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ANALYSIS OF MOTION STABILITY OF WHEELSET BASED ON LYAPUNOV FUNCTION METHOD

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Abstract- The characteristics of the kinematic vibrations of a free wheelset with a conical profile and the influence of the mathematical structure of forces on the stability of motion are considered. The qualitative analysis of the stability conditions of the hypothetical wheel module of the rail crew was carried out on the basis of the direct Lyapunov method using the concept of mathematical structure of forces of the mechanical system. On the one hand, it facilitates the process of constructing new quadratic Lyapunov functions in a matrix form, and on the other, it can serve as a benchmark for practical structural changes in the system in order to improve its passive stabilization. Obtained in the work necessary and sufficient conditions of stability of the wheel pair with a conical profile can be used in the study of the stability of modern designs of rail crews. A simple method is proposed for choosing the characteristics of the rigidity of the axle-box suspension of the wheelset, which provides the necessary stability margin for the parameter of the longitudinal speed of movement. In addition, the critical speed does not depend on the value of the creep coefficient (it is determined by the given stiffness characteristic of the axle-box suspension and geometric parameters of wheelset, including conical parameter).

Keywords: Wheelset Model, Condition Stability, Lyapunov Functions.

1. INTRODUCTION

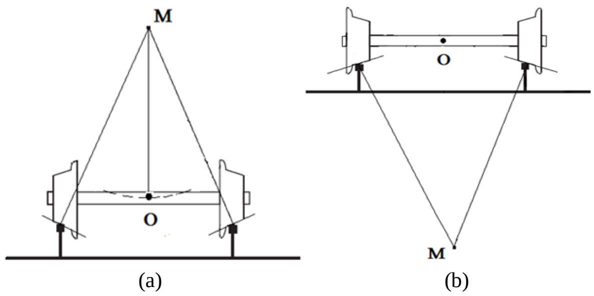
Dynamics of the wheelset continues to attract attention of numerous researchers [6, 7, 25, 33, 34], despite the considerable age of the subject of the study [8, 10, 18, 19, 20, 21, 26, 27]. The reason for this interest is simple: it is associated with the emergence of new types of high-speed rail vehicles and issues of ensuring safe railway operation. However, the very mechanical essence of the phenomenon, which serves as a reliable control system for the obtained numerical results, often can be lost in the practice of a specific comprehensive numerical analysis of the next model [1, 30]. Meanwhile, numerical testing of general analytical results on the basis of more complete or new refined mathematical models makes it possible to evaluate the area of results continuity.

Current article presents the results of an analytical analysis of the wheelset stability conditions in straight-line track sections. The results were obtained on the basis of Lyapunov function method [2, 3, 4, 5, 9, 14, 15, 31, 32]. Also, the fundamental possibility of constructive solutions (aimed at providing passive stabilization of a hypothetical model of a wheel module) is discussed on basis of concept of force structure [8, 11, 12, 13, 16, 17, 23, 28, 29].

The presence of conicity of the wheelset's rolling surface is the most significant factor (element) of its construction design and the main reason for its peculiar dynamic behavior. Introducing of conicity to the wheelset design is one of the intuitive methods of passive centering (stabilization) of straight-line motion of the wheelset with a cylindrical profile of rolling, which is "indifferent" to possible changes in the direction of motion (occurrence of yaw angles).

Figure 1,a illustrates the positive effect of passive stabilization of the wheelset with the introducing of the conic element - the wheelset's mass center is in unperturbed position (in the absence of lateral displacement); and in this case it takes the lowest of all possible positions. Since sufficiently small transverse deviations from the neutral position are realized due to rotation of relatively instantaneous center of velocity *M* in the vertical transverse plane. There is the opposite effect for the case (Figure 1,b) (in the unperturbed position, the mass center of the wheelset is in the highest position, which would lead to slippage).

It should be noted that the analysis of the dynamic stability of the wheelset is much more complicated.



(a) (b)

Figure 1. Illustration of the conicity effect impact, a- it is auspicious in terms of centering; b- it leads to "slippage" from the highest possible position of the mass center

2. KINEMATIC OSCILLATIONS OF A FREE WHEELSET

Assuming that there is no slippage at the points of contact between the wheelset and the rails, this leads to relations that are called equations of kinematic oscillations.

$$\dot{y} = V\psi;\ \dot{\psi} = -\frac{V\gamma}{dr_0} y \quad (1)$$

The first equation indicates the absence of transverse slippage. Total slippage is generated by the velocity $\dot{y}$ and the transverse component of the longitudinal velocity $-V\psi$ (Figure 2); the second equation indicates the absence of slippage in the longitudinal direction (indeed, if there is a transverse displacement y on the left sloping, the rolling radius will increase by a value $\Delta r = \gamma \cdot y$, and the linear longitudinal velocity will increase by a value $\Delta r \cdot \Omega = \gamma \cdot y \cdot V / r_0$, then the yawing angular velocity $\dot{\psi}$ is determined by the second equation of the system (1)). In this case, deviations of the variables satisfy the relation (2).

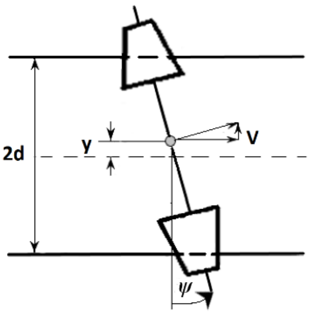


Figure 2. The position of the model is determined by two phase coordinates: y, ψ - transverse deviation of the wheelset's inertia center relative to the track centering and the wobble angle accordingly

The last one is obtained as a differential consequence of the system (1): let us make a linear combination of corresponding parts of the system (1) (a factor of the first equation is $\frac{\gamma}{dr_0} \cdot y$, a factor of the second equation is ψ); the left side of the resulting expression is the time derivative of the expression (2) (integral of motion):

$$\frac{\gamma}{d \cdot r_0} y^2 + \psi^2 = \text{const} \quad (2)$$

3. ANALYSIS OF STRAIGHT LINE MOTION STABILITY OF THE WHEELSET WITH ELASTIC ELEMENTS IN AXLE-BOX SUSPENSION

Let us make the analysis of straight line motion stability of the wheelset (Carter's creep model) based on the dynamic equations of motion [6, 10, 25].

If only the linear creep is taken into account, the equations of perturbed motion have following form [10]:

$$\begin{gathered} m\ddot{y} + 2k_y y + 2k_1\left(\frac{\dot{y}}{V} - \psi\right) = 0 \\ J\ddot{\psi} + 2k_x b^2 \psi + 2k_1 d^2\left(\frac{\dot{\psi}}{V} + \frac{\gamma \cdot y}{dr_0}\right) = 0 \end{gathered} \quad (3)$$

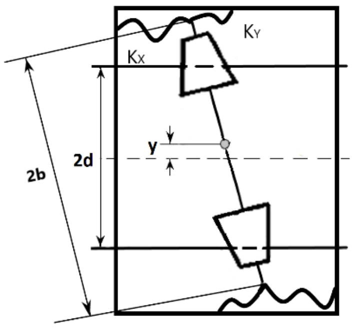


Figure 3. Inertial frame of reference (rigidly connected to the frame) moves at a constant speed along a straight line coinciding with the track centerline

We choose the coefficients of rigidity k_x, k_y of the elastic elements so that the subsystems (corresponding to the phase variables y, ψ) have coinciding fundamental frequencies.

$$\frac{2k_y}{m} = \frac{2k_x b^2}{J} = \tau$$

The unperturbed motion of the wheelset corresponds to zero values of the phase variables $y = 0$, $\psi = 0$.

Let us represent the equations of the wheelset perturbed motion relative to the new variables p and q (which are slipping's in the transverse and longitudinal directions), taking into account the nonlinear nature of the creep forces.

$$\begin{cases} \dot{y} = p + V\psi \\ \dot{p} = \left(\frac{V^2\gamma}{dr_0} - \tau\right) y - \frac{V}{d} q - 2\frac{k_1 p}{mV\sqrt{1 + (k_1\varepsilon / k_f P)^2}} \\ \dot{\psi} = -\frac{V\gamma}{dr_0} y + \frac{1}{d} q \\ \dot{q} = \frac{V\gamma}{r_0} p + \left(\frac{V^2\gamma}{r_0} - \tau \cdot d\right)\psi - 2\frac{k_1 d^2 q}{JV\sqrt{1 + (k_1\varepsilon / k_f P)^2}} \end{cases} \quad (4)$$

where, $\varepsilon = \sqrt{(p^2 + q^2)/V^2}$ is relative slip.

The Lyapunov function in the form of a linear combination of its differential consequences [5] can be constructed for the system (4): each equation is multiplied by the corresponding variable with some coefficient in order to construct an expression that would be: a) the time derivative of some quadratic form of the system variables; b) quadratic form with constant terms. Based on this approach, the canonical quadratic form was obtained.

$$V = \frac{1}{2}\left[\frac{\gamma}{r_0} p^2 + \frac{1}{d} q^2 + \frac{\gamma}{r_0}\left(\tau - \frac{V^2\gamma}{dr_0}\right) y^2 + d\left(\tau - \frac{V^2\gamma}{dr_0}\right)\psi^2\right] \quad (5)$$

Its time derivative is non-positive due to the system (4)

$$\dot{V} = -\frac{2k_1}{V\sqrt{1 + (k_1\varepsilon / k_f P)^2}}\left(\frac{p^2\gamma}{mr_0} + \frac{q^2 d}{J}\right)$$

We obtain the expression for critical velocity from the condition of positive definiteness of Lyapunov function:

$$V_{kr} = \sqrt{\frac{\tau dr_0}{\gamma}} \quad (6)$$

While $V < V_{kr} y$, the unperturbed motion is asymptotically stable, and the nonlinear nature of the creep forces does not place restrictions on the domain of attraction of the unperturbed motion; while $V > V_{kr}$, the unperturbed motion is unstable.

While τ=0, quadratic form (5) corresponds to the free wheelset; also, the structural instability occurs.

Conditions of single-point contact place some restrictions on the variables' disturbance values y, ψ; therefore, it is necessary to evaluate the domain of attraction of the unperturbed motion, which would guarantee the absence of undesired contact between the wheel flange and the rail.

Let us move to dimensionless variables. Since the expression of Lyapunov function has dimension of acceleration, we choose parameters V, d for nondimensionalization. Let $\bar{R}$ be a maximum radius of the circle which is lying entirely in some given domain D, i.e., $\bar{y}^2 + \psi^2 = \bar{R}^2$. Consider a four-dimensional ball of the same radius, i.e., $\bar{y}^2 + \psi^2 + \bar{p}^2 + \bar{q}^2 = \bar{R}^2$. Its projection into the plane $\bar{y}, \psi$ does not leave the circle of the radius $\bar{R}$.

Consider the domain of attraction of the unperturbed motion given by the relation:

$$\frac{\gamma}{\bar{r}_0}\bar{p}^2 + \bar{q}^2 + \frac{\gamma}{\bar{r}_0}(\tau - \frac{\gamma}{\bar{r}_0})\bar{y}^2 + (\tau - \frac{\gamma}{\bar{r}_0})\psi^2 < \alpha \bar{R}^2$$

It ensures that subsequent perturbations of variables do not leave the circle of radius $\bar{R}$, i.e., the single point contact will be realized here ($\alpha = \min\{\frac{\gamma}{\bar{r}_0}, 1, \frac{\gamma}{\bar{r}_0}(\tau - \frac{\gamma}{\bar{r}_0}), (\tau - \frac{\gamma}{\bar{r}_0})\}$).

4. ANALYZING POSSIBILITY OF STABILIZATION THE MOTION OF THE WHEELSET WITH THE BALANCER OF FINAL INERTIA

The linearized equations of the perturbed motion of the system are as follows:

$$m\ddot{y} + 2k_y(y - y_1) = 0$$

$$m_1\ddot{y}_1 + 2k_1(y_1 - y) + 2k_1(\frac{\dot{y}_1}{v} - \psi_1) = 0$$

$$J\ddot{\psi} + 2k_x b^2(\psi - \psi_1) = 0$$

$$J_1\ddot{\psi}_1 + 2k_x b^2(\psi_1 - \psi) + 2k_1 d^2(\frac{\dot{\psi}_1}{v} + \frac{\gamma y_1}{dr_0}) = 0$$

where, m, J, m_1, J_1 is mass and central vertical moment of inertia of the balancer and the wheelset accordingly. For the case of $\frac{k_y}{m_1} = \frac{k_x b^2}{J_1} = \tau$; $J_1 = m_1 d^2$; $J = mb^2$, we can use the previously constructed function as the basic Lyapunov function. The corresponding quadratic forms of the phase variables of the system are as follows:

$$\frac{1}{v}\ddot{y}_1\dot{y}_1 + \frac{2k_y}{m_1 v}(y_1 - y)\dot{y}_1 + \frac{2k_1}{m_1 v^2}\dot{y}_1^2 - \frac{2k_1}{m_1 v}\psi_1\dot{y}_1 = 0$$

$$\frac{dr_0}{v\gamma}\ddot{\psi}_1\dot{\psi}_1 + \frac{2k_x b^2 dr_0}{J_1 v\gamma}(\psi_1 - \psi)\dot{\psi}_1 + \frac{2k_1 d^3 r_0}{J_1 v^2 \gamma}\dot{\psi}_1^2 + \frac{2k_1 d^2}{J_1 v} y_1\dot{\psi}_1 = 0$$

$$\ddot{\psi}_1 y_1 - \ddot{y}_1\psi_1 + \frac{2k_x b^2}{J_1}(\psi_1 - \psi)y_1 + \frac{2k_1 d^2}{J_1 v}\dot{\psi}_1 y_1 + \frac{2k_1 d\gamma}{J_1 r_0} y_1^2 - \frac{2k_y}{m_1}(y_1 - y)\psi_1 - \frac{2k_1}{m_1 v}\dot{y}_1\psi_1 + \frac{2k_1}{m_1}\psi_1^2 = 0$$

Supplement it with the second group of equations (for the balancer):

$$\frac{m}{m_1 v}\ddot{y}\dot{y} + \frac{2k_y}{m_1 v}(y - y_1)\dot{y} = 0$$

$$\frac{Jdr_0}{J_1 v\gamma}\ddot{\psi}\dot{\psi} + \frac{2k_x b^2 dr_0}{J_1 v\gamma}(\psi - \psi_1)\dot{\psi} = 0$$

$$\frac{m}{m_1}(\ddot{\psi} y - \ddot{y}\psi) + \frac{2k_x}{m_1}(\psi - \psi_1)y - \frac{2k_y}{m_1}(y - y_1)\psi = 0$$

As a result, we obtain an auxiliary function as an alternating-sign quadratic form:

$$V = \frac{1}{2}\Big[\frac{m}{m_1 v}\dot{y}^2 + \frac{1}{v}\dot{y}_1^2 + \frac{2k_y}{m_1 v}(y - y_1)^2 + \frac{Jdr_0}{J_1 v\gamma}\dot{\psi}^2 + \frac{dr_0}{v\gamma}\dot{\psi}_1^2 + \frac{2k_x b^2 dr_0}{J_1 v\gamma}(\psi - \psi_1)^2 + \frac{2m}{m_1}(\dot{\psi} y - \dot{y}\psi) + 2(\dot{\psi}_1 y_1 - \dot{y}_1\psi_1)\Big]$$

Its derivative has a derivative of constant signs due to the equations of perturbed motion (7).

$$\dot{V} = -2k_1\left[\frac{1}{m_1}(\frac{\dot{y}_1}{v} - \psi_1)^2 + \frac{dr_0}{J_1\gamma}(\frac{d\dot{\psi}_1}{v} + \frac{\gamma y_1}{r_0})^2\right] \quad (7)$$

The latter indicates the impossibility of stabilizing the wheelset using one inertial balancer.

5. CONSTRUCTION OF THE QUADRATIC LYAPUNOV FUNCTION FOR LINEAR MECHANICAL SYSTEM

The Lyapunov functions method can be effective while analyzing the stability of complex mechanical systems and searching for meaningful solutions for their passive stabilization; namely, it can be effective in terms of meaningful techniques for constructing quadratic Lyapunov functions, which using the concept of the mathematical structure of the mechanical system forces.

Thomson-Tait-Chataev's theorems [28, 29] on the influence of dissipative and gyroscopic forces on the stability of a linear conservative system were the first results in that process; the results of *I*. Metelitsyn were also important for the development of this direction [17, 13].

Below are the results of [22, 24], in which a general meaningful approach to the choice of quadratic Lyapunov functions (which are presented in matrix form) is traced. In addition, their comparison will be useful for developing a strategy of searching for new function options as candidates for the role of quadratic Lyapunov function in matrix form.

For a linear dynamic system of general form

$$A\ddot{x}+D\dot{x}+G\dot{x}+Fx+Ex=0 \tag{8}$$

where, A, D, F are symmetric matrices of inertial, dissipative and potential forces; G and E are skew-symmetric matrices of gyroscopic and non-conservative positional forces, the following statement is true [22].

- Theorem. Let us suppose

$$A=\{a_{ij}\}; D-A=h\{d^*_{ij}\}; F=l\{f_{ij}\}$$

are positive definite matrices ($h>0$, $l>0$), then the zero solution to the system (8) is asymptotically stable. If $G\neq E$, then the zero solution to system (8) is asymptotically stable while sufficiently large $h>0$ or $l>0$ (regardless of gyroscopic and non-conservative forces).

- Proof of the theorem.

Let us choose Lyapunov function as following:

$$V=\frac{1}{2}[\dot{x}^T A\dot{x}+2\dot{x}^T Ax+x^T(F+D)x] \tag{9}$$

By virtue of the system (8), its derivative is determined by the expression of:

$$\dot{V}=\frac{1}{2}[\dot{x}^T A\ddot{x}+\dot{x}^T A\dot{x}+x^T A\ddot{x}+x^T(F+D)\dot{x}] \tag{10}$$

After substituting the expression $A\ddot{x}$ (8) in the expression (10) and combining the terms, we get a negative definite form:

$$\dot{V}=\frac{1}{2}[-\dot{x}^T(D-A)\dot{x}-x^T Fx] \tag{11}$$

Positive definiteness of the form (9) follows directly from bringing it to the form of:

$$V=\frac{1}{2}[\dot{x}^T A\dot{x}+2\dot{x}^T Ax+x^T Ax+x^T(F+D-A)x]$$

The form (9) is positive definite when the form $x^T(F+D-A)x$ is positive definite.

If $G\neq E$, then we have the following expression:

$$\dot{V}=\frac{1}{2}[-\dot{x}^T(D-A)\dot{x}-\dot{x}^T(E-G)x-x^T Fx] \tag{12}$$

It can be shown that the form (12) is negative definite while sufficiently large $h>0$ or $l>0$ (necessary and sufficient conditions for the sign-definiteness of bilinear forms $V(x,\dot{x}); \dot{V}(x,\dot{x})$ are related to the condition of positive definiteness of the corresponding block matrix. Thus, the condition of positive definiteness of a bilinear form is determined by the condition of positive definiteness of a block matrix, which is generating a bilinear form $V(x,\dot{x})$.

$$\begin{pmatrix} A & A \\ A^T & F+D \end{pmatrix}\succ 0 \Leftrightarrow F+D-A^T\cdot A^{-1}\cdot A\succ 0$$

The conditions of negative definiteness of a bilinear form $\dot{V}(x,\dot{x})$ coincide with the condition of positive definiteness of a block matrix, which is generating a bilinear form $\dot{V}(x,\dot{x})$.

$$\begin{pmatrix} D-A & 1/2(E-G) \\ 1/2(E-G)^T & F \end{pmatrix}\succ 0 \Leftrightarrow D-A-1/2(E-G)\cdot F^{-1}\cdot 1/2(E-G)^T\succ 0$$

Theorem is proved.

Thus, the choice of quadratic Lyapunov function in the form (8) indicates only the possibility of stabilization of a general system by increasing dissipative or potential forces; the possibility of stabilization of an unstable potential system by gyroscopic forces is not covered by this case. This gap can be filled by using the most general structure of quadratic Lyapunov function, which is proposed in [24]. It is proposed to search the quadratic Lyapunov function in the following form

$$V=x^T Fx+\dot{x}^T G\dot{x}+\dot{x}^T Hx \tag{13}$$

for a general system (corresponds to the cited work up to the designation of phase variables)

$$M\ddot{x}+C\dot{x}+Kx=0 \tag{14}$$

where, F and G are symmetric matrices, and the matrix H can have an arbitrary structure. The derivative of quadratic Lyapunov function (13) has the following form by virtue of the system (14):

$$\begin{aligned}\dot{V}&=-x^T K^T M^{-1}H\,x-\dot{x}^T\left[2GM^{-1}C-H\right]\dot{x}-\\&-\dot{x}^T\left[2F-2GM^{-1}K-C^T M^{-1}H\right]x\end{aligned} \tag{15}$$

As can be seen from the expression (15), the choice of a matrix $F=GM^{-1}K+{}^1\!/_2 C^T M^{-1}H$ can greatly simplify the analysis of sign-definiteness (provided that the expression on the right is a symmetric matrix). In fact, the structure of only two matrices (G and H) should be chosen. Although this fact is a poor consolation, since the considered quadratic Lyapunov function case (9) is "weakly" visible in this generalized form (it was necessary to choose $G = M$ and $H = M$).

Next, we consider two examples which illustrate the possibilities of using a generalized quadratic Lyapunov function of the form (13).

Example 1. Gyroscopic stabilization of unstable potential system

$$\ddot{x}_1-g\dot{x}_2-kx_1=0;\ M=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix};\ C=\begin{pmatrix}0 & -g\\ g & 0\end{pmatrix};$$

$$K=\begin{pmatrix}-k & 0\\ 0 & -k\end{pmatrix};\ \ddot{x}_2+g\dot{x}_1-kx_2=0$$

We choose $G = M$ and $H = C$, then:

$$K^T M^{-1}H=\begin{pmatrix}0 & g\cdot k\\ -g\cdot k & 0\end{pmatrix}$$

$$F=K+{}^1\!/_2 C^T M^{-1}C=\begin{pmatrix}-k+g^2/2 & 0\\ 0 & -k+g^2/2\end{pmatrix}$$

$$V=x^T Fx+\dot{x}^T G\dot{x}+\dot{x}^T Hx;\ \dot{V}=0$$

Condition of the positive definiteness of the quadratic Lyapunov function (14) as $g^2/4 > k$.

Example 2. Sufficient instability of mechanical system in the presence of potential and non-conservative positional forces

$$\ddot{x}_1 + k_1 x_1 - e x_2 = 0$$

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\ K = \begin{pmatrix} k_1 & -e \\ e & k_2 \end{pmatrix} \quad (16)$$

$$\ddot{x}_2 + e x_1 + k_2 x_2 = 0$$

Let us use $V = \dot{x}^T H x;\ H = -H^T$, as a quadratic Lyapunov functions, then:

$$\dot{V} = -x^T K^T H\, x = -\tfrac{1}{2} x^T (K^T H + H^T K) x$$

Statement: If there exists a skew-symmetric matrix H such that the matrix $K^T H$ has a sign-definite symmetric part, then the zero solution to the system (16) is unstable. For example, the condition of positive definiteness $K^T H + H^T K \succ 0$, while:

$$H = \tfrac{1}{2}(K - K^T)$$

$$\begin{vmatrix} 2e^2 & e(k_2 - k_1) \\ e(k_2 - k_1) & 2e^2 \end{vmatrix} > 0 \Rightarrow 2e > k_1 - k_2$$

(agreement $k_1 > k_2$ is adopted).

6. CONCLUSION

The procedure of constructing a quadratic Lyapunov function was carried out for a certain relation between the stiffness characteristics parameters of axle-box suspension of the wheelset model with a conical profile of the rolling surface. Quadratic Lyapunov function provides the necessary and sufficient conditions for the stability of the unperturbed straight-line motion of the wheelset. Moreover, the nonlinear nature of the creep forces does not impose restrictions on the domain of attraction. In addition, the critical speed does not depend on the value of the creep coefficient (it is determined by the given stiffness characteristic of the axle-box suspension and geometric parameters of wheelset, including conical parameter).

The results of analysis regarding the force structure influence on the stability of linear mechanical systems of a general form are presented, which make it possible to use the possibilities of structural changes in order to passively stabilize the hypothetical model of the wheel module. New options of quadratic Lyapunov functions for model mechanical systems are proposed.

NOMENCLATURES

1. Symbols / Parameters

A: matrix of inertia forces
D: matrix of dissipation forces
G: matrix of gyroscopic forces
F: matrix of potential forces
E: matrix of non - conservative forces
V: scalar Lyapunov function
h, l: constants > 0
m_1: mass of the wheel pair
J_1: moment of inertia of the wheel pair
v: longitudinal velocity
k_1: creep ratio
k_x: stiffness coefficient of axle box in longitudinal direction
k_y: stiffness coefficient of axle box in transverse direction
y: lateral bearing of the wheel pair
r_0: radius of the rolling surface
ψ: yaw angle of the frame
ψ_1: yaw angle of the wheel pair
γ: conicity of the rolling surface

REFERENCES

[1] A. Lau, I. Hoff, "Simulation of Train-Turnout Coupled Dynamics Using a Multibody Simulation Software", Modelling and Simulation in Engineering, Vol. 2018, Article ID 8578272, p. 10, 2018.

[2] S.A. Agafonov, "Stability and Motion Stabilization of Nonconservative Mechanical Systems", Journal of Mathematical Sciences, Vol. 112, pp. 4419–4497, 2002.

[3] A.A. Agafonov, "On the stability of nonconservative systems", Vestn. MGU Ser. Mat., Mekh., No. 4, pp. 87-90, 1972.

[4] M. Ahmadian, D.J. Inman, "On the Stability of General Dynamic Systems Using a Liapunov's Direct Method Approach", Computers & Structures, Vol. 20, pp. 287-292, 1985.

[5] A.V. Banshchikov, L.A. Burlakova, V.D. Irtegov, T.N. Titorenko, "Symbolic computations in modeling and qualitative analysis of dynamic systems", Vychislitelnyye tekhnologii, Vol. 19, No. 6, pp. 3-18, 2014.

[6] F.W. Carter, "On the stability of running of locomotives", Proc. R. Soc., A121, pp. 585-611, 1928.

[7] A.D. de Pater, "The equations of motion of railway wheelset moving over tangent track", LTM Report 985, Delft University of Technology, 1992.

[8] D. Hao, Z. Jing, W. Liang, D. Huanyun, "Analysis of the Gyroscopic Stability of the Wheelset. Shock and Vibration", Hindawi Publishing Corporation, Vol. 1, 7 p. 2014.

[9] V.I. Goncharenko, "Stabilization of motion of an unstable linear mechanical system", Prikl. Mekh., Vol. 26, No. 4, pp. 79-85, 1990.

[10] S.E. Iwnicki, "Handbook of Railway Vehicle Dynamics", CRC Press, Taylor & Francis Group, p. 552, Boca Raton, London, New York, 2006.

[11] V. Karapetyan, "On the stability of nonconservative systems", Vestn. MGU, Ser. Mat. Mekh., No. 4, pp. 109-113, 1975.

[12] O.N. Kirillov, "Nonconservative Stability Problems of Modern Physics", Vol. 14, 528 p., De Gruyter, Berlin, Germany, March, 2021.

[13] W. Kliem, A. P. Seyranian, "Metelitsyn's inequality and stability criteria for mechanical systems", Journal of Applied Mathematics and Mechanics, Vol. 68, No. 2, pp. 199-205, 2004.

[14] H.G. Kwatny, L.Y. Bahar, A.K. Pasrija, "Energy-like Lyapunov functions for power system stability analysis", The 23rd IEEE Conference on Decision

and Control, Vol. 32, No. 11, pp. 1140-1149, 1985.
[15] V.M. Lakhadanov, "The effect of the structure of forces on the stability of motion", Prikl. Mat. Mekh., Vol. 38, No. 2, pp. 246-253, 1974.
[16] J. Li, "On the stability of dissipative mechanical systems with circulatory forces", J. Appl. Maths. & Phys. (ZAMP) No. 48, pp. 161-164, 1997.
[17] I.I. Metelitsyn, "The problem of gyroscopic stabilization", Reports of the USSR Academy of Sciences, Vol. 86, No. 1, pp. 31-34, 1952.
[18] K. Popp, W. Schiehlen, "Ground vehicle dynamics", Springer-Verlag, 348 p., Berlin, Heidelberg, 2010.
[19] H. True, "Dynamics of a rolling wheelset", App. Mech. Rev., pp. 438-444, 1993.
[20] H. True, "On the critical speed of high-speed railway vehicles", Noise and Vibration on High-Speed Railways, FEUP Porto Portugal, pp. 149-166, 2008.
[21] U. von Wagner, D. Hochlenert, W. Martens, "Attractors of Nonlinear Wheelset Models", The 8th International Conference on Structural Dynamics, EURODYN 2011, Leuven, Belgium, pp. 699-704, 4-6 July 2011.
[22] V.G. Verbitskii, "The effect of the structure of forces on the stability of a linear system", Prikl. Mekh., Vol. 18, No. 12, pp. 119-121, 1982.
[23] J.A. Walker, "A note on stabilizing damping configurations for linear nonconservative systems", Int. J. Solids Struct., Vol. 9, No. 12, pp. 1543-1545, 1973.
[24] J.A. Walker, "On the Stability of Linear Discrete Dynamic Systems", ASME Transact., Ser. E, J. Appl. Mech. Vol. 37, No. 2, pp. 271-275, 1970.
[25] A.H. Wickens, "The dynamics of railway vehicles – From Stephenson to Carter", Proc. Instn. Mech. Engrs., Vol. 212 (Part F), pp. 209-217, 1999.
[26] A.H. Wickens, "The dynamics of railway vehicles on straight track: Fundamental considerations of lateral stability", Proc. Instn. Mech. Engrs., Vol. 180 (Part 3F), pp. 29-44, p. 150, 1965-1966.
[27] H. Yoshino, T. Hosoya, H. Yoshino, S. Lin, Y. Suda, "Theoretical and experimental analyses on stabilization of hunting motion by utilizing the traction motor as a passive gyroscopic damper", Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, Vol. 229, No. 4, pp. 395-401, 2015.
[28] E.E. Zajac, "The Kelvin-Tait-Chetaev theorem and extensions," J. Astronaut Sci., Vol. 11, No. 2, pp. 46-49, 1964.
[29] J.A Ramos Hernanz, J.J. Campayo, J. Larranaga, E. Zulueta, O. Barambones, J. Motrico, U. Gamiz Fernandez, I. Zamora, "Two Photovoltaic Cell Simulation Models in Matlab/Simulink", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 10, Vol. 4, No. 1, pp. 45-51, March 2012.
[30] A.B. Perlman, C.L. Dym, "A Note on the Lyapunov Stability Analysis of a Linear Railway Wheelset", Vehicle Systems Dynamics, Vol. 9, No. 3, pp. 61-68, Mar. 1980.
[31] L.D. Clive, "Stability Theory and its Applications to Structural Mechanics", Noordhoff International, Publishing Company, Leyden, Netherlands, 1974. (Republished by Dover Publications, New York, 2002).
[32] K. Knothe, F. Boehm, History of Stability of Rail-way and Road Vehicles, Vehicle System Dynamics, Vol. 31, No. 5-6, pp. 283-323,1999.
[33] J. Guo, H. Shi, R. Luo, J. Zeng, "Bifurcation Analysis of a Railway Wheelset with Nonlinear Wheel-Rail Contact," Nonlinear Dynamics, Vol. 104, pp. 989-1005, 2021.

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