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DEVELOPMENT OF PULSE METHOD IN TRANSITION PROCESSES OF LINEAR ELECTRIC CIRCUITS

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Abstract- Based on operator calculus of generalized functions the general operational method of solution in distributions has been developed for differential and integro-differential equations of state and analysis of transient processes in linear electric pulse circuits with concentrated elements. The applicability of this method for the linear electric pulse circuits with concentrated parameters at different sources of discontinuous actions is shown.

Keywords: Operational Calculus, Convolution of Generalized Functions, Function of Singularities, Regular Composition of Generalized Function, Pulse Circuits, Transient Process.

1. INTRODUCTION

If there are pulse sources, as well at some commutations, voltages and currents are not limited in magnitude. Reducing of the time of occurrence of real perturbations to infinitesimals and the transition to an idealized, instantly arising perturbation lead to the exclusion of all accompanying phenomena in the construction of models of transient processes [1, 2, 3].

The situation changed only after the construction of theory of generalized functions [4]. The main provisions of the well-known method of drawing up and solving of equations of electric circuits within the apparatus of continuous functions were subjected to critical analysis and revision from the positions opened by generalized functions. The generalized Kirchhoff laws for currents (KLC) and voltages (KLV) were formulated, which determine the relationships between the instantaneous voltages and currents directly before and after the moment of disturbance of discontinuity of these values, from which the equations of instantaneous values are obtained as effects of equations in generalized functions.

In paper [5] methods of research of transient processes in linear pulse circuits with concentrated parameters are considered on the basis of use of two modifications of one-sided Laplace transform $L_+\left[f\{t\}\right]=F_+(p)$ and $L_-\left[f\{t\}\right]=F_-(p)$ on originals being generalized functions of type $f\{t\}=f_-(t)1(-t)+f_+(t)1(t)+\sum_{k=o}^{q}Ck\delta^{(k)}(t)$, where

The $f_-(t), f_+(t)$ are smooth functions, having continuous derivatives of any order; q is arbitrary integral positive number; $\delta(t)$ is delta function; $1(t)$ is Heaviside unit function; L_+ and L_- are Laplace transforms with lower limit 0_+ and 0_- respectively, at the same time it is accepted that $L_+\left[\delta(t)\right]=0$ and $L_-\left[\delta(t)\right]=1$. Image reversal $F_-(p)$ coincides with image reversal $F_+(p)$, at the same time $L_-^{-1}\left[F_-(p)\right]=L_+^{-1}\left[F_+(p)\right]$, which $F(t)$, $L(t)$ define only a regular composition of the solution and for the finding of coefficients C_k of functions of singularities one has to use a limit value of image $F_-(p)$ at $|p|\to\infty$, $Cq=\lim\frac{F_-(p)}{p_q}$, $C_{q-1}=\lim\left[\frac{F_-(p)}{p^{q-1}}-Cq.p\right]$, etc.

Peculiar method of construction of operator calculus was proposed by Y. Mikusinsky [6]. In Y. Mikusinsky's operator calculus, the ring of "operators" of f/g type with convolution as a multiplication operation is considered, where f and g are functions on [0, ∞), having no more than finite number of break points. Despite the formal similarity of the Laplace transform and Mikusinsky's operator calculus, they are not equivalent. The Laplace transform method restricts the application limits of the operator calculus to the class of functions $f(t)$ for which the Laplace transform converges. If generalized functions as regards Schwarz distribution [12] with carriers on semiaxis [0, ∞), for which the addition function is defined as a convolution, are understood by f and g, then we obtain K'_+ ring without zero divisor. In the K'_+ ring set of all relations f/g coincides with set D'_+ of all generalized functions with the carrier bounded from the left and with these relations one can operate according to ordinary rules of algebra.

The generalized functions $f/g\in D'_+$ may be called as operators, and the above-mentioned algebra on them – operator calculus for generalized functions. First results in this direction are obtained by Schwarz [7]. Application of this theory to the solution of some equation classes, containing generalized functions, is considered in [8,9].

The substantiation of operator method of solution of linear ordinary differential equations and systems with constant coefficients, the right parts of which are generalized functions with the carrier bounded from the left, is given in paper [10], where the solution presentation in the form of generalized function from K'_+ for both the equations and systems themselves, and initial value problem (Cauchy problem) is obtained. Differential equations with pulse actions were studied in several papers (see, for example, [11, 14]), papers [5, 15-21] are dedicated to pulse electric circuits.

The paper objective is to show that the application of the operator calculus in the space of generalized functions gives a solution of distribution for the wide class of, generally speaking, differential and integra-differential equations non-transformed by Laplace, describing the transient processes in the linear electric pulse circuits with concentrated parameters, being time-constant or time-variable.

2. OPERATOR CALCULUS IN THE SPACE OF GENERALIZED FUNCTIONS

If K' is a set of all generalized functions $f\{t\}$, then the set of all generalized functions is designated through K'_+, carriers (i.e. set of points t, in which $f\{t\} \neq 0$) of which are located on the $[0, \infty)$ semiaxis, and through D'_+ is set of all generalized functions with the carrier bounded from the left. Each of sets K'_+ and D'_+ is linear subspace of K' space. By convolution $f*g$ of two generalized functions f and g is understood the generalized function, determined on any primary function $\varphi(t)$ by the following equation:

$$(f*g,\varphi)=(g(t),(f(\xi),\varphi(t+\xi))) \tag{1}$$

If in the K'_+ and D'_+ spaces along with the addition operation the multiplication operation as convolution (1) is determined, then K'_+ and D'_+ will be commutative rings without zero divisors. Generalized function u, satisfying the equation $f=u*g$, where f and g belong to K'_+, is designated through f/g. Set of all such generalized functions $u=f/g$ coincides with D'_+. These functions are named *operators*. It is proved in [34] that the D'_+ ring is isomorphic (i.e., one-to-one correspondence) to subring of M field of Mikusinsky's operators. Thus, the field of Mikusinsky's operators is wider than D'_+. Let $\Phi_\lambda\{t\}=\dfrac{t_+^{\lambda-1}}{\Gamma(\lambda)}$ is generalized function from [4] (the so-called λ-order Heaviside function), where $\Gamma(.)$ is gamma function and function t_+^λ is equal to t^λ, at $t > 0$ and 0 at $t \leq 0$, $\Phi_1=\eta\{t\}$ is λ=1 order Heaviside function, $\Phi_0(t)=\delta(t)$. At any complex λ the generalized function $\Phi_\lambda\{t\}$ belongs to K'_+. On the basis of properties of convolution of two generalized functions from D'_+, for any generalized function $g\{t\}\in D'_+$ the following convolution is defined:

$$g_\lambda\{t\}=g\{t\}*\Phi_\lambda\{t\} \tag{2}$$

Along with the Heaviside function $\eta\{t\}=\Phi_1\{t\}$, we will also use the so-called *Heaviside jump* $\eta_\lambda\{t\}=\begin{cases}1\ ,\ t>0\\ 0\ ,\ t<\lambda'\end{cases}$ and Dirac pulse $\delta_\lambda(t)=\delta(t-\lambda)=D\eta_\lambda\{t\}$. Using operators $s^\lambda=\delta^{(\lambda)}(t)$ and $l^\lambda=\Phi_\lambda\{t\}$, $sl=\delta(t)$, the differentiation formula of order λ of generalized function $g\{t\}\in D'_+$ one can write in the form:

$$D^\lambda g\{t\}=s^\lambda g\{t\}=g\{t\}*\delta^{(\lambda)}(t) \tag{3}$$

and formula of integration of order λ is in the form:

$$D^{-\lambda}g\{t\}=l^\lambda g\{t\}=\Phi_\lambda\{t\}*g\{t\} \tag{4}$$

Differentiation of product of smooth function $\omega(t)$ and generalized function $g\{t\}\in D'_+$ is performed according to:

$$D(\omega(t)f)=\frac{d\omega(t)}{dt}.f+\omega(t)Df \tag{5}$$

being an analog of Leibniz formula for the differentiation of product of two ordinary functions. Having taken into account the relation $\eta\{t\}*\delta(t)=\eta\{t\}$ and filtering feature of the gamma function: $\omega(t)\delta(t)=\omega(0)\delta(t)$, from (5) at $f\{t\}=\eta\{t\}$. Let's we'll obtain Equation (6):

$$s(\omega(t).\eta\{t\}=\omega'(t).\eta\{t\}+\omega'(0)\delta(t) \tag{6}$$

3. SOLUTION IN DISTRIBUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

Let's consider nonhomogeneous system of equations

$$x'\{t\}-Ax\{t\}=f\{t\},\ t>0 \tag{7}$$

where, $f\{t\}=\left(f_0\{t\},f_1\{t\},\dots,f_{n-1}\{t\}\right)$ is generalized vector-function (column) with components $f_j\{t\}\in D'_+\ (j=0,1,\dots,n-1)$ and A is matrix of constant coefficients $a_{kj}\,(k,j=0,1,\dots,n-1)$. Let's put the following algebraic equation in operator area in correspondence with the system (7)

$$(E_s-A)x\{t\}=f(t) \tag{8}$$

where, s is operator of differentiation in D'_+; E is unity matrix. Equation (8) is a result of application of the differentiation Equation (6) to (7). Solution $x\{t\}=\left(x_0\{t\},x_1\{t\},\dots,x_{n-1}\{t\}\right)$ of system (8) is presented as $x\{t\}=(E_s-A)^{-1}f\{t\}$, hence scalar Equations (9):

$$x^i\{t\}=\sum_{j=0}^{n-1}\frac{Q_{n-1}^{(i,j)}(s)}{P_n(s)}f_j\{t\}\ ,\ (i=0,1,...,n-1) \tag{9}$$

where, $P_n(s)$ is polynom of power n from s, equal to determinant $|E_s-A|$, and $Q_{n-1}{}^{(i,j)}(s)$ is polynom of power n-1 from s, equal to coefficient at $f_j\{t\}$ in determinant obtained from the determinant $|E_s-A|$, replacing its $(i+1)$ column by $f\{t\}$ column.

In this case transfer functions $W^{(ij)}(s)=\dfrac{Q_{n-1}^{(i,j)}(s)}{P_n s}$ are proper rational fractions. Any rational fraction with real coefficients:

$$F(s)=\frac{Q_m(s)}{P_n(s)}=\frac{b_0 s^m+b_1 s^{m+1}+...+b_n}{s^n+a_1 s^{n-1}+...+a_n} \quad (10)$$

is presented in the form sum:

$F(s)=\sum_{k=0}^{m-n} C_k \cdot s^k+\dfrac{Q_{n-1}(s)}{P_n(s)}$, where C_k is real numbers and $Q_{n-1}(s)$ is polynom of power n-1 from s with real coefficients. Realization of rational fraction $F(s)$ with real coefficients from operator s of differentiation in D'_+ is presented by real generalized function from K'_+ of form [10]

$$w(t)\cdot\eta\{t\}+\sum_{k=0}^{m-n} C_k\delta^{(k)}(t) \quad (11)$$

where, $\omega(t)$ is smooth function equal to weighted sum of exponential and trigonometric functions with weighting coefficients, being polynoms from t with real coefficients determined from formulas (11). At $m < n$ the sum on k in (12) is absent. Based on (11), the expression (9) for components $x_i\{t\}$ of solution $x\{t\}$ of system (7) will be written as:

$$x_i\{t\}=\sum_{j=0}^{n-1}\omega^{(i,j)}(t)(\eta\{t\}*f_i\{t\}),\ (i=0,1,\ldots,n-1) \quad (12)$$

where, $\omega^{(i,j)}(t)$ is smooth functions representing the sum of coefficients at function $\eta\{t\}$ in realization of partial fractions, being a part of fractions $Q_{n-1}^{(i,j)}(s)/P_n(s)$ from (9), the multiplication operation in the space D'_+ is defined as the convolution $f*g$, and multiplication by $\omega^{(i,j)}(t)$ is determined according to rule of multiplication of generalized function by infinitely differentiable function.

If the right part of the system (7) is an ordinary vector-function $f(t)=(f_0(t), f_1(t),\ldots, f_{n-1}(t))$, where $f_i(t)(i=0,1,\ldots,n-1)$ are functions, measurable and majorized by functions summable on [0, ∞), then there is a unique solution $x(t)=(x_0(t), x_1(t),\ldots,x_{n-1}(t))$ [22], determined at $t \geq 0$ and complying with the system at $t > 0$ in common (classic) sense:

$$x'(t)-Ax(t)=f(t) \quad (13)$$

and initial condition $x(0)=(x_0°, x_1°, \ldots, x_{n-1}°)$, where $x_i°(i=0,1,\ldots,n-1)$ are desired numbers. Let's designate through $f\{t\}$ a vector-valued functional $f\{t\}=(f_0\{t\}, f_1\{t\},\ldots, f_{n-1}\{t\})$, where $f_i\{t\}(i=0,1,\ldots,n-1)$ are regular functionals from K'_+ type functions, equal to 0 at $t < 0$ and equal to $f_i\{t\}$ at $t > 0$.

Let's construct an auxiliary system of differential equations in generalized functions $x'\{t\}-A_x(t)=f(t)+x(0)\delta(t)$, being a system of form (7) with the right part $f(t)+x(0)\delta(t)$ from the K'_+. According to Equation (9), the solution $x\{t\}=(x_0\{t\}, x_1\{t\},\ldots,x_{n-1}\{t\})$ of Cauchy problem for the Equation (13) will be written as:

$$x_i\{t\}=\sum_{i=0}^{n-1}\frac{Q_{n-1}^{(i,j)}}{P_n(s)}(fj(t)+x_j(0)\delta(t)),\ (i=0,1,\ldots,n-1) \quad (14)$$

Considering the formula $\eta\{t\}*\delta(t)$, (14) can be written in the form:

$$x_i\{t\}=\sum_{i=0}^{n-1}\omega^{(i,j)}\lfloor\eta\{t\}*(f_i\{t\}+x_j(0))\rfloor \quad (15)$$

where, $\omega^{(i,j)}$ is smooth functions from (12).

4. SOLUTION IN DISTRIBUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

Let's designate through $D'(t_0)$ the space of generalized functions f, so that $f(t)=0$ at $t < t_0$, i.e. on each primary function $\varphi(t)$ with $\varphi(t)$=0 at $t \geq 0$, value $(f,\varphi)=0$. Let's consider a system of linear differential equations:

$$Dx=A(t).x+f\{t\},\ t>0 \quad (16)$$

where, n×n is assumed smooth for matrix $A(t)$, $x(t)=(x_1(t),\ldots,x_n(t)^{tr})$ and $f\{t\}\in D't_0\ (i=1,\ldots,n)$, tr is transposition sign. Let's designate through $Y(.)$ is n×n matrix, being the solution of Cauchy problem:

$$Y'=-YA(t),\ Y(t_0)=E \quad (17)$$

where, E is unity n×n matrix; and $t_0 \geq 0$.

From existence theorem and uniqueness [23] of solution $x(t)$ of the system of equations

$$x(t)=A(t)x,\ t>0 \quad (18)$$

with matrix $A(t)=\|a_{ij}(t)\|$, $(i,j=1,\ldots,n)$, definite and continuous on (0, ∞), the $x(t)$ solution is definitely determined at desired "initial" value $x(t_0)=x°$.

Fundamental matrix $x(t)$, composed from linear-independent solutions of system (18), corresponding to various vectors $x^{0(1)}, x^{0(2)},\ldots,x^{0(n)}$ of values x(t0) with nonsingular matrix $X_0=(x^{0(1)}, x^{0(2)},\ldots,x^{0(n)})$, $|X_0|\neq 0$, is called [24] *integral matrix*, and at $X_0=E$ is normalized integral matrix $X(t)$ or matrix $\Omega_{t_0}^t(A)$ or just $\Omega_{t_0}^t$. Determinant of any integral matrix $X(t)$, according to Jacobi identity, is represented in the form $|X(t)|=c.\exp\left\{\int_{t_0}^t spAdt\right\}$, where c is constant matrix, and

$spA = a_{11} + a_{22} + \ldots + a_{nn}$ is spur of matrix *A*. For the normalized integral matrix *c*=*E*. Hence it follows that *Y*(*t*) at any *t* is nonsingular matrix.

Integral matrix $X(t) = \Omega_{t_0}^{t}(A)$ is defined by convergence method using recursion formula $X_k(t) = \int_{t_0}^{t} A(t)X_{k-1}(t)$, $k = 1,2...,X_0(t) = E$ and is represented in the form of absolutely and uniformly convergent series on $(a,b) = (0,\omega)$

$$X(t) = E + \int_{t_0}^{t} A(\xi)d\xi + \int_{t_0}^{t} A(\xi)(\int_{t_0}^{t} A(\xi_1)d\xi_1)d\xi + ... \quad (19)$$

matrix $\Omega_{t_0}^{t}$ can be calculated approximately in the following manner [24]. Let's divide the main interval (t_0,t) into n parts, introducing passing points $t_1,\ t_2,\ \ldots,\ t_{n-1}$ and assume $\Delta t_k = t_k - t_{k-1}(k = 1,2,\ldots,n;\ t_0 = t)$.

Based on the property of matrix $\Omega_{t0}^{t} = \Omega_{t_0}^{t_1} \cdot \Omega_{t_0}^{t_1}(t_0, t_1, t \in (a,b))$ we will obtain $\Omega_{t_0}^{t} = \Omega_{tn-1}^{t} ... \Omega_{t_1}^{t_2} \cdot \Omega_{t_0}^{t_1}$. Let's choice passing point $\tau_k\ (k = 1,2,\ldots n)$. Then, assuming Δt as first order small value (designating them by asterisk (*)), when calculating $\Omega_{tk-1}^{t_k}$ with accuracy to second order small values one can accept $A(t) \approx \text{const} = A(\tau_k)$ (because of continuity of function *A*(*t*) on (*a*,*b*)).

Then $\Omega_{tk-1}^{tk} = e^{A(\tau_k)\cdot \Delta t_k} + (**) = E + A(\tau_k)\cdot \Delta t_k + (**)$ (by property of integral matrix for linear system of differential equations with constant coefficients), where symbol (**) designates a sum of members, starting from the second order infinitesimal. Hence, we find:

$$\Omega_{t0}^{t} = e^{A(tn)\cdot \Delta tn} ... e^{A(t_2)\Delta t_2} \cdot e^{A(t_1)\Delta t_1} + (*) \quad (20)$$

$$\Omega_{t_0}^{t} = [E + A(t_n)\Delta t_n]...[E + A(t_2)\Delta t_2][E + A(t_1)\Delta t_1] + (*) \quad (21)$$

The equation (12) can be used for the approximate calculation of matrix at quite small Δt_k. By means of Leibniz rule (7) one can establish that for any vector-distribution $g \in D'_{t_0}$ the generalized function $D(Y_x)$ coincides with the distribution $Y(D_x – A(t)x)$. Therefore, any solution of system (16) simultaneously satisfies to the system:

$$D(Y_x) = Y_f \quad (22)$$

Since at any *x* the matrix *Y*(.) is nonsingular, then it is true also converse proposition. In turn, the system (22) is equivalent to system of algebraic equations $Y_x = c + \eta * Y_f$, where *c* is arbitrary scalar vector. Hence follows the formula of general solution of the system (16)

$$X = Y^{-1}.c + Y^{-1}(\eta * Y_f) \quad (23)$$

where, Y^{-1} is matrix inverse for *Y*. Analysis of the formula (14) shows that the system (16) in class D'_{t_0} has a single solution described with generalized function $x_f = Y - 1(\eta * Y_f)$, called the system (16) response to perturbation *f* [13]. By solution of the system (16) "with initial condition $x°=x(t_0)$" one can understand this system response to perturbation $x°\delta_{t0}+f$. This response by principle of superposition is a sum of responses to components $x°\delta_{t0}$ and *f* and, therefore, is described with generalized function $x=Y^{-1}(\eta*Yx°\delta_{t0})Y^{-1}(\eta*Y_f)$. Since the distribution $Yx°\delta_{t0}$ is the generalized function $Y(t_0)x°\delta_{t0}=x°\delta_{t0}$ and because of first equation in (3) $\eta*x°\delta_{t0}=x°\eta_{t0}$, then finally we obtain formula:

$$x_{f,t0} = Y^{-1}x°\eta_{t0} + Y^{-1}(\eta * Y_f) \quad (24)$$

If the distribution *f* is regular, then $\eta*Y_f$ is also a regular functional, originated by ordinary function $\int_{-\infty}^{\infty} \eta(t-\lambda)Y(\lambda)f(\lambda)d\lambda = \int_{-\infty}^{t} Y(\lambda)f(\lambda)d\lambda$. Hence, it is seen that in this case the regular vector-distribution (24) corresponds to ordinary vector-function, which on [t_0, ∞) coincides with classic solution of Cauchy problem for the system (7) with assumed initial condition *x*°. Matrix *Y*(*t*) is defined also by convergence method $Y_k(t) = \int_{t0}^{t} (-A(t))Y_{k-1}(t)dt(k = 1,2,...), Y_0(t) = E$. Hence, considering equation $Y = \Omega_{t0}^{t}(-A)$, we find:

$$Y_{-1} = \Omega_{t0}^{t}(A) \quad (25)$$

and for the calculation of value $Y_{-1}(t)$ at any fixed $t \in (a,b) \subset [0,\infty)$ one can use the appropriate Equation (21).

5. APPLICATION OF OPERATOR CALCULUS TO PULSE CIRCUITS

5.1. Action of Pulsed Circuits

Analysis of transient processes in linear electric circuits, one of main stages of which is a solution of differential or integro-differential equations, can be performed by means of operator method on the basis of operator calculus for generalized functions. Peculiarities of its application for the solution of equations of linear circuits with sources of pulse voltage we will consider on the example of single-loop electric circuits represented by Figures 1 and 2 from paper [5]. Circuit capacitance voltage (Figure 1) and inductance current were equal to 0 prior to the moment of action of voltage source $U°\delta(t)$. It is necessary to determine currents of such circuits, formed by the action of pulse voltage sources. Current of the circuit in Figure 1 is described by Equation (26) in generalized functions:

$$R_i\{t\} + \frac{1}{C}\int_{-0}^{t} i\{t\}dt = U^{\circ}\delta(t)\ ,\ -\infty < t < +\infty \quad (26)$$

In operator calculus for generalized functions D'_+ Equation (26) will be written as $(s + \frac{1}{RC})i\{t\} = \frac{U^{\circ}}{R}s\delta(t)$. Hence we found the representation of the solution in operator form $i\{t\}=\dfrac{s}{s+(1/RC)}.\dfrac{U^{\circ}}{R}s\delta(t)$ or $i\{t\}=(1-\dfrac{1/RC}{s+1/RC})\dfrac{U^{\circ}}{R}\delta(t),$ where $1=\delta(t)$.

Applying the relation $\delta(t)*\delta(t)=\delta(t),\ \eta\{t\}*\delta(t)=\eta\{t\}$ and $\dfrac{1}{s+a}=e^{-at}\eta(t)$ we will obtain solution in distributions of Equation (26):

$$i\{t\}=\frac{U^{\circ}}{R}\delta(t)\frac{U^{\circ}}{R^{2}C}e^{\frac{1}{RC}}\eta\{t\} \tag{27}$$

Circuit current in Figure 2 can be determined by operator method of solution of equations in generalized functions

$$LDi\{t\}+Ri\{t\}=U^{\circ}\delta(t)\ ,\ -\infty<t<+\infty \tag{28}$$

The Equation (28) is written in operator form corresponds equation $Lsi\{t\}+Ri\{t\}=U^{\circ}\delta(t)$, solving it, we find $i\{t\}=\dfrac{U^{\circ}}{L}e\dfrac{R}{L}t\cdot\eta\{t\}$.

5.2. Action of Intermittent Sources

Let in the circuit input (Figure 3 in [5]), which represents a capacitance voltage divider with load in the form of resistance, the voltage source $u\{t\}=Ue^{\omega t}\eta\{t\}$ starts to operate. Assuming that voltages on capacitances up to the moment of start of action of intermittent source were equal to zero, it is necessary to determine the source current and voltage in the divider output.

Equation of loop currents in this circuit will be written in generalized functions as

$$\begin{aligned}&S_{11}\int_{0-}^{t}i_{1}\{t\}dt+S_{12}\int_{0-}^{t}i_{2}\{t\}dt=u\{t\}\\&S_{21}\int_{0-}^{t}i_{1}\{t\}dt+S_{22}\int_{0-}^{t}i_{2}\{t\}dt+Ri_{2}\{t\}=0\end{aligned} \tag{29}$$

where, $S_{11}=\dfrac{1}{c_{1}}+\dfrac{1}{c},\ S_{22}=\dfrac{1}{C},\ S_{12}=S_{21}=-\dfrac{1}{C}$ are self-capacitance and general inverse capacitance of the circuit. Using the Equation (11), we will write operator equations, corresponding to Equations (29):

$$\begin{aligned}&\frac{1}{S}[S_{11}i_{1}\{t\}+S_{12}i_{2}\{t\}]=\frac{U}{S-W}\\&\frac{1}{S}[S_{22}i_{1}\{t\}+S_{22}i_{2}\{t\}+Ri_{2}\{t\}]+Ri_{2}\{t\}=0\end{aligned} \tag{30}$$

Solving the system (30) of linear algebraic equations relative to $i_1\{t\}$ and $i_2\{t\}$, we find:

$$\begin{aligned}&i_{1}\{t\}=UC_{1}\frac{s}{s-w}.\frac{1+sRC}{1+sR(C+C_{1})}\\&i_{2}\{t\}=UC_{1}\frac{s}{s-w}.\frac{1}{1+sR(C+C_{1})}\end{aligned} \tag{31}$$

After realization of operator functions, we will obtain

$$\begin{aligned}&i_{1}\{t\}=\frac{CC_{1}}{C+C_{1}}U\delta(t)+\left[\frac{U\omega C_{1}}{1+\omega R(C+C_{1})}e\omega t+\right.\\&\left.+\frac{UC_{1}^{2}}{R(C+C_{1})^{2}}\times\frac{1}{1+\omega R(C+C_{1})}\cdot e\frac{-t}{R(C+C_{1})}\right]\eta\{t\}\end{aligned} \tag{32}$$

Voltage on resistance is calculated by $u_{R}\{t\}=i_{2}\{t\}\cdot R$, where $i_2\{t\}$ is determined by expression (32).

6. CIRCUITS WITH MULTIPLE COMMUTATOR

There is voltage source $u\{t\}$ (Figure 4 in [5]), containing regular and pulse compositions, which operate in the moments t_s of commutator actuations:

$$\begin{aligned}&u\{t\}=u_{s-}(t)l(t_{s}-t)+u_{s}(t)l(t-t_{s})+\\&+U_{s}^{\circ}\delta(t-t_{s})\ ,\ t_{s}-1\le t<t_{s}+1\end{aligned} \tag{33}$$

At selected positive directions of voltages and currents the equations of loop currents in generalized functions are written as follows:

$$\left.\begin{aligned}&u\{t\}=D(L(t)i(t))+R_{1}(t)(u\{t\}+ik\{t\})\\&uk\{t\}=R_{1}(t)i\{t\}+(R_{1}(t)+R_{2}(t)\cdot i_{k}\{t\}\end{aligned}\right\},\ (-\infty<t<\infty) \tag{34}$$

Excluding commutator current $i_k\{t\}$ from (34), we will obtain equation:

$$\begin{aligned}&u\{t\}=L(t)Di\{t\}+\left(L'(t)+\frac{R_{1}(t)\cdot R_{2}(t)}{R_{1}(t)+R_{2}(t)}\right).\\&.i(t)+\frac{R_{1}(t)}{R_{1}(t)+R_{2}(t)}u_{k}\{t\}\end{aligned} \tag{35}$$

In accordance with the commutator actuation mode, its voltage $u_k\{t\}$ is expressed as $u_k\{t\}=R_1(t).i(t)$ in the first interval of commutation cycle, when commutator takes up the "off" position, and $u_k\{t\}=0$ is in the second interval, when the commutator takes up the "on" position. Substituting $u_k\{t\}$ value in (35), after division of general parts of this equation by $L(t)$, we will obtain (7) type equation relative to $y\{t\}=i\{t\}$. Assuming that functions are smooth and positive at $t\in(0,\infty)$, for the solution of equation in generalized functions obtained for $y\{t\}$ one can use the Equation (15).

7. COMPARISON WITH METHODS OF ONE-SIDED LAPLACE TRANSFORMS AND CONCLUSIONS

Differential or integro-differential equations for the momentary values represent the "voltage-current" relationship directly after occurrence of transient processes, which is described on the basis of general provisions on the continuity of cumulative charge of capacities of each node and cumulative flux linkage of inductances of each closed loop. Based on these provisions the transition can be performed from initial conditions 0_- to initial conditions 0_+ implemented by means of set-up and solution of Kirchhoff equations for pulse amplitudes.

The solutions of equations for the momentary values can be obtained [5] only based on transform L_+ and require the knowledge of initial conditions 0_+. However, this transform defines only regular composition of solution.

The difference of the transform L_-, used for the equations in generalized functions, is a possibility for the determination both regular and singular compositions of the solution with direct use of initial conditions 0_-. As it was mentioned in the introduction, the image reversal $F_-(p)=L_-[f(t)]$ coincides with the image reversal $F_+(p)=L_+[f(t)]$ and gives only regular composition $f_+(t)\eta(t)$.

Amplitudes of functions of singularities of the solution are obtained in [5] only by means of sequential calculation of coefficients C_q, C_{q-1}, …, C_0 at $\delta^{(q)}(t)$, $\delta^{(q-1)}(t)$, …, $\delta^{(0)}(t)=\delta(t)$ relatively.

In this paper the advantages of use of operator calculus in the space D'_+ of all generalized functions with bounded support for the solution of differential equations, describing the transient processes in linear electric pulse circuits with concentrated parameters, being time-constant or time-variable, are shown.

At pulse actions, containing Dirac delta function and its product, the explicit solution representation in the form of vector-distribution (6) for the system of linear differential equations (1) in generalized functions with constant coefficients and the solution in the form of vector-distribution (15) for the system of equations (7) with variable coefficients are obtained. For the numerical solution of the system (7) the approximate formula (12) with the accuracy to first order infinitesimals is obtained.

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BIOGRAPHY

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