}\right)\frac{w_i}{R_i^2}\frac{\partial \vartheta_i}{\partial \theta_i}+2b_{12i}\frac{1}{R_i^2}\frac{\partial u_i}{\partial x_i}\frac{\partial \vartheta_i}{\partial \theta_i}+b_{66i}\frac{1}{R_i^2}\left(\frac{\partial u_i}{\partial \theta_i}\right)^2+b_{66i}\left(\frac{\partial \vartheta_i}{\partial x_i}\right)^2+b_{66i}\frac{1}{R}\frac{\partial \vartheta_i}{\partial x_i}\frac{\partial u_i}{\partial \theta_i}\right\}dx_i d\theta_i \quad (1)$$

Kinetic energies of cylindrical shells:

$$K_i==\frac{\rho_i h_i}{2R_i\left(1-v_i^2\right)}\iint_{s_i}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2\right]dx_i d\theta_i \quad (2)$$

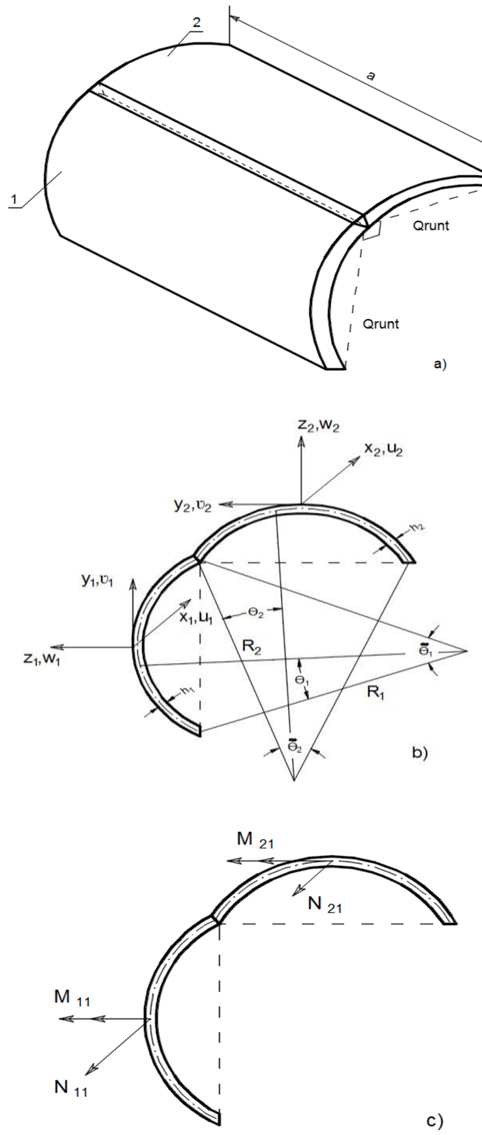

Figure 1. Scheme of the viscous-elastic heterogenous retaining wall composed of the joint of cylindrical shells

Influence of soil to cylindrical shells are replaced by the external forces q_{xi}, q_{yi}, q_{zi}. The work done by these forces in displacements is determined by expressions (3).

$$A_i = -R\int_0^a \int_0^{3\pi/4} \left(q_{xi}u_i + q_{yi}\vartheta_i + q_{zi}w_i\right)dxd\theta \qquad (3)$$

where, $i=1$ corresponds to the first cylindrical plate forming the support, $i=2$ corresponds to the second cylindrical plate (graph 2.1.1); u_i, ϑ_i, w_i are displacements of the points of cylindrical plates; R_i, h_i are curvature radii of thickness of cylindrical plates; $b_{11i}, b_{22i}, b_{12i}, b_{66i}$ are elasticity module of orthotropic cylindrical plates; E_{1i}, E_{2i} are elasticity module of orthotropic cylindrical plates in the direction of coordinate axis x_i and θ_i; ν_{1i}, ν_{2i} are Poisson ratios; q_{xi}, q_{yi}, q_{zi} are the components of forces acting on cylindrical plates as viewed from soil; t is time; s_i are surfaces of cylindrical plates. The elasticity module of orthotropic cylindrical plates are expressed by the constants $b_{11i}, b_{22i}, b_{12i}, b_{66i}$ $E_{1i}, E_{2i}, \nu_{1i}, \nu_{2i}$ as follows:

$$b_{11i} = \frac{E_{1i}}{1-\nu_{1i}\nu_{2i}}$$

$$b_{22i} = \frac{E_{2i}}{1-\nu_{1i}\nu_{2i}}$$

$$b_{12i} = \frac{\nu_{2i}E_{1i}}{1-\nu_{1i}\nu_{2i}} = \frac{\nu_{1i}E_{2i}}{1-\nu_{1i}\nu_{2i}}$$

For taking into account the heterogeneity of soil, we will consider that the rigidity coefficients p_i, k_{si} at compression and shift is the function of x coordinate alternating along the generatrix of cylindrical plates. As a result, for the pressure force components q_{xi}, q_{yi}, q_{zi} acting on cylindrical plates as viewed from soil we can write:

$$q_{xi} = q_{yi} = 0$$

$$q_{z1} = p_1 w_1 + k_{s1}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) - \int_0^t \Gamma(t-\tau)w_1(\tau)d\tau$$

$$q_{z2} = p_2 w_2 + k_{s2}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) - \int_0^t \Gamma(t-\tau)w_2(\tau)d\tau \qquad (4)$$

In expression (4) we will consider that the rigidity coefficients p_i, k_{s_i} of soil at compression and shift change according to the following rule:

$$p_1(x) = p_{10}\left(1+\alpha\frac{x}{a}\right)$$

$$p_2(x) = p_{20}\left(1+\beta\frac{x}{a}\right)$$

$$k_{s1}(x) = k_{s10}(x)\left(1+\gamma\frac{x}{a}\right) \qquad (5)$$

$$k_{s2}(x) = k_{s20}\left(1+\mu\frac{x}{a}\right)$$

where, $\alpha, \beta, \gamma, \mu \in [0,1]$ and p_{i0}, k_{si0} are rigidity coefficients of soil at compression and shift, $\Gamma(t) = Ae^{-\psi t}$ is a relaxation core, A, ψ are empirical constants.

As a result, the total energy of the system is in the form:

$$\Pi = \sum_{i=1}^{2}\left(G_i + K_i + A_i\right) \qquad (6)$$

To the expression (6), we add contact and boundary conditions. We assume that cylindrical shells are elastically joined with each other, i.e. in the contact

$$w_1(x)\Big|_{\theta_1=\tilde{\theta}_1} = \vartheta_2(x)\Big|_{\theta_2=0};\ \vartheta_1(x)\Big|_{\theta_1=\tilde{\theta}_1} = w_2(x)\Big|_{\theta_2=0}$$

the conditions

$$u_1(x)\Big|_{\theta_1=\tilde{\theta}_1} = u_2(x)\Big|_{\theta_2=0};\ \frac{\partial w_1(x)}{\partial x}\Big|_{\theta_1=\tilde{\theta}_1} = \frac{\partial \vartheta_2(x)}{\partial x}\Big|_{\theta_2=0} \qquad (7)$$

are satisfied.

It is accepted that cylindrical shells are highly supported on the ideal diaphragms along the lines $x=0$ and $x=a$ that in this case the boundary conditions are expressed as

$$u_i=0\,,\; w_i=0 \qquad T_1=0\,,\; M_1=0 \tag{8}$$

where, T_1, M_1 are force and moment acting on the cross-section of the cylindrical shell (Figure 1,c).

Using the Ostrogradsky-Hamilton stationarity condition, we can obtain a frequency equation for determining natural vibrations frequency of a retaining wall formed by the joint of cylindrical shells:

$$\delta W=0 \tag{9}$$

where, $W=\int_{t_0}^{t_1'}\Pi dt$ is Hamilton's action. It we realize variation operation in the equality $\delta W=0$ and take into account that the variations $\delta u_1, \delta\vartheta_1, \delta w_1$ are not arbitrary and dependent, for finding natural frequencies of retaining walls composed of the joint of cylindrical shells dynamically contacting with soil, we get a frequency equation. So, the solution of a problem of vibrations of retaining walls formed from the joint of cylindrical shells dynamically contacting with soil is reduced to joint integration of total energy (6) of the construction within contact (7) and boundary conditions (8).

3. PROBLEM SOLUTION

We look for the displacements of the points of the cylindrical plate in the following form:

$$\begin{aligned} u_i &= u_{0i}\cos\chi\xi_i\left(\cos n\theta_i+\sin n\theta_i\right)\sin\omega_1 t_1\\ \vartheta_i &= \vartheta_{0i}\sin\chi\xi_i\left(\cos n\theta_i+\sin n\theta_i\right)\sin\omega_1 t_1\\ w_i &= w_{0i}\sin\chi\xi_i\left(\cos n\theta_i+\sin n\theta_i\right)\sin\omega_1 t_1 \end{aligned} \tag{10}$$

where, $u_{0i}\vartheta_{0i}, w_{0i}$ are unknown constants, $\xi_i=\frac{x_i}{a}$, $t_1=\omega_{01}t, \chi, n$ are wave numbers of the cylindrical plate along the generatrix and in circular direction, $0\le\theta_1\le\tilde{\theta}_1$. $0\le\theta_2\le\tilde{\theta}_2$

$$\omega_1=\sqrt{\frac{\left(1-v_{11}^2\right)\rho_1 R_1^2\omega^2}{E_{11}}}\,,\; \omega_{01}=\sqrt{\frac{E_{11}}{\left(1-v_{11}^2\right)\rho_1 R_1^2}}$$

Using the expressions (3), (4) and (10), we calculate the influence of soil on cylindric shells as viewed from soil as the work A_i done by external forces q_{xi}, q_{yi}, q_{zi} in displacements of the points of the cylindrical shell:

$$A_i=w_{0i}^2 a\left(p_i+\frac{\chi^2}{a^2}k_{si}+\frac{n^2}{R^2}k_{si}+A\frac{\psi e^{-\psi t}\sin\omega t+\psi\sin^2\omega t}{\psi^2+\omega^2}\right)\times \left(\tilde{\theta}_i-\frac{\cos 2n\tilde{\theta}_i}{2n}+\frac{1}{2n}\right)\left(\frac{1}{2}-\frac{\sin 2\chi}{4\chi}\right) \tag{11}$$

Using contact conditions (7) and solution (10), we can express the constants $u_{02}, \vartheta_{02}, w_{02}$ by the constants $u_{01}, \vartheta_{01}, w_{01}$:

$$\begin{aligned} u_{02} &= u_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\\ \vartheta_{02} &= w_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right)\\ w_{02} &= \vartheta_{01}\left(\cos n\tilde{\theta}_1+\sin n\tilde{\theta}_1\right) \end{aligned} \tag{12}$$

Since the retaining wall consists of two cylindrical plates, the total energy of the system will be

$$\Pi=\sum_{i=1}^{2}G_i \tag{13}$$

If we substitute the solutions (10) in (1) and (2), and implement the integration operation, for the total energy of the *i*th plate we get the following expression:

$$\begin{aligned} G_1 = \Bigg\{ & \left[\frac{h_1R_1b_{111}\chi^2q_0q_{2i}}{2}+\frac{\rho_1h_1\omega_1^2\omega_{01}^2q_1}{2R_1\left(1-v_1^2\right)}+b_{661}\frac{n^2}{R_1^2}q_1q_{31}\right]u_{01}^2+\\ &+\frac{b_{221}n^2q_0q_{31}}{R_1^2}\vartheta_{01}^2+\left[\frac{h_1a}{2R_1}\left(b_{111}+2b_{121}+b_{221}\right)q_0q_{21}+\right.\\ &\left.+\frac{\rho_1h_1\omega_1^2\omega_{01}^2q_0}{2R_1\left(1-v_1^2\right)}\right]w_{01}^2+\left[\frac{2b_{121}}{aR_1^2}\chi nq_0q_{41}+\frac{b_{661}\chi n}{aR_1}q_1q_{31}\right]u_{01}\vartheta_{01}-\\ &-h_1aR_1\left(b_{111}+b_{121}\right)q_0q_{21}u_{01}w_{01}+\\ &+\left[-\frac{h_1a}{R_1}\left(b_{121}+b_{221}\right)nq_0q_{41}\right]\vartheta_{01}w_{01}\Bigg\}\times\\ &\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]+\\ &+\Bigg\{aq_0q_{21}\left[p_{10}+\frac{\chi^2}{a^2}k_{s10}+\frac{n^2}{R_2^2}k_{s10}+\left(\beta p_{10}+\delta k_{s10}\right)\times\right.\\ &\times\left(\frac{1}{2}-\frac{\sin 2\chi}{4\chi}+\frac{\sin^2\chi}{4\chi^2}\right)\left(t_1'-t_0\right)+aq_0q_{21}\frac{A}{\left(\psi^2+\omega^2\right)^2}\times\\ &\times\left(-\psi\omega\left(e^{-\psi t_1'}\cos\omega t_1'-e^{-\psi t_0}\cos\omega t_0\right)-\right.\\ &\left.-\psi^2\left(e^{-\psi t_1'}\sin\omega t_1'-e^{-\psi t_0}\sin\omega t_0\right)\right)+aq_0q_{21}\frac{A\psi}{\psi^2+\omega^2}\times\\ &\left.\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\right]\Bigg\}w_{01}^2 \end{aligned} \tag{14}$$

$$\begin{aligned} G_2 = \Bigg\{ & \left[\frac{h_2R_2b_{112}\chi^2q_0q_{22}}{2}+\frac{\rho_2h_2\omega_1^2\omega_{01}^2q_1}{2R_2\left(1-v_2^2\right)}+b_{662}\frac{n^2}{R_2^2}q_1q_{32}\right]u_{01}^2+\\ &+\left[\frac{b_{222}n^2q_0q_{32}}{R_1^2}+\frac{\rho_2h_2\omega_1^2\omega_{01}^2q_0}{2R_2\left(1-v_2^2\right)}\right]\vartheta_{01}^2+\\ &+\left[\frac{h_2a}{2R_2}\left(b_{112}+2b_{122}+b_{222}\right)q_0q_{22}+\frac{\rho_2h_2\omega_1^2\omega_{01}^2q_0}{2R_2\left(1-v_2^2\right)}\right]w_{01}^2+\\ &+\left[\frac{2b_{122}}{aR_2^2}\chi nq_0q_{42}+\frac{b_{662}\chi n}{aR_2}q_1q_{32}\right]u_{01}w_{01}+ \end{aligned} \tag{15}$$

$$\left.-h_2 aR_2\left(b_{112}+b_{122}\right)q_0 q_{22}u_{01}\vartheta_{01}-\frac{h_2 a}{R_2}\left(b_{122}+b_{222}\right)nq_0 q_{42}\vartheta_{01}w_{01}\right\}\times$$

$$\times\left(1+\sin 2n\tilde{\theta}_1\right)+\left\{aq_0 q_{21}\left[p_{20}+\frac{\chi^2}{a^2}k_{s20}+\frac{n^2}{R_2^2}k_{s20}+\right.\right.$$

$$\left.+\left(\beta p_{20}+\delta k_{s20}\right)\frac{1}{2}-\frac{\sin 2\chi}{4\chi}+\frac{\sin^2\chi}{4\chi^2}\right]\left(t_1'-t_0\right)+$$

$$+aq_0 q_{21}\frac{A}{\left(\psi^2+\omega^2\right)^2}\left(-\psi\omega\left(e^{-\psi t_1'}\cos\omega t_1'-e^{-\psi t_0}\cos\omega t_0\right)-\right.$$

$$\left.-\psi^2\left(e^{-\psi t_1'}\sin\omega t_1'-e^{-\psi t_0}\sin\omega t_0\right)\right)+aq_0 q_{22}\frac{A\psi}{\psi^2+\omega^2}\times$$

$$\left.\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\right\}\left(1+\sin 2n\tilde{\theta}_1\right)\vartheta_{01}^2$$

where,

$$q_0=\frac{1}{2}-\frac{\sin 2\chi}{4\chi}$$

$$q_1=\frac{1}{2}+\frac{\sin 2\chi}{4\chi}$$

$$q_{2i}=\tilde{\theta}_i+\frac{1}{2n}-\frac{\cos 2n\tilde{\theta}_i}{2n}$$

$$q_{3i}=\tilde{\theta}_i+\frac{1}{2n}+\frac{\cos 2n\tilde{\theta}_i}{2n}$$

$$q_{4i}=\frac{\sin 2n\tilde{\theta}_i}{2n}$$

$$q_{5i}=1+\sin\frac{2R_i\tilde{\theta}_i}{k_i+1}$$

$$q_{6i}=1-\sin\frac{2R_i\tilde{\theta}_i}{k_i+1}$$

$$q_{7i}=\cos\frac{2R_i\tilde{\theta}_i}{k_i+1}$$

As can be seen from expressions (14) and (15), the total energy of cylindrical shells forming the retaining walls are second degree polynomials with respect to the constants $u_{01},\vartheta_{01}w_{01}$. We show them in following form:

$$G_1=\varphi_{11}u_{01}^2+\varphi_{22}\vartheta_{01}^2+\varphi_{33}w_{01}^2+\varphi_{44}u_{01}\vartheta_{01}+ +\varphi_{55}u_{01}w_{01}+\varphi_{66}\vartheta_{01}w_{01} \tag{16}$$

where,

$$\varphi_{11}=\left(\frac{h_1 R_1 b_{111}\chi^2 q_0 q_{2i}}{2}+\frac{\rho_1 h_1\omega_1^2\omega_{01}^2 q_1}{2R_1\left(1-v_1^2\right)}+b_{661}\frac{n^2}{R_1^2}q_1 q_{31}\right)\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$\varphi_{22}=\left[\frac{b_{221}n^2 q_0 q_{31}}{R_1^2}+\frac{\rho_1 h_1\omega_{01}^2 q_0}{2R_1\left(1-v_1^2\right)}\right]\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$\varphi_{33}=\left\{\frac{h_1 a}{2R_1}\left(b_{111}+2b_{121}+b_{221}\right)q_0 q_{21}+\frac{\rho_1 h_1\omega_1^2\omega_{01}^2 q_0}{2R_1\left(1-v_1^2\right)}\right\}\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]+$$

$$+aq_0 q_{21}\left[p_{10}+\frac{\chi^2}{a^2}k_{s10}+\frac{n^2}{R_2^2}k_{s10}+\right.$$

$$\left.+\left(\beta p_{10}+\delta k_{s10}\right)\frac{1}{2}-\frac{\sin 2\chi}{4\chi}+\frac{\sin^2\chi}{4\chi^2}\right]\left(t_1'-t_0\right)+$$

$$+aq_0 q_{21}\frac{A}{\left(\psi^2+\omega^2\right)^2}\left(-\psi\omega\left(e^{-\psi t_1'}\cos\omega t_1'-e^{-\psi t_0}\cos\omega t_0\right)-\right.$$

$$\left.-\psi^2\left(e^{-\psi t_1'}\cos\omega t_1'-e^{-\psi t_0}\cos\omega t_0\right)\right)+$$

$$+aq_0 q_{21}\frac{A\psi}{\psi^2+\omega^2}\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$\varphi_{44}=\left[\frac{2b_{121}}{aR_1^2}\chi nq_0 q_{41}+\frac{b_{661}\chi n}{aR_1}q_1 q_{31}\right]\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$\varphi_{55}=-h_1 aR_1\left(b_{111}+b_{121}\right)q_0 q_{21}\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$\varphi_{66}=-\frac{h_1 a}{R_1}\left(b_{121}+b_{221}\right)nq_0 q_{41}\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]$$

$$G_2=\psi_{11}u_{01}^2+\psi_{22}\vartheta_{01}^2+\psi_{33}w_{01}^2+\psi_{44}u_{01}\vartheta_{01}+ +\psi_{55}u_{01}w_{01}+\psi_{66}\vartheta_{01}w_{01} \tag{17}$$

where,

$$\psi_{11}=\left[\frac{h_2 R_2 b_{112}\chi^2 q_0 q_{22}}{2}+\frac{\rho_2 h_2\omega_1^2\omega_{01}^2 q_1}{2R_2\left(1-v_2^2\right)}+b_{662}\frac{n^2}{R_2^2}q_1 q_{32}\right]\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right)$$

$$\psi_{22}=\left[\frac{h_2 a}{2R_2}\left(b_{112}+2b_{122}+b_{222}\right)q_0 q_{22}+\frac{\rho_2 h_2\omega_1^2\omega_{01}^2 q_0}{2R_2\left(1-v_2^2\right)}\right]\times$$

$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right)+$$

$$+\left\{aq_0 q_{21}\left[p_{20}+\frac{\chi^2}{a^2}k_{s20}+\frac{n^2}{R_2^2}k_{s20}+\left(\beta p_{10}+\delta k_{s20}\right)\frac{1}{2}-\right.\right.$$

$$\left.-\frac{\sin 2\chi}{4\chi}+\frac{\sin^2\chi}{4\chi^2}\right]\left(t_1'-t_0\right)+aq_0 q_{21}\frac{A}{\left(\psi^2+\omega^2\right)^2}\times$$

$$\times\left(-\psi\omega\left(e^{-\psi t_1'}\cos\omega t_1' - e^{-\psi t_0}\cos\omega t_0\right)-\right.$$
$$\left.-\psi^2\left(e^{-\psi t_1'}\sin\omega t_1' - e^{-\psi t_0}\sin\omega t_0\right)\right)+aq_0q_{21}\frac{A\psi}{\psi^2+\omega^2}\times$$
$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\Bigg\}\left(1+\sin 2n\tilde{\theta}_1\right)$$
$$\psi_{55}=\left[\frac{2b_{122}}{aR_2^2}\chi nq_0q_{42}+\frac{b_{662}\chi n}{aR_2}q_1q_{32}\right]\times$$
$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right)$$
$$\psi_{44}=-h_2aR_2\left(b_{112}+b_{122}\right)q_0q_{22}\times$$
$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right);$$
$$\psi_{33}=\left[\frac{b_{222}n^2q_0q_{32}}{R_1^2}+\frac{\rho_2h_2\omega_1^2\omega_{01}^2q_0}{2R_2\left(1-v_2^2\right)}\right]\times$$
$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right)$$
$$\psi_{66}=-\frac{h_2a}{R_2}\left(b_{122}+b_{222}\right)nq_0q_{42}\times$$
$$\times\left[\frac{1}{2}\left(t_1'-t_0\right)+\frac{\sin 2\omega t_0-\sin 2\omega t_1'}{4\omega}\right]\left(1+\sin 2n\tilde{\theta}_1\right).$$

According to expressions (16) and (17) we can write

$$W=\left(\varphi_{11}+\psi_{11}\right)u_{01}^2+\left(\varphi_{22}+\psi_{22}\right)\vartheta_{01}^2+$$
$$+\left(\varphi_{33}+\psi_{33}\right)w_{01}^2+\left(\varphi_{44}+\psi_{44}\right)u_{01}\vartheta_{01}+ \qquad (18)$$
$$+\left(\varphi_{55}+\psi_{55}\right)u_{01}w_{01}+\left(\varphi_{66}+\psi_{66}\right)\vartheta_{01}w_{01}$$

$$W=\tilde{\varphi}_{11}u_{01}^2+\tilde{\varphi}_{22}\vartheta_{01}^2+\tilde{\varphi}_{33}w_{01}^2+\tilde{\varphi}_{44}u_{01}\vartheta_{01}+ \qquad (19)$$
$$+\tilde{\varphi}_{55}u_{01}w_{01}+\tilde{\varphi}_{66}\vartheta_{01}w_{01}$$

where,

$$\tilde{\varphi}_{11}=\varphi_{11}+\psi_{11}$$
$$\tilde{\varphi}_{22}=\varphi_{22}+\psi_{22}$$
$$\tilde{\varphi}_{33}=\varphi_{33}+\psi_{33}$$
$$\tilde{\varphi}_{44}=\varphi_{44}+\psi_{44}$$
$$\tilde{\varphi}_{55}=\varphi_{55}+\psi_{55}$$
$$\tilde{\varphi}_{66}=\varphi_{66}+\psi_{66}$$

Using expression (19), performing variation operation in the equality $\delta W=0$ and taking into account that the variations $\delta u_{01},\delta\vartheta_{01},\delta w_{01}$ are arbitrary, independent, we get a system of homogeneous linear equations with respect to the constants $u_{01},\vartheta_{01},w_{01}$

$$\begin{cases}2\tilde{\varphi}_{11}u_{01}+\tilde{\varphi}_{44}\vartheta_{01}+\tilde{\varphi}_{55}w_{01}=0\\ \tilde{\varphi}_{44}u_{01}+2\tilde{\varphi}_{22}\vartheta_{01}+\varphi_{66}w_{01}=0\\ \tilde{\varphi}_{55}u_{01}+\tilde{\varphi}_{66}\vartheta_{01}+2\varphi_{33}w_{01}=0\end{cases} \qquad (20)$$

Since the system (20) is linear homogeneous, the necessary and sufficient condition for the existence of its nonzero solution is the equality of its principle determinant to zero.

As a result, for finding natural frequencies of retaining walls formed from joint of cylindrical shells dynamically contacting with soil, we get the following frequency equation:

$$\begin{vmatrix}2\tilde{\varphi}_{11} & \tilde{\varphi}_{44} & \tilde{\varphi}_{55}\\ \tilde{\varphi}_{44} & 2\tilde{\varphi}_{22} & \tilde{\varphi}_{66}\\ \tilde{\varphi}_{55} & \tilde{\varphi}_{66} & 2\tilde{\varphi}_{33}\end{vmatrix}=0 \qquad (21)$$

We write equation (21) as follows:

$$4\tilde{\varphi}_{11}\tilde{\varphi}_{22}\tilde{\varphi}_{33}+\tilde{\varphi}_{44}\tilde{\varphi}_{55}\tilde{\varphi}_{66}-\tilde{\varphi}_{22}\tilde{\varphi}_{55}^2-\tilde{\varphi}_{33}\tilde{\varphi}_{44}^2-\tilde{\varphi}_{11}\tilde{\varphi}_{66}^2=0$$

4. CONCLUSIONS

The roots of equation (22) were calculated by the numerical method. For the parameters contained in the problem solution we take the followings:

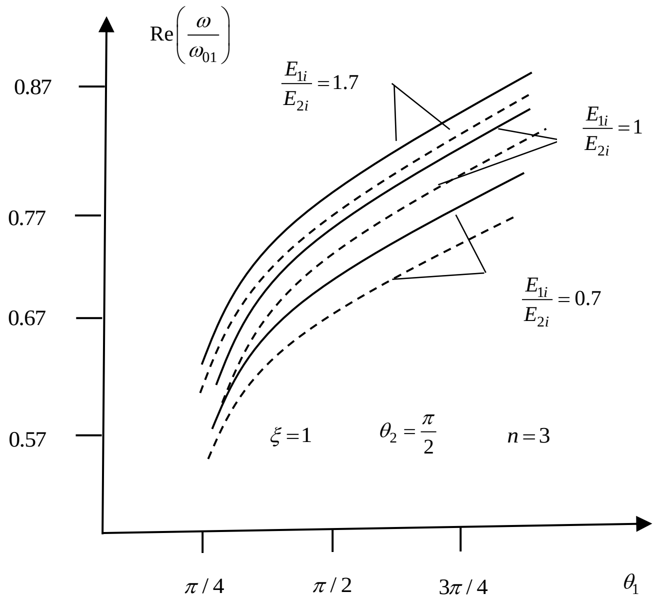

Figure 2. Dependence of frequency parameter on θ_1

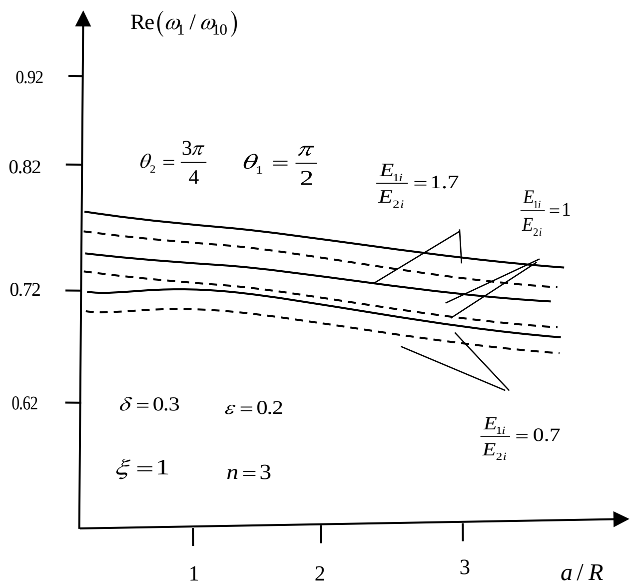

Figure 3. Dependence of frequency parameter on the ratio a/R

$p_{10} = p_{20} = 7\times10^8$ N/m^2 , $k_{si0} = 11\times10^6$ N/m^2

$\frac{a}{R_i} = 3$, $\nu_{1i} = \nu_{2i} = 0.35$, $R_i = 160$ mm

$b_{11i} = 18.3$ QPa , $b_{12i} = 2.77$ QPa

$b_{22i} = 25.2$ QPa , $b_{66i} = 3.5$ QPa

$\rho_i = 1850$ kg/m^3 , $\chi = 1$, $n = 8$

$h_i = 0.45$ mm , $\Psi = 0.05$, $A = 0.0897$

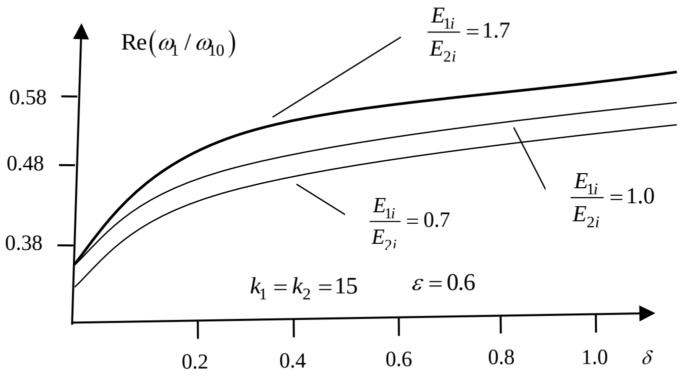

Figure 4. Dependence of frequency parameter on homogeneity parameter δ

The results of calculation were given in Figure 2 in the form of dependence of frequency parameter on θ_1, in Figure 3 on the ratio of a/R, in Figure 4 on homogeneity parameter δ. As can be seen from figure 2, as the value of the angle θ_1 increase, the value of the frequency parameter also increases.

As the length of the shell increases, vice versa the value of the frequency parameter decreases (Figure 3). Furthermore, the value of frequency parameter increases due to enhancement of orthotropic property of the shell material. Broken lines in Figures 2 and 3 correspond to the case of elastic plates, viscous-elastic soil. As can be seen from both figures, account of heterogeneity of the soil material causes to increase the value of frequency parameters of vibrations of the retaining wall compared with the case when the soil material is homogeneous viscous-elastic.

Unlike elastic soil, in the case of viscous-elastic inhomogeneous soil, the value of frequency parameter is a complete complex number. The real part of this complex number is the frequencies of vibrations of the retaining wall, the coefficient of the imaginary part characterizes damping of vibrations with respect to time. Figure 4 shows that as the value of the inhomogeneity parameter α of soil increases, the value of frequency parameter of vibrations of the retaining wall also increases. This is explained by the fact that the rigidity of the soil material increases due to its heterogeneity.

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BIOGRAPHY

Dilgam Seyfeddin Qaniyev was born in Goyler, Shamakhi, Azerbaijan, in 1981. He graduated from Faculty of Transportation, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan in 2002. In the same year, he received his M.Sc. degree from the same university. During 2004-2007, he got his Ph.D. education at the same university. In 2004, he defended his Ph.D. thesis, earning his Ph.D. degree in Technical Sciences. In 2005, he started his career and worked as a leading engineer in several projects of Azerbaijan. He also worked as a bridge engineer in Akin Project company. He was awarded with "Tereqqi" medal in 2018. He was elected as a member of International Academy of Transport in 2017. He was awarded with the Jubilee Medal of Azerbaijan "100 Years of Azerbaijan Automobile Roads (1918-2018)" in 2018. Currently, he works as a Chief Engineer in Institute of Azerbaijan State Agency of Automobile Roads, Baku, Azerbaijan. He is also a lecturer in Azerbaijan University of Architecture and Construction.