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NATURAL VIBRATIONS OF AN ORTHOTROPIC CYLINDRICAL SHELL-SOLID-FLUID STIFFENED WITH ANNULAR RIBS

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Abstract- For ensuring stability of medium-contacting cylindrical shells subjected to the action of various forces, the designers have to stiffen them with ribs. Since cylindrical shape constructions and structural elements are widely used in industry and civil engineering fields, the problems related to their dynamical rigidity characteristics still retain their relevance. The goal of the present paper is to find natural vibrations frequency of an orthotropic cylindrical shell contacting with fluid-filled inner channel and stiffened with rings, to study influence of parameters characterizing this medium to these frequencies. In the paper we consider a rigid contact that takes into consideration that the shift is impossible without leaving tangential surfaces of an orthotropic shell and medium each other. Using the Hamilton-Ostrogradsky variational principle in the solution of the problem, we make use the system of motion equations of the obtained cylindrical shell, the system of motion equations of the elasticity system in displacements. For finding natural vibrations frequencies of the system using the contact conditions between the cylindrical shell solid medium-fluid, the obtained equation was studied by the analytical method. In the calculation process, the properties of the Bessel function were used. The graphs of dependence of the natural vibration frequencies of the system of the radius of the channel and density of fluid were structured.

Keywords: Shell, Solid Medium, Liquid, Friction Force, The Variational Principle, Frequency, Orthotropic.

1. INTRODUCTION

Vibrations and stability problems of medium-contacting smooth cylindrical shells were studied in the monograph [1]. The monograph [2] studies the solution of the problems of vibrations and stability of stiffened isotropic cylindrical shells. Vibrations and stability of smooth cylindrical shells were considered in [3]. The stability of a medium-contacting, stiffened isotropic cylindrical shell subjected to the action of compressive force was considered in [4, 5]. The monograph [6] was devoted to vibrations and stability of fluid and solid medium-contacting smooth cylindrical shells.

In [7-9], for studying free and forced vibrations of a fluid-filled cylindrical shell stiffened with rods and subjected to the compressive force a physical and mathematical model was structured. In axially-symmetric and asymmetric cases of vibrations, the frequency equation of the system was structured, its approximate roots were found and influence of geometrical, physical, mechanical parameters characterizing this system on these roots were studied. Free vibrations of an isotropic inhomogeneous, moving fluid-contacting cylindrical shell stiffened with cross system of ribs were studied in [10].

In the paper [11] natural vibrations of flowing fluid interacting, cylindrical shell inhomogeneous in thickness, are studied. Using the Hamilton-Ostrogradsky variation principle in the solution of the problem, for studying free vibrations of a flowing-fluid-contacting cylindrical shell inhomogeneous in thickness and stiffened with rings, a system of equations was constructed. Homogeneity of the thickness of the cylindrical shell was taking into account accepting the Young modulus and density of the material as a function of coordinate alternating along the thickness.

When studying vibrations of a cylindrical shell inhomogeneous along the thickness and stiffened with annular ribs and dynamically interacting with flowing fluid, we considered two cases: a) fluid is at rest inside the cylindrical shell b) fluid moves with constant velocity inside the cylindrical shell. In both cases, the frequency equation was structured and its roots were found. In the calculation process, linear and parabolic cases of alternation of inhomogeneity function with respect to the coordinate were considered.

2. PROBLEM STATEMENT

The system of motion equations of fluid and solid medium-contacting cylindrical shell with inner channel, stiffened with rings, located in the plane perpendicular to the axis in the displacements is in the following form:

$$\left[a_1\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\theta^2}\right]u+\left(1+a_{12}\right)\frac{\partial^2\vartheta}{\partial\xi\partial\theta}-a_{12}\frac{\partial w}{\partial\xi}=\frac{R^2q_x}{G_{12}h}$$

$$\left(1+a_{12}\right)\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{\frac{1-v}{2}\frac{\partial^2}{\partial\xi^2}+\right.$$

$$\left.+\left[1+\left(1-\frac{h_s}{R}\right)^2\gamma_s^{(2)}+a_2\right]\frac{\partial^2}{\partial\theta^2}\right\}\vartheta-$$

$$-\left[a_2+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial w}{\partial\theta}=\frac{R^2q_y}{G_{12}h}$$

$$-a_{12}\frac{\partial}{\partial\xi}u-\left\{\left[a_2+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\frac{\partial^3}{\partial\theta^3}\right\}\vartheta+$$

$$+\left\{a_2+\gamma_s^{(2)}+\eta_{s1}^{(2)}+2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)\frac{\partial^2}{\partial\theta^2}+\right.$$

$$+a^2\left[a_1\frac{\partial^4}{\partial\xi^4}+2(a_{12}+2)\frac{\partial^4}{\partial\xi^2\partial\theta^2}+\right. \qquad (1)$$

$$\left.\left.+\left(a_{12}+\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)\frac{\partial^4}{\partial\theta^4}\right]+p\frac{\partial^2}{\partial\xi^2}\right\}w=\frac{R^2}{G_{12}h}q_z$$

$$a^2=\frac{h^2}{12R^2},\ \xi=\frac{x}{R},\ \gamma_s^{(2)}=\frac{E_s}{G_{12}}\bar{\gamma}_s^{(2)},\ \bar{\gamma}_s^{(2)}=\frac{F_s(k_2+1)}{L_1h},$$

$$\bar{\delta}_s^{(2)}=\frac{h_s}{R}\bar{\gamma}_s^{(2)},\qquad \bar{\eta}_s^{(2)}=\left(\frac{h_s}{R}\right)^2\bar{\gamma}_s^{(2)},\qquad \delta_s^{(2)}=\frac{h_s}{R}\bar{\gamma}_s^{(2)},$$

$$\eta_{s2}^{(2)}=\frac{E_s}{G_{12}}\bar{\eta}_s^{(2)},\qquad \bar{\eta}_{s1}^{(2)}=\frac{(1+k_1)E_sJ_{xs}}{G_{12}L_1hR^2},\qquad \theta=\frac{y}{R},$$

$$\bar{\mu}_s^{(2)}=\frac{(k_1+1)J_{kps}}{L_1hR^2},\ L_1=x_2-x_1,\ a_{12}=a_1\nu_{21}=a_2\nu_{12}$$

In the system (1) u,ϑ,w are displacements of the points of the shell, R,h are radius and thickness of the cylindrical shell, respectively, E_s is an elasticity modulus of ring-shaped ribs, F_s is the area of the cross-section of the shell, J_{xs}, $J_{kp.s}$ are inertia moments of the cross-section of the shell, q_x,q_y,q_z are pressure force components acting on the cylindrical shell as viewed from fluid, k_2 is the amount of ring-shaped ribs, G_s is elasticity modulus of ribs in shift, ρ_0,ρ_s are densities of the materials of cylindrical shell and ring-shaped ribs, G_{12} is an elasticity modulus of the cylindrical shell in shift, E_i are elasticity modulus of the cylindrical shell in the direction of coordinate axes, ν_{12},ν_{21} is the Poison ratio of the materials of the cylindrical shell, h_s is the thickness of the ring-shaped shell, and L_1 is the length of the cylindrical shell.

The system of motion equations of the medium is written in the cylindrical coordinates as follows [6]:

$$\left(\lambda_s+2\mu_s\right)\frac{\partial\theta}{\partial r}-\frac{2\mu_s}{r}\frac{\partial\omega_x}{\partial\varphi}+2\mu_s\frac{\partial\omega_\varphi}{\partial x}-\rho_s\frac{\partial^2s_x}{\partial t^2}=0$$

$$\left(\lambda_s+2\mu_s\right)\frac{1}{r}\frac{\partial\theta}{\partial\varphi}-2\mu_s\frac{\partial\omega_r}{\partial x}+2\mu_s\frac{\partial\omega_x}{\partial x}-\rho_s\frac{\partial^2s_\varphi}{\partial t^2}=0 \qquad (2)$$

$$\left(\lambda_s+2\mu_s\right)\frac{\partial\theta}{\partial x}-\frac{2\mu_s}{r}\frac{\partial}{\partial r}\left(r\omega_\varphi\right)+\frac{2\mu_s}{r}\frac{\partial\omega_r}{\partial\varphi}-\rho_s\frac{\partial^2s_r}{\partial t^2}=0$$

where, s_x, s_φ, s_r are the components of displacements vector of the medium, λ_s, μ_s are Lame coefficients of the medium, ρ_s is medium's density, x,r,φ are longitudinal, radial and circular coordinates and $a_t=\sqrt{\frac{\lambda_s+2\mu_s}{\rho_s}}$, $a_e=\sqrt{\frac{\mu_s}{\rho_s}}$.

The volume expansion θ and the components ω_x, ω_φ, ω_r are calculated by means of the expressions:

$$\theta=\frac{\partial s_r}{\partial r}+\frac{s_r}{r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi}+\frac{\partial s_x}{\partial x},\qquad 2\omega_x=\frac{1}{r}\left[\frac{\partial\left(rs_\varphi\right)}{\partial r}-\frac{\partial s_r}{\partial\varphi}\right],$$

$$2\omega_\varphi=\frac{\partial s_r}{\partial x}-\frac{\partial s_x}{\partial r},\ \ 2\omega_r=\frac{1}{r}\frac{\partial s_x}{\partial\varphi}-\frac{\partial s_\varphi}{\partial x}.$$

The stresses arising in the medium are expressed by the displacements s_x, s_φ, s_r as follows:

$$\sigma_{rx}=\mu_s\left(\frac{\partial s_x}{\partial r}+\frac{\partial s_r}{\partial x}\right)$$

$$\sigma_{r\varphi}=\mu_s\left[r\frac{\partial}{\partial r}\left(\frac{s_\varphi}{r}\right)+\frac{1}{r}\frac{\partial s_r}{\partial\varphi}\right] \qquad (3)$$

$$\sigma_{rr}=\lambda_s\left(\frac{\partial s_x}{\partial x}+\frac{1}{r}\frac{\partial\left(rs_r\right)}{\partial r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi}\right)+2\mu_s\frac{\partial s_r}{\partial r}$$

The motion of fluid moving with velocity U with respect to the potential $\tilde{\varphi}$ is as follows [7]:

$$\Delta\tilde{\varphi}-\frac{1}{a_0^2}\left(\frac{\partial^2\tilde{\varphi}}{\partial t^2}+2U\frac{\partial^2\tilde{\varphi}}{R\partial\xi\partial t}+U^2\frac{\partial^2\tilde{\varphi}}{R^2\partial\xi^2}\right)=0 \qquad (4)$$

To the systems (1), (2) and Equation (4) we add contact conditions. Rigid contact between solid medium and cylindrical shell is considered. In this case the contact conditions consist of the followings:

- Equality of displacements

$$s_x=u,\ s_\varphi=\upsilon,\ s_r=w\quad (r=R) \qquad (5)$$

- Equality of pressure forces

$$q_x=-\sigma_{rx},\ q_\theta=-\sigma_{r\theta},\ q_r=-\sigma_{rr}\quad (r=R) \qquad (6)$$

Equality of velocity and pressure in the radial direction in the contact of medium-fluid is satisfied:

$$\vartheta_r\Big|_{r=a}=\frac{\partial\varphi}{\partial r}\Big|_{r=a}=-\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right) \qquad (7)$$

$$\sigma_{rx}=0,\ \sigma_{r\theta}=0,\ \sigma_{rr}=-p\quad (r=a) \qquad (8)$$

To the contact conditions (5)-(9), we add the following boundary conditions. It is considered that the cylindrical shell was highly supported at the edges, i.e. in the sections $\xi=0$ and $\xi=\xi_1\ (\xi_1=L_1/R)$

- The conditions

$$\vartheta=w=0,\ T_1=M_1=0 \qquad (9)$$

- For the medium the conditions

$$\sigma_{xx}=0;\ \ s_\varphi=s_r=0 \qquad (10)$$

are satisfied.

Thus, the solution of the stated problem is reduced to finding natural vibrations of a construction with flowing fluid, with a channel in the domain and stiffened with rings, to joint integration of the system of motion equations of the cylindrical shell (1), of solid medium (2), of fluid (4) within the contact conditions (5)-(8) and boundary conditions (9) and (10).

3. PROBLEM SOLUTION

We look for the potential $\tilde{\varphi}$ of perturbations in fluid in the following form:

$$\tilde{\varphi}(\xi,r,\varphi,t_1)=f(r)\cos n\varphi\sin kx\sin\omega_1 t_1 \tag{11}$$

Using (11), (7) and (8), we get:

$$\begin{aligned}&\varphi=-\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right)\\ &p=\Phi^*_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2}+2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1}+U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right)\end{aligned}\tag{12}$$

where,

$$\begin{aligned}&\Phi_{\alpha n}=\begin{cases}I_n(\beta r)/I'_n(\beta R), & M_1<1\\ J_n(\beta_1 r)/J'_n(\beta_1 R), & M_1>1\\ \dfrac{r^n}{nR^{n-1}}, & M_1=1\end{cases}\\ &\Phi^*_{\alpha n}=\begin{cases}I_n(\beta R)/I'_n(\beta R), & M_1<1\\ J_n(\beta_1 R)/J'(\beta_1 R), & M_1>1\\ \dfrac{R}{n}, & M_1=1\end{cases}\end{aligned}\tag{13}$$

In equalities (13) $M_1=\dfrac{U+\omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2=R^{-2}\times\times\left(1-M_1^2\right)\chi^2$, $\beta_1^2=R^{-2}\left(M_1^2-1\right)\chi^2$, I_n is the n-th order modified, J_n is n-th order first kind Bessel functions.

Since the point zero is not the inner point of medium, when the iteration effect of the medium on the vibration process is weak, the displacement vector components of the medium are in the following form [6]:

$$\begin{aligned}&s_x=\left[\left(-kr\frac{\partial I_n(kr)}{\partial r}-4(1-v_s)kI_n(kr)\right)A_s+kI_n(kr)B_s+\right.\\ &\left.+\left(-kr\frac{\partial K_n(kr)}{\partial r}-4(1-v_s)kK_n(kr)\right)\tilde{A}_s+kK_n(kr)\tilde{B}_s\right]\times\\ &\times\cos n\theta\cos\chi\xi\sin\omega t\\ &s_\theta=\left[-\frac{n}{r}I_n(kr)B_s-\frac{\partial I_n(kr)}{\partial r}C_s-\right.\\ &\left.-\frac{n}{r}K_n(kr)\tilde{B}_s-\frac{\partial K_n(kr)}{\partial r}\tilde{C}_s\right]\sin n\theta\sin\chi\xi\sin\omega t\\ &s_r=\left[-k^2 rI_n(kr)A_s+\frac{\partial I_n(kr)}{\partial r}B_s+\frac{n}{r}I_n(kr)C_s-\right.\\ &\left.-k^2 rK_n(kr)A_s+\frac{\partial K_n(kr)}{\partial r}\tilde{B}_s+\frac{n}{r}K_n(kr)\tilde{C}_s\right]\times\\ &\times\cos n\theta\cos\chi\xi\sin\omega t\end{aligned}\tag{14}$$

We will look for the solutions of the shell as follows:

$$\begin{aligned}&u=A\cos n\theta\cos\chi\xi\sin\omega_1 t_1\\ &\upsilon=B\sin n\theta\sin\chi\xi\sin\omega_1 t_1\\ &w=C\cos n\theta\sin\chi\xi\sin\omega_1 t_1\end{aligned}\tag{15}$$

By means of the contact and boundary conditions (5)-(8) for determining the constants $A_s,B_s,C_s,\tilde{A}_s,\tilde{B}_s,\tilde{C}_s$ we get the following system of equations:

$$\begin{aligned}&\left(-k^2 rI_n(kR)-4(1-v_s)kI_n(kr)\right)A_s+kI_n(kR)B_s+\\ &+\left(-k^2 rK_n-4(1-v_s)kK_n(kr)\right)\tilde{A}_s+kK_n(kr)\tilde{B}_s=A\\ &-\frac{n}{r}I_n(kr)B_s-kI'_n(kR)C_s-\frac{n}{r}K_n(kr)\tilde{B}_s-\\ &-kK'_n(kR)\tilde{C}_s=B\\ &-k^2 RI_n(kR)A_s-k^2 RK_n(kR)A_s+kI'_n(kR)B_s+\\ &kK'_n(kR)\tilde{B}_s+\frac{n}{R}I_n(kR)C_s+\frac{n}{R}K_n(kR)\tilde{C}_s=C\\ &\left(akI_n(ka)+kaI''_n(ka)+(5-4v_s)I'_n(ka)k^2A_s-\right.\\ &-2k^2a^2I'_n(ka)B_s-nkaI_n(ka)C_s+\\ &+\left(akK_n(ka)+kaK'_n(ka)+(5-4v_s)K'_n(ka)\right)k^2\tilde{A}s-\\ &-2k^2a^2K'_n(ka)\tilde{B}_s-nkaK_n(ka)\tilde{C}_s=0\\ &-a^2nk^2I_n(ka)A_s+2n\left(kaI'_n(ka)-I_n(ka)\right)B_s+\\ &+\left(k^2a^2I''_n(ka)-kaI'_n(ka)+n^2I_n(ka)\right)C_s-\\ &-a^2nk^2K_n(ka)\tilde{A}_s+2n\left(kaK'_n(ka)-K(ka)\right)\tilde{B}_s+\\ &+\left(k^2a^2K''_n(ka)-kaK'_n(ka)+n^2K_n(ka)\right)\tilde{C}_s=0\\ &\left(2(1-2v_s)I_n(ka)+2kaI'_n(ka)\right)k^2a^2A_s-\\ &-2k^2a^2I''_n(ka)B_s+2n\left(I_n(ka)-kaI'_n(ka)\right)C_s+\\ &+\left(2(1-2v_s)K_n(ka)+2kaK'_n(ka)\right)k^2a^2\tilde{A}_s-\\ &-2k^2a^2K''_n(ka)\tilde{B}_s+2n\left(K_n(ka)-kaK'_n(ka)\right)\tilde{C}_s=\\ &=\Phi_{\alpha n}(a)\rho_m(\omega_0^2\omega_1^2-2U\chi\omega_1+U^2\chi^2R^{-2})C\end{aligned}\tag{16}$$

By means of this system we express the quantities $A_s,B_s,C_s,\tilde{A}_s,\tilde{B}_s,\tilde{C}_s$ by the constants A, B, C:

$$\begin{aligned}&A_s=\Delta^{-1}(\Delta_1^{(1)}A+\Delta_1^{(2)}B+\Delta_1^{(3)}C)\\ &B_s=\Delta^{-1}(\Delta_2^{(1)}A+\Delta_2^{(2)}B+\Delta_2^{(3)}C)\\ &C_s=\Delta^{-1}(\Delta_3^{(1)}A+\Delta_3^{(2)}B+\Delta_3^{(3)}C)\\ &\tilde{A}_s=\Delta^{-1}(\Delta_4^{(1)}A+\Delta_4^{(2)}B+\Delta_4^{(3)}C)\\ &\tilde{B}_s=\Delta^{-1}(\Delta_5^{(1)}A+\Delta_5^{(2)}B+\Delta_5^{(3)}C)\\ &\tilde{C}_s=\Delta^{-1}(\Delta_6^{(1)}A+\Delta_6^{(2)}B+\Delta_6^{(3)}C)\end{aligned}\tag{17}$$

where, Δ is the principal determinant of the system (16), $\Delta_i^{(j)}$ are auxiliary determinants. Taking into account the expressions (17) of the constants $A_s,B_s,C_s,\tilde{A}_s,\tilde{B}_s,\tilde{C}_s$ in the last equality, we get:

$$\sigma_{rx} = -\mu_s \Delta^{-1}\Big[\Big(q_{11}\Delta_1^{(1)} + q_{12}\Delta_2^{(1)} + q_{13}\Delta_3^{(1)} + q_{14}\Delta_4^{(1)} + q_{15}\Delta_5^{(1)} + q_{16}\Delta_6^{(1)}\Big)\cdot A + \Big(q_{11}\Delta_1^{(2)} + q_{12}\Delta_2^{(2)} + q_{13}\Delta_3^{(2)} + q_{14}\Delta_4^{(2)} + q_{15}\Delta_5^{(2)} + q_{16}\Delta_6^{(2)}\Big)\cdot B + \Big(q_{11}\Delta_1^{(3)} + q_{12}\Delta_2^{(3)} + q_{13}\Delta_3^{(3)} + q_{14}\Delta_4^{(3)} + q_{15}\Delta_5^{(3)} + q_{16}\Delta_6^{(3)}\Big)\cdot C\Big]\cos n\theta \cos \chi\xi \sin \omega t$$

$$\sigma_{r\theta} = -\mu_s \Delta^{-1}\Big[\Big(p_{11}\Delta_1^{(1)} + p_{12}\Delta_2^{(1)} + p_{13}\Delta_3^{(1)} + p_{14}\Delta_4^{(1)} + p_{15}\Delta_5^{(1)} + p_{16}\Delta_6^{(1)}\Big)\cdot A + \Big(p_{11}\Delta_1^{(2)} + p_{12}\Delta_2^{(2)} + p_{13}\Delta_3^{(2)} + p_{14}\Delta_4^{(2)} + p_{15}\Delta_5^{(2)} + p_{16}\Delta_6^{(2)}\Big)\cdot B + \Big(p_{11}\Delta_1^{(3)} + p_{12}\Delta_2^{(3)} + p_{13}\Delta_3^{(3)} + p_{14}\Delta_4^{(3)} + p_{15}\Delta_5^{(3)} + p_{16}\Delta_6^{(3)}\Big)\cdot C\Big]\sin n\theta \sin \chi\xi \sin \omega t \quad (18)$$

$$\sigma_{rr} = -\mu_s \Delta^{-1}\Big[\Big(r_{11}\Delta_1^{(1)} + r_{12}\Delta_2^{(1)} + r_{13}\Delta_3^{(1)} + r_{14}\Delta_4^{(1)} + r_{15}\Delta_5^{(1)} + r_{16}\Delta_6^{(1)}\Big)\cdot A + \Big(r_{11}\Delta_1^{(2)} + r_{12}\Delta_2^{(2)} + r_{13}\Delta_3^{(2)} + r_{14}\Delta_4^{(2)} + r_{15}\Delta_5^{(2)} + r_{16}\Delta_6^{(2)}\Big)\cdot B + \Big(r_{11}\Delta_1^{(3)} + r_{12}\Delta_2^{(3)} + r_{13}\Delta_3^{(3)} + r_{14}\Delta_4^{(3)} + r_{15}\Delta_5^{(3)} + r_{16}\Delta_6^{(3)}\Big)\cdot C\Big]\cos n\theta \sin \chi\xi \sin \omega t$$

$$q_{11} = \left(\chi I_n(\chi) + \chi I_n''(\chi) + (5-4v_s)I_n'(\chi)\right)\chi^2;$$
$$q_{12} = -2\chi^2 I_n'(\chi);\ q_{13} = -n\chi I_n'(\chi);$$
$$q_{14} = \left(\chi K_n(\chi) + \chi K_n'(\chi) + (5-4v_s)K_n'(\chi)\right)\chi^2;$$
$$q_{15} = -2\chi^2 K_n'(\chi);\ q_{16} = -n\chi K_n(\chi);$$
$$p_{11} = -n\chi^2 I_n(\chi);\ p_{12} = 2n\left(\chi I_n'(\chi) - I_n(\chi)\right);$$
$$p_{13} = \chi^2 I_n''(\chi) - \chi I_n'(\chi) + n^2 I_n(\chi);$$
$$p_{14} = -n\chi^2 K_n(\chi);\ p_{15} = 2n\left(\chi K_n'(\chi) - K_n(\chi)\right);$$
$$p_{16} = \chi^2 K_n''(\chi) - \chi K_n'(\chi) + n^2 K_n(\chi)$$
$$r_{11} = \left(2(1-2v_s)I_n(\chi) + 2\chi I_n'(\chi)\right)\chi^2;$$
$$r_{12} = -2\chi^2 I_n''(\chi);\ r_{13} = 2n\left(I_n(\chi) - \chi I_n'(\chi)\right);$$
$$r_{14} = \left(2(1-2v_s)K_n(\chi) + 2\chi K_n'(\chi)\right)\chi^2;$$
$$r_{15} = -2\chi^2 K_n''(\chi);\ r_{16} = 2n\left(I_n(\chi) - \chi I_n'(\chi)\right)$$

Using equalities (18) and contact conditions (6), we can find the pressure components q_x, q_θ, q_r. We show these expressions in the following form: Taking into account

$$q_x = \left(\tilde{C}_{x1}A + \tilde{C}_{x2}B + \tilde{C}_{x3}C\right)\cos n\theta \sin \chi\xi$$
$$q_\theta = \left(\tilde{C}_{\theta 1}A + \tilde{C}_{\theta 2}B + \tilde{C}_{\theta 3}C\right)\sin n\theta \sin \chi\xi \quad (19)$$
$$q_r = \left(\tilde{C}_{r1}A + \tilde{C}_{r2}B + \tilde{C}_{r3}C\right)\cos n\theta \sin \chi\xi$$

with respect to the constants A, B, C we get the following system of homogeneous equations:

$$\begin{cases} \alpha_{11}\mathrm{A} + \alpha_{12}B + \alpha_{13}C = 0 \\ \alpha_{21}\mathrm{A} + \alpha_{22}B + \alpha_{23}C = 0 \\ \alpha_{31}\mathrm{A} + \alpha_{32}B + \alpha_{33}C = 0 \end{cases} \quad (20)$$

Necessary and sufficient condition for the existence of non-zero solution of the system is the equality of its principal determinant to zero:

$$\det\left\|\alpha_{\mathrm{ij}}\right\| = 0\ ,\ i, j = 1, 2, 3 \quad (21)$$

4. CONCLUSIONS

The Equation (21) was studied by the numerical method. The following values were taken for the parameters:

$E_s = 6.67\times 10^9$ N/m^2; $h_s = 1.39$ mm; $R = 160$ mm

$q_z = 25$ N/mm^2; $v_{12} = v_{21} = 0.3$; $a_t = 308$ m/s

$h = 0.45$; $F_s = 5.75$ mm^2; $J_{xs} = 19.9$ mm^4

$J_{kp.s} = 0.48$ mm^4; $m = 1$; $n = 4$; $L_1 = 5$ m; $a_l = 2.25a_t$

The results of calculations were given in figure 1 in the form of dependence of natural vibrations frequency on the channel radius, in Figure 2 in the form of dependence of natural vibrations frequency on fluid density for various ratios of elasticity constants of the shell material. As can be seen from the figure, natural vibrations frequencies of the system increase according to the increase of the channel radius and strengthening of orthotropic properties of the cylindrical shell. Figure 2 shows that as the fluid density increases, the system's natural vibrations frequencies decrease. In both graphs, $k_2 = 20$ corresponds to broken lines $k_2 = 15$ corresponds to solid lines. As can be seen from the figure, natural vibration frequencies of the system increase according to the number of rings.

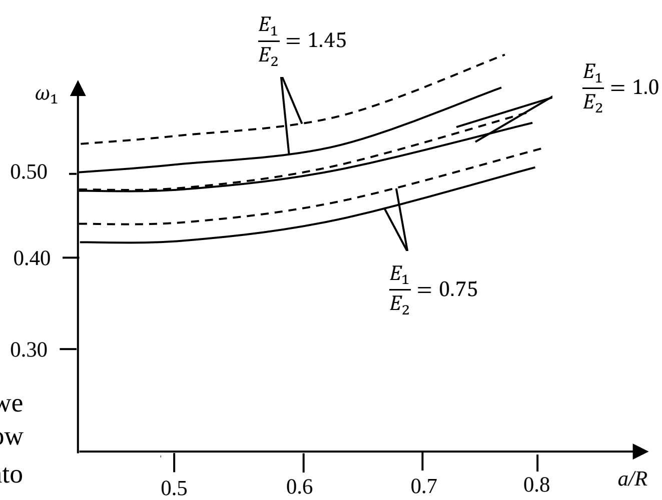

Figure 1. Dependence of the system's natural vibrations frequency on the medium's channel

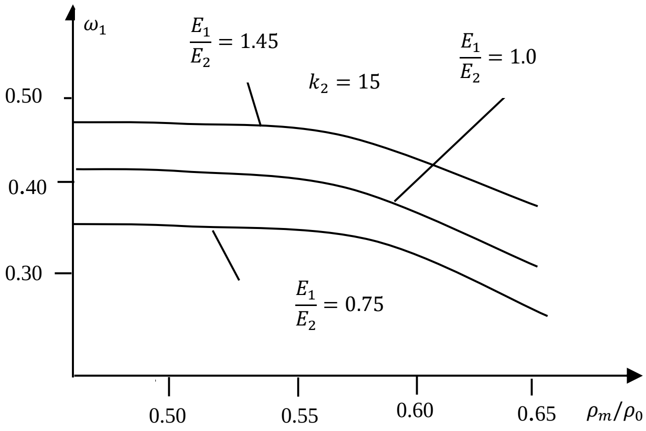

Figure 2. Dependence of the system's natural vibration frequency on the fluid density

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