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FREE VIBRATION SYSTEM: RETAINING WALL-GROUND

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Abstract- Wide use of thin walled constructions or structural elements in mechanical engineering, in transmission systems, in the field of construction does urgent calculation of their dynamical strength characteristics and choice of their optimal variants. The bases of retaining walls are composed of open profile joined cylindrical shells, and their forms are accepted as optimal. An empty system is obtained by replacing massive retaining walls by thin-walled shells. To provide stability of this system, empty parts are filled with soil and this results in savings of concrete. Taking into account that the Republic of Azerbaijan is in the active seismic zone, calculation of natural vibrations and finding of resonance frequencies of such constructions of a great practical importance.

Keywords: Orthotropic Shell, Variation Principle, Free Oscillation, Visco-Elastic Soil.

1. INTRODUCTION

In this paper, based on the Ostrogradskiy-Hamilton principle, we study natural vibrations of retaining walls composed of two orthotropic cylindrical shells under dynamical interaction with visco-elastic soil, find resonance frequencies of the wall under consideration and construct typical dependence curves.

Note that analysis of retaining walls composed of two isotropic cylindrical shells was carried out for statical cases in [1-3]. Statical deformations of retaining walls consisting of two isotropic shells were studied in the paper [1]. When solving the problems, the method represented in the paper [4] is used. Analysis of retaining walls consisting of three isotropic cylindrical shells under plain deformation was given in the paper [2]. The problem is reduced to the solution of ordinary differential equations, and their solutions are constructed. The paper [3] was devoted to design method of retaining walls consisting of isotropic cylindrical shells with regard to the soil behavior in compression and shear. Calculations were conducted on the base of moment theory of cylindrical shells. In the scientific works [5-8], considering different variants of combination of open profile cylindrical shells, differential equations of moment theory were constructed under different mating conditions.

In paper [9] appropriate to select supporting in the form of formations consisting of soil-filled cylindrical panels when bridges over the mountain rivers. These supporting should be constructed such that one of the control lines would stand against the river flow. Under the action of the river flow there happens a vibrational process in this construction. The present paper was devoted to the study of natural vibrations of such supports.

Pased on the Hamilton-Ostrogradsky variational principle, we study forced vibrations of a vertical retaining wall consisting of three orthotropic cylindric panels contacting with viscous-elastic, heterogeneous soil, obtain analytic expressions to calculate the displacements of the points of cylindrical panels and structure characteristical curves. Account of heterogeneity of soil is performed by accepting its rigidity coefficients as a function of coordinate was given in the paper [10]. . It is assumed that the Poisson ratio is constant.

2. PROBLEM STATEMENT

We write potential and kinetic energies of cylindrical retaining walls [11,12]:

$$\begin{aligned}\Im_i=\frac{h_iR_i}{2}\iint_{s_i}\Bigg\{&b_{11i}\left(\frac{\partial u_i}{\partial x}\right)^2-2\left(b_{11i}+b_{12i}\right)\frac{w_i}{R}\frac{\partial u_i}{\partial x}+\\&+\frac{w_i^2}{R^2}\left(b_{11i}+2b_{12i}+b_{22i}\right)+\frac{b_{22i}}{R^2}\left(\frac{\partial \vartheta_i}{\partial \theta}\right)^2-\\&-2\left(b_{12i}+b_{22i}\right)\frac{w_i}{R^2}\frac{\partial \vartheta_i}{\partial \theta}+2b_{12i}\frac{1}{R^2}\frac{\partial u_i}{\partial x}\frac{\partial \vartheta_i}{\partial \theta}+\\&+b_{66i}\frac{1}{R^2}\left(\frac{\partial u_i}{\partial \theta}\right)^2+b_{66i}\left(\frac{\partial \vartheta_i}{\partial x}\right)^2+b_{66i}\frac{1}{R}\frac{\partial \vartheta_i}{\partial x}\frac{\partial u_i}{\partial \theta}\Bigg\}dxd\theta\end{aligned} \quad (1)$$

$$K_i=\frac{E_ih_i}{2R_i^2(1-\nu_i^2)}\times\iint_{s_i}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2\right]dxdy$$

where, i=1 corresponds to the first cylindrical shell in retaining walls, i=2 corresponds to the second cylindrical shell in retaining walls (Figure 1); u_i, ϑ_i, w_i are displacements of shells, R_i, h_i are the radius and thickness of cylindrical shells, respectively, b_{11}, b_{22}, b_{12}, b_{66} are the main module of elasticity of orthotropic materials of cylindrical shells and are expressed by the module of elasticity in coordinate directions E_{1i}, E_{2i} by the Poisson ratio ν_{1i}, ν_{2i} as follows: $b_{11i}=\dfrac{E_{1i}}{1-\nu_{1i}\nu_{2i}}$;

$b_{22i} = \dfrac{E_{2i}}{1-\nu_{1i}\nu_{2i}}$; $b_{12i} = \dfrac{\nu_{2i}E_{1i}}{1-\nu_{1i}\nu_{2i}} = \dfrac{\nu_{1i}E_{2i}}{1-\nu_{1i}\nu_{2i}}$, and s_i are the surfaces of cylindrical shells appearing in retaining walls.

The influence of the soil on cylindrical shells is replaced by the external forces q_{xi}, q_{yi}, q_{zi}. The work done by these forces in displacements of shells is in the form:

$$A_i = -\int_0^{x_1}\int_0^{2\pi}\left(q_{xi}u_i + q_{yi}\vartheta_i + q_{zi}w_i\right)dxdy \tag{2}$$

As a result, total energy of retaining walls is:

$$\Pi = \sum_{i=1}^{2}\left(\text{Э}_i + K_i + A_i\right) \tag{3}$$

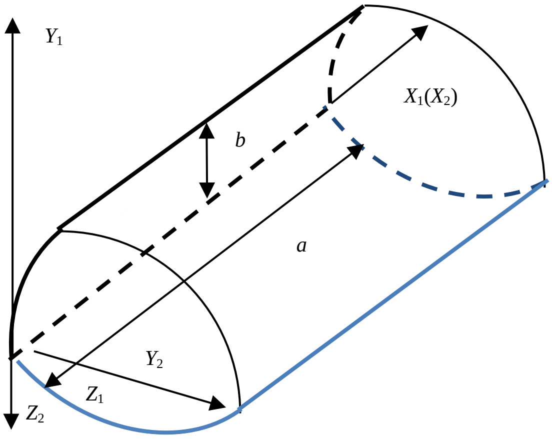

Figure 1. The scheme of a retaining wall consisting of orthotropic cylindrical shells

To expressions (1) and (2) we add contact and boundary conditions. We will assume that cylindrical shells are elastically connected between themselves, i.e. in the contact

$$w_1(x)\big|_{y_1=0} = \vartheta_2(x)\big|_{y_2=0}\ ;\ \vartheta_1(x)\big|_{y_1=0} = w_2(x)\big|_{y_2=0}$$
$$u_1(x)\big|_{y_1=0} = u_2(x)\big|_{y_2=0}\ ;\ \left.\frac{\partial w_1(x)}{\partial x}\right|_{y_1=0} = \left.\frac{\partial \vartheta_2(x)}{\partial x}\right|_{y_2=0} \tag{4}$$

are fulfilled.

It is assumed that cylindrical shells are located on ideal diaphragms as on hinges along the lines $x=0$ and $x=a$. In this case, the boundary conditions have the form:

$$u=0,\ w=0,\ T_1=0,\ M_1=0 \tag{5}$$

where, T_1, M_1 are the force and moments in the cross-section of the cylindrical shell.

Using the Ostrogradsky-Hamilton stationarity principle, one can get frequency equation of vibrations of retaining walls consisting of open profile cylindrical shells

$$\delta W = 0 \tag{6}$$

where, $W = \int_{t_0}^{t_1} \Pi dt$ is the Hamilton action.

Fulfilling the variation operation in the equality $\delta W = 0$ and taking into account independence and arbitrariness of the variations $\delta u = 0, \delta\vartheta, \delta w$ we get a frequency equation of vibrations of retaining walls consisting of open profile soil-contacting cylindrical shells. Thus, construction of a frequency equation of vibrations of retaining walls consisting of connection of two soil-contacting cylindrical shells, is reduced to joint integration of total energy of the system (3) under contact (4) and boundary conditions (5).

The forces from the viscoelastic soil on cylindrical shells q_{xi}, q_{yi}, q_{zi} contained in (1) will be represented in the form:

$$q_{xi} = q_{yi} = 0\ ;\ q_{z1} = k_1 w_1 - \int_0^t \Gamma(t-\tau)w_1(\tau)d\tau$$
$$q_{z2} = k_2 w_2 - k_s\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) - \int_0^t \Gamma(t-\tau)w_2(\tau)d\tau \tag{7}$$

where, k_1, k_2, k_s are factors of rigidity of the soil in compression and share.

3. PROBLEM SOLUTION

We will look for displacements of cylindrical shells in the form:

$$u_i = u_{0i}\cos\chi\xi\left(\cos n\theta_i + \sin n\theta_i\right)\sin\omega_1 t_1$$
$$\vartheta_i = \vartheta_{0i}\sin\chi\xi\left(\cos n\theta_i + \sin n\theta_i\right)\sin\omega_1 t_1 \tag{8}$$
$$w_i = w_{0i}\sin\chi\xi\left(\cos n\theta_i + \sin n\theta_i\right)\sin\omega_1 t_1$$

where, $u_{0i}, \vartheta_{0i}, w_{0i}$ are unknown constants, $\xi = \dfrac{x}{a}$, $t_1 = \omega_0 t$, $\chi, n = 2k+1$ are wave numbers in the direction of generator and circular direction of cylindrical shells, $\theta_i = \dfrac{y_i}{R}$, $0 \le \theta_i \le \dfrac{3\pi}{4}$. Subject to these conditions on the boundaries $\theta_i = \dfrac{3\pi}{4}$ of cylindrical shells the hinge support conditions are fulfilled.

Substituting the solutions (8) in (3), taking into account boundary conditions (4) and expression (7), expressing the constants $u_{02}, \vartheta_{02}, w_{02}$ by the constants $u_{01}, \vartheta_{01}, w_{01}$, for total energy (3) we get a second order polynomial with respect to the constants $u_{01}, \vartheta_{01}, w_{01}$:

$$\Pi = \breve{\varphi}_{11}u_{01}^2 + \breve{\varphi}_{22}\vartheta_{01}^2 + \breve{\varphi}_{33}w_{01}^2 + \breve{\varphi}_{44}u_{01}\vartheta_{01} +$$
$$+\breve{\varphi}_{55}u_{01}w_{01} + \breve{\varphi}_{66}\vartheta_{01}w_{01}$$

Because of the complexity of the coefficients $\breve{\varphi}_{11}, \breve{\varphi}_{22}, \breve{\varphi}_{33}, \breve{\varphi}_{44}, \breve{\varphi}_{55}, \breve{\varphi}_{66}$ we don't give here the expression. Performing variation operations in the expression Π with respect to independent constants $u_{01}, \vartheta_{01}, w_{01}$ and equating the coefficients of independent variations to zero, we get the following system of algebraic homogeneous equations:

$$\begin{cases} 2\breve{\varphi}_{11}u_{01} + \breve{\varphi}_{44}\vartheta_{01} + \breve{\varphi}_{55}w_{01} = 0 \\ \breve{\varphi}_{44}u_{01} + 2\breve{\varphi}_{22}\vartheta_{01} + \breve{\varphi}_{66}w_{01} = 0 \\ \breve{\varphi}_{55}u_{01} + \breve{\varphi}_{66}\vartheta_{01} + 2\breve{\varphi}_{33}w_{01} = 0 \end{cases} \tag{9}$$

As the system (9) is linear and homogeneous, then for the existence of its nontrivial solutions, equality of its principal determinant to zero is a necessary and sufficient condition. As a result, we get the frequency equation:

$$\begin{vmatrix} 2\breve{\varphi}_{11} & \breve{\varphi}_{44} & \breve{\varphi}_{55} \\ \breve{\varphi}_{44} & 2\breve{\varphi}_{22} & \breve{\varphi}_{66} \\ \breve{\varphi}_{55} & \breve{\varphi}_{66} & 2\breve{\varphi}_{33} \end{vmatrix} = 0 \quad (10)$$

We write equation (10) in the form:

$$4\breve{\varphi}_{11}\breve{\varphi}_{22}\breve{\varphi}_{33} + \breve{\varphi}_{44}\breve{\varphi}_{55}\breve{\varphi}_{66} - \breve{\varphi}_{55}^2\breve{\varphi}_{22} - \breve{\varphi}_{66}^2\breve{\varphi}_{11} - \breve{\varphi}_{44}^2\breve{\varphi}_{33} = 0 \quad (11)$$

4. NUMERICAL RESULTS

Equation (10) was realized numerically. The parameters in the solution of the problem are:

$k_1 = k_2 = 7\times10^8$ N/m^2, $k_s = 11\times10^6$ N/m^2, $\frac{a}{R} = 3$, $\frac{h}{R} = \frac{1}{6}$, $\nu_1 = \nu_2 = 0.35$

The results of calculations are given in Figure 2 in the form of dependence of the frequency parameter on θ_1, in Figure 3, of frequency parameter on the ratio a/R. From Figure 2 it is seen that with an increase in the angle θ_1 the value of the frequency parameter increases. With an increase in the length of the shells, the value of the frequency parameter decreases. Furthermore, with increase in orthotropic in the shell material, the values of frequency parameter increase.

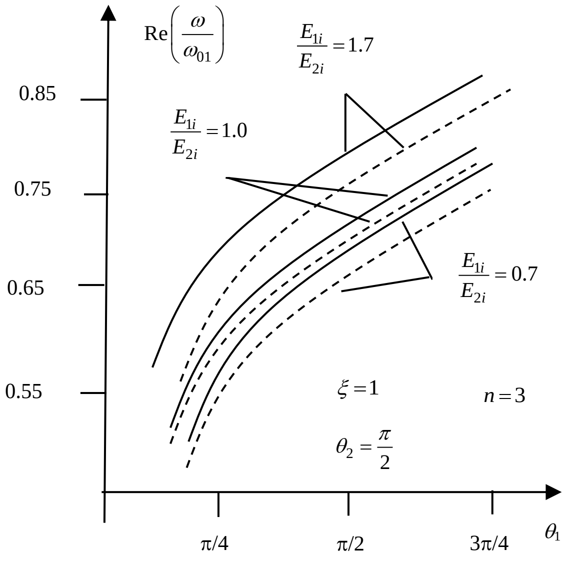

Figure 2. Dependence of the frequency parameter on θ_1

In Figures 2 and 3, the dashed lines denote the case of viscoelastic soil. It is seen that account of viscosity of the soil material, reduces to a reduction in natural vibrations of the studied retaining wall in comparison with frequency of vibration of the same retaining wall, when the soil material is elastic. Unlike elastic soil, in the case of viscoelastic soil, the frequency of vibrations is a complete complex number. The real part of this complex number corresponds to real frequency of vibrations of a retaining wall, and the imaginary part characterizes the damping of the vibrations of the retaining wall.

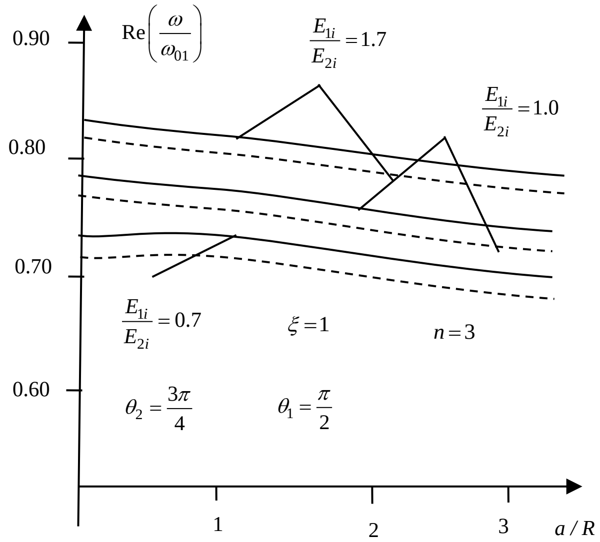

Figure 3. Dependence of the frequency parameter on a/R

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Fuad Seyfeddin Latifov was born in Ismayilly, Azerbaijan, in 1955. He graduated from Faculty of Mechanics and Mathematics, Azerbaijan State University, Baku, Azerbaijan in 1977. In 1983, he received his M.Sc. degree in Physics-Mathematics from Saint Petersburg State University, Saint Petersburg, Russia. In 2003, he got the Ph.D. degree in Physics-Mathematics. He is a Professor and the Chief of Department of Higher Mathematics in Azerbaijan University of Architecture and Construction Baku, Azerbaijan. He has written more than 90 scientific articles and 11 monographs. He is the coauthor of an encyclopedia on mathematics.

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