# VIBRATIONS OF AN ORTHOTROPIC, HETEROGENEOUS CYLINDRICAL SHELL DYNAMICALLY CONTACTING WITH VISCOUS LIQUID AND STIFFENED WITH BARS 

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#### Abstract

This paper deals with vibrations of an orthotropic, heterogeneous, viscous liquid-filled cylindrical shell stiffened with bars. The OstrogradskyHamilton principle is used to solve the problem. For taking into account heterogeneity, it was considered that the modulus of elasticity and density of the cylindrical shell material is a function of the normal coordinate. A frequency equation for studying joint vibrations of an orthotropic, heterogeneous, viscous liquid-filled cylindrical shell was structured and its roots were found. In the calculation process, linear and parabolically changing cases of the heterogeneity function were considered. The characteristic curves were structured.


Keywords: Stiffened Shell, Variational Principle, Viscous Liquid, Free Vibrations, Anisotropic Shell.

## 1. INTRODUCTION

Note that vibrations of a homogeneous cylindrical shell stiffened with anisotropic bars, and ideal liquid were studied in [1]. In the paper [2], free vibrations of an orthotropic, laterally stiffened, ideal fluid-filled cylindrical shell inhomogeneous in thickness and in circumferential direction is studied. Using the Ostrogrdasky-Hamilton variational principle, the systems of equations of the motion of an orthotropic, ideal fluid filled cylindrical shell stiffened in thickness and circumference, are constructed. In order to calculate inhomogeneity of the shell material in thickness and circumference, it is accepted that the Young modulus and the density of the material of the shell are the functions of normal and circumferential coordinates. Frequency equations are constructed and free vibrations of an orthotropic, ideal fluid-filled, laterally stiffened cylindrical shell inhomogeneous in thickness and in circumference are numerically implemented. The characteristically dependence curves were constructed.

In the paper [3], a problem of parametric vibration of an external elastic medium-contacting, longitudinally stiffened orthotropic cylindrical shell under the action of inner pressure in the geometrically nonlinear statement was solved by means of the variational principle. The influence of the external medium was taken into account by means of the Pasternak model. Amplitude frequency
dependences of parametric vibrations of a medium-filled, cylindrically stiffened shell were constructed.

In [4-9], deformation waves in elastic shells and elastic coaxial shells contacting with viscous incompressible liquid, are studied. In these papers it was shown that the wave amplitude in its motion process may increase or decrease depending on the value of the Poisson ratio of the shell material, i.e. presence of liquid leads to destruction of the solitary wave. This time, the equation describing the wave process has no exact solution, therefore this phenomenon was discovered and studied by means of computer simulation. The availability of liquid layer between coaxial elastic shells leads to the coupled system of equations describing the wave process in the shells and with an exact solution. However, the development of deformation waves in the inner shell in the availability of perturbations in the outer shell and that requires a computer simulation is of special interest. Passage from a continuous model to a discrete one is implemented by means of the basis technique.

In the paper [10], natural vibrations frequency of the system is studied that consisting of a solid medium-filled elastic-plastic orthotropic cylindrical shell strengthened with discretely distributed rings established on a plane perpendicular to its axis. Utilizing the OstrogradskyHamilton principle, a frequency equation for determining vibration frequencies of the system following consideration was created; its roots were obtained by mathematical method.

In the paper [11], natural vibrations of flowing fluid interacting, cylindrical shell inhomogeneous in thickness, are studied. Using the Ostrogradsky-Hamilton variation principle in the solution of the problem, for studying free vibrations of a flowing-fluid-contacting cylindrical shell inhomogeneous in thickness and stiffened with rings, a system of equations was constructed. Homogeneity of the thickness of the cylindrical shell was taking into account accepting the Young modulus and density of the material as a function of coordinate alternating along thickness.

When studying vibrations of a cylindrical shell inhomogeneous along the thickness and stiffened with annular ribs and dynamically interacting with flowing fluid, we considered two cases: a) fluid is at rest inside the cylindrical shell b) fluid moves with constant velocity
inside the cylindrical shell. In both cases, the frequency equation was structured and its roots were found. In the calculation process, linear and parabolic cases of alternation of inhomogeneity function with respect to the coordinate were considered.

The analysis of the works close to the topic of the problem in the paper shows that the study of a viscous fluid-filled orthotropic cylindrical shell stiffened with bars has not found its solution.

## 2. PROBLEM STATEMENT

Under the orthotropic, heterogeneous cylindrical shell stiffened with bars we understand a cylindrical shell and a system consisting of bars rigidly attached to it along coordinate lines (Figure 1). It is considered that the coordinate axes coincide with the principle curvature lines of the shell and the ribs are in contact with the shell along these lines.

When solving the problems, we will use the Ostrogradskiy-Hamilton variational principle. For that we will write the total energy of the construction consisting of orthotropic, heterogeneous cylindrical shell, rings subjected to the action of viscous liquid. Allowing for heterogeneity along the thickness of the cylindrical shell, we will use three-dimensional functional.


Figure 1. Viscous liquid-contacting orthotropic cylindrical shell stiffened with bars

In this case, the total energy of the cylindrical shell is as Equation (1).
$U=\frac{1}{2} \iint_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{\alpha} e_{\alpha}+\sigma_{\beta} e_{\beta}+\tau_{\alpha \beta} e_{\alpha \beta}+\right.$
$\left.+\rho(z)\left(\frac{\partial u}{\partial t}\right)^{2}+\left(\frac{\partial \vartheta}{\partial t}\right)^{2}+\left(\frac{\partial w}{\partial t}\right)^{2}\right) d \alpha d \beta d z$
There are various methods for taking account the heterogeneity. One of them is to accept the Young modulus and material density as a function of coordinate changing along the thickness [12]: $E=E(z), \rho=\rho(z)$. We assume that the Poisson ratio is constant. In this case the stress-strain ratios are written as follows:
$\sigma_{11}=b_{11}(z) \varepsilon_{11}+b_{12}(z) \varepsilon_{22}$
$\sigma_{22}=b_{12}(z) \varepsilon_{11}+b_{22}(z) \varepsilon_{22}$
$\sigma_{12}=b_{66}(z) \varepsilon_{12}$
$\varepsilon_{11}=\frac{\partial u}{\partial x}, \varepsilon_{22}=\frac{\partial \vartheta}{\partial y}+w, \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x}$
We can write the expression (1) as follows:
$V=\frac{1}{2} \iint\left\{\tilde{b}_{11} \varepsilon_{11}^{2}+2 \tilde{b}_{12} \varepsilon_{11} \varepsilon_{22}+2 \tilde{b}_{26} \varepsilon_{12} \varepsilon_{22}+\right.$
$\left.+2 \tilde{b}_{16} \varepsilon_{11} \varepsilon_{12}+\tilde{b}_{22} \varepsilon_{22}^{2}+\tilde{b}_{66} \varepsilon_{12}^{2}\right\} d x d y+$
$+\iint\left(\tilde{\rho}\left(\frac{\partial u}{\partial t}\right)^{2}+\left(\frac{\partial \vartheta}{\partial t}\right)^{2}+\left(\frac{\partial w}{\partial t}\right)^{2}\right) d x d y$
where
$\tilde{b}_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{11}(z) d z ; \quad \tilde{b}_{12}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{12}(z) d z ;$
$\tilde{b}_{22}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{22}(z) d z ; \tilde{b}_{66}=\int_{-\frac{h}{2}}^{\frac{h}{2}} b_{66}(z) d z ; b_{11}=\frac{E_{1}(z)}{1-v_{1} v_{2}} ;$
$\tilde{b}_{22}=\frac{E_{2}(z)}{1-v_{1} v_{2}} ; b_{12}(z)=\frac{v_{2} E_{1}(z)}{1-v_{1} v_{2}}=\frac{v_{1} E_{2}(z)}{1-v_{1} v_{2}} ;$
$b_{66}(z)=G_{12}(z)=G(z)$ are the principal elasticity module of the orthotropic material and $\rho_{0}=\int_{-h}^{h} \rho(z) d z$. Write the potential and kinetic energies of the $j$ th ring [13]:

$$
\begin{align*}
& \Pi_{j}=\frac{R}{2} \int_{0}^{2 \pi}\left[E_{j} F_{j}\left(\frac{\partial \vartheta}{\partial y}-\frac{w_{j}}{R}\right)^{2}+E_{j} J_{x j}\left(\frac{\partial^{2} w_{j}}{\partial x^{2}}+\frac{w_{j}}{R^{2}}\right)^{2}+\right. \\
& \left.+E_{j} J_{z j}\left(\frac{\partial^{2} u_{i}}{\partial y^{2}}-\frac{\varphi_{k p j}}{R}\right)^{2}+G_{j} J_{k p j}\left(\frac{\partial \varphi_{k p i}}{\partial y}+\frac{1}{R} \frac{\partial u_{j}}{\partial y}\right)^{2}\right] d \varphi \tag{5}
\end{align*}
$$

$$
K_{j}=\rho_{j} F_{j} \int_{0}^{2 \pi}\left[\left(\frac{\partial u_{j}}{\partial t}\right)^{2}+\left(\frac{\partial \vartheta_{j}}{\partial t}\right)^{2}+\right.
$$

$$
\begin{equation*}
\left.+\left(\frac{\partial w_{j}}{\partial t}\right)^{2}+\frac{J_{k p j}}{F_{j}}\left(\frac{\partial \varphi_{k p j}}{\partial t}\right)^{2}\right] d \varphi \tag{6}
\end{equation*}
$$

In the expressions (2)-(6) $u, \vartheta, w$ are the displacements of the points of the cylindrical shell, $R, h$ is a radius and thickness of the cylindrical shell, respectively, $E_{j}$ is the elasticity modulus of the $j$ th ring, $F_{j}$ is the cross section area of the $j$ th ring, $I_{x j}, I_{k p . j}$ are inertia moments of the cross-section of the $I_{x j}, I_{k p}, j$ th ring, $G_{j}$ is the elasticity modulus of the $j$ th ring at the shift, $u_{j}, \vartheta_{j}, w_{j}$ are displacements of the points of the $j$ th ring, $\rho_{j}$ is the density of the material of the $j$ th ring.

The work done by the pressure force $q_{x}, q_{y} q_{z}$ acting on the cylindrical shell as viewed from viscous liquid in the displacements of the shell is written as follows:
$A_{0}=-\int_{0}^{L} \int_{0}^{2 \pi}\left(q_{x} u+q_{y} \vartheta+q_{z} w\right) d x d y$
As a result, for the total energy of the heterogeneous orthotropic cylindrical shell dynamically interacting with liquid and stiffened with rings along the thickness we get:

$$
\begin{equation*}
J=V+\sum_{i=1}^{k_{1}}\left(\Pi_{i}+K_{i}\right)+A_{0} \tag{8}
\end{equation*}
$$

where, $k_{2}$ is the amount of the rings.
The motion of the liquid is written by means of the Navier-Stocks Equation [14]:
$\rho_{m} \frac{\partial \vec{\vartheta}}{\partial t}=-\operatorname{grad} p+\frac{1}{3} \operatorname{graddiv} \vec{\vartheta}+\vec{\mu} \nabla^{2} \vec{\vartheta}$
To the expression (8), we add the boundary and contact boundary conditions. We assume that the edges of the shell are highly supported, i.e. in $x=0$ and $x=L$ the conditions $\quad N_{x}=0 ; M_{x}=0 ; w=0 ; \tilde{\vartheta}=0 \quad$ are satisfied. $N_{x}, M_{x}, \tilde{\vartheta}, w$ are displacements of the longitudinal force, bending moment and the shell points in the circular and normal direction.

At the inner points of the shell surface $\left(r=R-\frac{h}{2}\right)$
$\vartheta_{x}=\frac{\partial u}{\partial t}, \vartheta_{\theta}=\frac{\partial \tilde{\vartheta}}{\partial t}, \vartheta_{r}=\frac{\partial w}{\partial t}$
$q_{x}=-\sigma_{r x}, q_{y}=-\sigma_{r \theta}, q_{z}=-p$
where, $\vec{\vartheta}\left(\vartheta_{x}, \vartheta_{r}, \vartheta_{\theta}\right)$ is the velocity of the points of the liquid, $p$ is pressure at any point of the liquid, $q_{x} . q_{y}, q_{z}$ are the projections of the force acting on the cylindrical shell as viewed from liquid, $\sigma_{r x}, \sigma_{r \theta}$ are viscosity forces [14].

The motion equation of the construction consisting of an orthotropic, heterogeneous shell and ring subjected to the action of the viscous liquid is determined from the stationarity condition of Ostrogradsky-Hamilton action: $\delta W=0$
where, $W=\int_{t^{\prime}}^{t^{\prime \prime}} J d t$ is Hamilton's action, $t^{\prime}$ and $t^{\prime \prime}$ is any moment of time.

## 3. PROBLEM SOLUTION

We find the solution of the Navier-Stocks equation
$\vec{\vartheta}=\operatorname{grad} \varphi+\operatorname{rot} \vec{\psi}$
with respect to the scalar $\varphi$ and vector $\vec{\psi}$ functions from the following equations:
$\rho_{m} \frac{\partial \varphi}{\partial t}+p-\frac{4}{3} \bar{\mu} \Delta \varphi=0$
$-\bar{\mu} \Delta \vec{\psi}+\rho_{m} \frac{\partial \vec{\psi}}{\partial t}=0$
In the case of Newton liquid, to the Equations (13) and (14) we add the continuity equation $\frac{\partial \rho}{\partial t}+\rho_{m} \operatorname{div} \vec{\vartheta}=0$ and state equations $\frac{\partial p}{\partial p}=a_{*}^{2}$ as well.

From the state equation, continuity equation and by the Navier-Stocka equations, for finding the pressure $p$ we get the following equation:
$\frac{1}{a_{*}^{2}} \frac{\partial^{2} p}{\partial^{2} t}=\nabla^{2}\left(p+\frac{4 \bar{\mu}}{3 \rho_{m} a_{*}^{2}} \frac{\partial p}{\partial t}\right)$
The solution of the Equation (15) is in the following form:

$$
\begin{equation*}
p=\left(p_{0} J_{n}(\lambda r)+c_{0} Y_{n}\left(\lambda_{r}\right)\right) \exp i(k x+n \theta+\omega t) \tag{16}
\end{equation*}
$$

where, $\lambda=\sqrt{\frac{\omega^{2}}{a_{*}^{2}\left(1+i \frac{4 \bar{\mu} \omega}{3 \rho_{m} a_{*}^{2}}\right)}-k^{2}}, J_{n}, Y_{n}$ are first and second kind $n$th order Bessel functions, $n$ is the amount of waves in the circular direction, $k=\frac{m \pi}{L}, m$ is the amount of longitudinal waves, $\bar{\mu}$ is the dynamic viscosity factor, $\rho_{m}$ is the density of the liquid in the perturbed form, $a_{*}$ is propagation speed of small perturbations in liquid, $\omega$ is frequency, $p_{0}, c_{0}$ are constants. Since at the point $r=0$ the function $p$ is bounded, we take $c_{0}=0$ and at the end for $p$ we get:

$$
\begin{equation*}
p=p_{0} J_{n}(\lambda r) \exp i(k x+n \theta+\omega t) \tag{17}
\end{equation*}
$$

From (13) for $\varphi$, we get:
$\Delta \varphi-\frac{3 \rho_{m}}{4 \bar{\mu}} \frac{\partial \varphi}{\partial t}=p_{0} J_{n}(\lambda r) \exp i(k x+n \theta+\omega t)$
The solution of the homogeneous differential equation corresponding to the Equation (18) is as follows:
$\varphi=C_{1} I_{n}(\tilde{k} r)=C_{2} K_{n}(\tilde{k} r)$
where, $\quad k=\sqrt{k^{2}+\frac{3 i \omega \rho_{m}}{4 \bar{\mu}}}, I_{n}(\tilde{k r}), K_{n}(\tilde{k r})$ are modified, first and second kind Bessel functions, $C_{1}, C_{2}$ are constants.

Using the method of variation of a constant, we can write Equation (18):
$\varphi(r)=p_{0} f(r)+\mu_{1} I_{n}(\tilde{k} r)$
$\Delta(r)=I_{n}(\tilde{k} r) K_{n}^{\prime}(\tilde{k} r)-I_{n}^{\prime}(k r) K_{n}(\tilde{k} r)$
With respect to the vector function $\vec{\psi}\left(\psi_{1}, \psi_{2}, \psi_{3}\right)$ the Equation (14) takes the form:
$\psi_{i}^{\prime \prime}(r)+\frac{1}{r} \psi_{i}^{\prime}(r)-\left(k^{2}+\frac{i \omega \rho_{m}}{\bar{\mu}}\right) \psi_{i}(r)=0$
The solution of the Equation (20) is as follows:
$\psi_{i}=\mu_{2} J_{n}(q r) \quad(i=1,2,3)$
where
$q=\sqrt{k^{2}+\frac{i \omega}{\bar{\mu}}}$.
Using (19), (21), from the expression (12), for the velocity components we get:
$v_{x}=\left[-\frac{k \omega}{\rho_{m} a_{*}^{2}} p_{0} f(r)+i k J_{n}(k r) \mu_{1}+\left(i n J_{n}(q r)-q J_{n}^{\prime}(q r)\right) \mu_{2}\right] \times$
$\times \exp i(k x+n \theta+\omega t)$
$v_{\theta}=\left[-\frac{n \omega}{\rho_{m} a_{*}^{2}} p_{0} f(r)+i n J_{n}(k r) \mu_{1}+i\left(k-\frac{n}{R}\right) J_{n}(q r) \mu_{2}\right] \times$
$\times \exp i(k x+n \theta+\omega t)$
$v_{r}=\left[\frac{i \omega}{\rho_{m} a_{*}^{2}} p_{0} f^{\prime}(r)+k J_{n}^{\prime}(k r) \mu_{1}+\left(q J_{n}^{\prime}(q r)-i k J_{n}(q r)\right) \mu_{2}\right] \times$
$=\exp i(k x+n \theta+\omega t)$
For the viscosity forces we obtain [12]:
$\sigma_{r x}=\bar{\mu}\left[-\frac{2 k \omega}{\rho_{m} a_{*}^{2}} f^{\prime}(r) p_{0}+2 i k J_{n}^{\prime}(k r) \mu_{1}+\right.$
$\left.+\left(-k\left(k-\frac{n}{R}\right) J_{n}(q r)+\frac{i n}{R} J_{n}^{\prime}(q r)-J_{n}^{\prime \prime}(q r)\right) \mu_{2}\right] \times$
$\times \exp i(k x+n \theta+\omega t)$
$\sigma_{r \theta}=\bar{\mu}\left[-\frac{2 n \omega}{R \rho_{m} a_{*}^{2}} f^{\prime}(r) p_{0}+\frac{2 i n}{R} J_{n}^{\prime}(k r) \mu_{1}+\right.$
$\left.+\left(i\left(k-\frac{n}{R}\right) J_{n}(q r)-i k J_{n}^{\prime}(q r)+J_{n}^{\prime \prime}(q r)\right) \mu_{2}\right] \times$
$\times \exp (k x+n \theta+\omega t)$
$\sigma_{r r}=p_{0} J_{n}(\lambda r) \exp i(k x+n \theta+\omega t)$
Using the expressions (10) and (23), we can find the forces $q_{x}, q_{y}, q_{z}$ acting on the cylindrical shell as viewed from the viscous liquid:
$q_{x}=\bar{\mu}\left[-\frac{2 k \omega}{\rho_{m} a_{*}^{2}} f^{\prime}(R) p_{0}+2 i k J_{n}^{\prime}(k R) \mu_{1}+\right.$
$\left.+\left(-k\left(k-\frac{n}{R}\right) J_{n}(q R)+\frac{i n}{R} J_{n}^{\prime}(q R)-J_{n}^{\prime \prime}(q R)\right) \mu_{2}\right] \times$
$\times \exp i(k x+n \theta+\omega t)$
$q_{z}=p_{0} J_{n}(\lambda R) \exp i(k x+n \theta+\omega t)$
$q_{y}=\bar{\mu}\left[-\frac{2 n \omega}{R \rho_{m} a_{*}^{2}} f^{\prime}(R) p_{0}+\frac{2 i n}{R} J_{n}^{\prime}(k R) \mu_{1}+\right.$
$\left.+\left(i\left(k-\frac{n}{R}\right) J_{n}(q R)-i k J_{n}^{\prime}(q R)+J_{n}^{\prime \prime}(q R)\right) \mu_{2}\right] \times$
$\times \exp i(k x+n \theta+\omega t)$
By means of these forces, we can calculate the work of $A_{0}$ contained in (8).

We look for the displacements of the shell in the following form:

$$
\begin{align*}
& u=u_{0 k n} \exp i(k x+n \theta+\omega t) \\
& \vartheta=\vartheta_{0 k n} \exp i(k x+n \theta+\omega t)  \tag{25}\\
& w=w_{0 k n} \exp i(k x+n \theta+\omega t)
\end{align*}
$$

Considering in the functional (8) $x_{1}=0, x_{2}=1$, $y_{1}=0, y_{2}=2 \pi, \quad t^{\prime}=0, t^{\prime \prime}=\frac{\pi}{\omega}, \quad$ we integrate the expressions (11) with respect to the variables $x, y, t$ and instead of the functional (8) we get a function with respect to the constants $u_{0 k n}, \vartheta_{0 k n}, w_{0 k n}$. From the stationarity condition of this function, we get the following system of equations:
$\frac{\partial W}{\partial u_{0 k n}}=0 ; \frac{\partial W}{\partial \vartheta_{0 k n}}=0 ; \frac{\partial W}{\partial w_{0 k n}}=0$
Since the system (26) is homogeneous, necessary and sufficient condition for existence of its nonzero solution is equality of its determinant to zero. As a result, with respect to $\omega_{1}$, we get following transcendental equation:

$$
\begin{equation*}
\operatorname{det}\left\|a_{i j}\right\|=0, i, j=1,3 \tag{27}
\end{equation*}
$$

## 4. CONCLUSIONS

The Equation (27) was solved by the numerical method. For the parameters included into the solution of the problem we take the following quantities:


Figure 2. Dependence of the frequency parameter on the dynamic viscosity factor
$l=0.8 \mathrm{~m} ; \rho_{0}=\rho_{i}=7800 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}, \tilde{E}_{i}=6.67 \times 10^{9} \mathrm{~Pa} ; v=0.11$
$\left|h_{j}\right|=0.1375 \times 10^{-1} \mathrm{R} ; k_{2}=10 ; \bar{\mu}=0.355$
$J_{x j}=300.8 \mathrm{~mm}^{4} ; J_{k p j}=0.35 \mathrm{~mm}^{4} ; \alpha=\beta=\gamma=0.5$
The linear case of changes in heterogeneity was considered:
$E_{1}(z)=E_{1}\left[1+\alpha\left(\frac{z}{h}\right)\right], E_{2}(z)=E_{2}\left[1+\beta\left(\frac{z}{h}\right)\right]$
$\rho(z)=\rho_{0}\left[1+\alpha\left(\frac{z}{h}\right)\right]$


Figure 3. Dependence of the frequency parameter on the amount of bars
The results of calculations were given in Figure 2 in the form of dependence of the frequency parameter on dynamical viscosity factor, in Figure 3 in the form of dependence of the frequency parameter on the amount of bars. As can be seen from Figure 2 as the value of the dynamic viscosity increases, the value of the frequency parameter decreases. As can be seen from Figure 3, as the amount of bars increases, the value of the frequency parameter at first increases, and after certain value decreases. This is explained by the fact that as the amount of bars increases, the strength of the construction increases, after certain value due to the strengthening of the inertia effect begins to decrease.

As a result of calculations it was determined that if vibrations prevail in a normal plane, then stiffening with rings, and vice versa, if vibrations prevail in tangential plane, then stiffening with bars is more affordable.

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