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N-DOF DISCRETE MODEL TO INVESTIGATE FREE VIBRATIONS OF CRACKED TAPERED BEAMS AND RESTING ON WINKLER ELASTIC FOUNDATIONS

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Abstract- A new discrete N-DOF (degree of freedom) model is utilized for the first time to analyses free vibrations of tapered beams with multiple transverse cracks resting on elastic Winkler foundations. The description of the mechanical system considered is detailed. After construction of the new mass and rigidity tensors, including the crack and the Winkler foundation contributions, Lagrange's equations are used and the dimensionless frequencies and associated mode shapes are determined using different boundary conditions and shapes of the tapered beams. The comparisons of the results obtained here in a systematic and a unified manner with those published in previous works ensure the reliability and applicability of the present model.

Keywords: Free Vibration, N-DOF, Crack, Tapered Beam, Winkler Foundation.

1. INTRODUCTION

Cracks in structures used in engineering, such as bars, beams, plates, shells..., can have a considerable influence on their dynamical behavior, and if left undealt with, may lead to fractures and other catastrophic defects [1]. Detection of crack initiation can be achieved by analyzing the changes induced in dynamic response of these structures.

Several articles examined beam, plate and shell structures with and without racks [2, 3, 4, 5] etc. Also, beams resting on various types of foundations have been considered. Some of them are discussed below.

S.E. Motaghian, et al. [6] developed an analytical method using Fourier combination to calculate the frequencies of a tapered beam under a Winkler variable foundation. A. Ghannadiasl and S. Khodapanah Ajirlou [7] used Green's dynamic functions to solve the equation governing the vibration of a cracked uniform beam. [8] Yijiang Ma and al proposed a theoretical method to determine the vibration frequencies of a particular class of tapered beams containing multiple cracks.

The above-mentioned articles only apply to either a tapered beam under a foundation or to cracked tapered beams with specific end conditions. The case of a cracked tapered beam supported on a foundation is not treated.

The objective of this paper is to examine free vibrations of a cracked tapered beam supported by a variable elastic Winkler foundation using the discrete model used by Moukhliss in [9] to treat uncracked tapered beams. The definition of the relative flexibility considers the crack as a spiral spring whose stiffness depends on its depth and position. The Winkler elasticity is described by a distribution of linear vertical springs, while the beam mentioned is presented in the present discrete model by N+1 identical bars of equal lengths and negligible mass. The total beam mass is divided into N point masses, expressing the beam inertia, located at the ends of the bars. The mass distribution is defined by the law of variation of the tapered beam cross section area along the beam length. The bending stiffness of the beam is presented by N+2-n spiral springs, the elasticity of which varies with their location due to the non-uniformity of the beam, and as previously stated, each crack is similar to a spiral spring, the stiffness of which varies with the crack depth and location.

After the definition of the N-DOF system mass tensor m_{ij}^{a}, beam rigidity tensor k_{ij}^{s} (taking into account the presence of the crack) and foundation rigidity tensor k_{ij}^{f}, Lagrange's equations are applied and the problem is expressed in a matrix form, allowing the natural frequencies and modes to be calculated for several combinations of end conditions, different shapes of tapered beams, various functions presenting the change in elasticity of the foundation along the beam length, and different numbers of degrees of freedom used in the discretization procedure.

2. GENERALE FORMULATION

This paper extends Rahmouni and Moukhliss [9, 10] discrete mechanical model to investigate the vibration of tapered beams with n cracks positioned at n distinct places along the x-axis and resting on an elastic Winkler base.

2.1. Presentation of the Discrete Model and Nomenclature

Figures 1(a) and 1(b) depict a typical fractured tapered continuous beam under a Winkler foundation and the corresponding model, respectively.

In this research, the beam is assumed to be homogeneous and isotropic, with Young's modulus E, density ρ and length L. The width and height of the cross section are given by $b(x)$ and $h(x)$. The cross-section height and width at the right and left ends of the beam, corresponding to x=0 and x=L, are denoted respectively by $b(0)=b_0$, $b(L)=b_L$, $h(0)=h_0$, $h(L)=b_L$ (Figure 1(b)).

As described in the general introduction of this paper, the model involves N point masses $m_1^a,....,m_N^a$ located at the ends of (N+1) bars assumed to be rigid and of negligible mass, connected by (N+2-n) spiral springs simulating the flexural stiffness of the tapered beam, n being the number of cracks. The average rigidity coefficient of the spring is noted by C_r^a for r=1 to N+2-1. The values of $C_{left}=C_1^a$ and $C_{right}=C_{N+2}^a$ depend on the beam end conditions [10].

- For an embedded beam $C_{left}=C_{right}=\infty$.
- For a simply supported beam $C_{left}=C_{right}=0$
- For an embedded beam-Free $C_{left}=\infty$ and $C_{N+1}=C_{right}=0$.

The n cracks are modelled by n spiral springs C_r^{ca} for r=1,…,n having a stiffness depending on the position $X_1,...,X_n$ due to the non-uniformity of the beam and the depth a of the cracks. The elastic Winkler foundation is described by a distribution of linear vertical springs k_r^f for r=1,…,N of a constant or a variable stiffness as Figure 1(b).

The notion of average value implies here that the stiffness of the spiral spring is related to the position of the center of the corresponding bar in the continuous beam, Figure 1.

The bending strain energy of the N-DOF discrete system V^S, the kinetic energy T, and the strain energy V^f due to the Winkler flexible foundation are given by the following relations:

$$V^s=\frac{k_{ij}^s}{2}y_i y_j \text{ for } i=j=1,...,N \tag{1}$$

$$T=\frac{m_{ij}^a(x)}{2}\dot{y}_i\dot{y}_j \text{ for } i=j=1,...,N \tag{2}$$

$$V^f=\frac{k_{ij}^f}{2}y_i y_j \text{ for } i=j=1,...,N \tag{3}$$

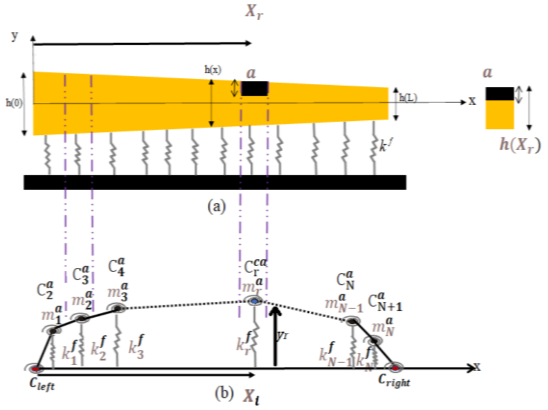

Figure 1. (a) A typical tapered crack beam, (b) the new equivalent discrete model

where, $m_{ij}^a(x)$, $k_{ij}^{sa}(x)$ and $k_{ij}^f(x)$ are the mass tensor, the stiffness tensor of the tapered beam and the stiffness tensor corresponding to the Winkler foundation, respectively.

The total strain energy of the discrete system presented in Figure 1 is given by:

$$V=V^s+V^f \tag{4}$$

We apply the Lagrangian formalism. Inspired by the work presented by Rahmouni [10] and Moukhliss [9], an algebraic system of N equations and N unknowns are obtained in the following matrix form:

$$\left[M_{Ndof}(x)\right]\{\ddot{y}\}+\left(\left[K_{Ndof}^s(x)\right]+\left[K_{Ndof}^l(x)\right]\right)\{y\}=0 \tag{5}$$

$$\left[M_{Ndof}(x)\right]\{\ddot{y}\}+\left[K_{Ndof}(x)\right]\{y\}=0 \tag{6}$$

where, $\left[M_{Ndof}(x)\right]$ and $\left[K_{Ndof}(x)\right]$ are the mass and linear stiffness matrices of the N-DOF system, respectively. The free response of the beam is considered to be harmonic [7]:

$$y_i=A_i\cos(\omega_{Ndof}^l t) \tag{7}$$

by replacing (7) in (6), the matrix system (6) can be written as follows:

$$-\left(\omega_{Ndof}^l\right)^2\left[M_{Ndof}(x)\right]\{A\}+\left[K_{Ndof}(x)\right]\{A\}=0 \tag{8}$$

where, ω_{Ndof}^l is the vibration pulsation of the new discrete system.

2.2. Calculation of Tensors $m_{ij}^a(x)$, $k_{ij}^{sa}(x)$ and $k_{ij}^f(x)$ Related to Discrete N-DOF System

The tapered beam used in this work are of three types, shown in Figure 2. The total mass of the beam is divided into N point masses in the N-DOF system and this distribution is made according to the type of the beam considered.

Based on the above remarks [10], the mass tensor of the new system is given by:

$$m_{ij}^a(x)=m_i(x)\delta_{ij}=\frac{\rho L}{N+1}S(x_i)\delta_{ij} \tag{9}$$

$$m_{ij}^a(x)=\frac{\rho S_0 L}{N+1}S^*(x_i)\delta_{ij}\ ,\ i=j=1,...,N \tag{10}$$

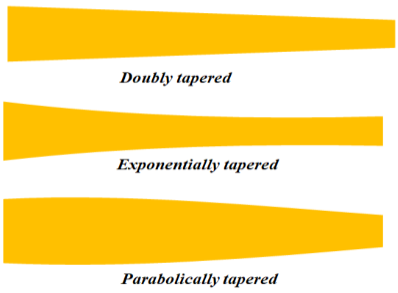

Figure 2. types of the tapered beams examined in this work; (a) Doubly tapered; (b) Exponentially tapered (c) Parabolically tapered

in which δ_{ij} is the Kronecker index given by:

$$\delta_{ij}=\begin{cases}1 \text{ if } i=j\\ 0 \text{ if } i\neq j\end{cases} \tag{11}$$

Note that $S(x_i)=h(x_i)b(x_i)$ presents the area of the tapered beam cross-section located at the abscissa $x_i=(i-1)L/(N+1)$, with $i=1,\dots,N+2$ and N the number of degrees of freedom used in the discretization process. This function can also be written as

$S(x_i)=S_0S^*(x_i)$

where, S_0 is the cross-section corresponding to x=0 i.e., the cross-section located at the node i=1, $S^*(x_i)$ is the dimensionless form of a cross-section of the tapered beam.

Expression (10) can be written in the following matrix form:

$$\left[M_{Ndof}(x)\right]=\frac{\rho S_0L}{N+1}\begin{bmatrix}S^*(x_1) & 0 & \cdots & 0\\ 0 & S^*(x_2) & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & 0 & \cdots & S^*(x_N)\end{bmatrix}= \tag{12}$$

$$=\frac{\rho S_0L}{N+1}\left[M^*_{Ndof}(x)\right]$$

where, $\left[M^*_{Ndof}(x)\right]$ denotes the dimensionless form of the mass matrix.

The strain energy of the N+2 spiral springs presenting the elasticity of the tapered beam is given by [6, 7]:

$$V^s=\frac{1}{2l^2}\sum_{r=1}^{r=N+2}C_r^a\left(y_r-2y_{r-1}+y_{r-2}\right)^2$$
$$y_{N+2}=y_{N+1}=y_0=y_{-1}=0 \tag{13}$$

following the steps in [10], the expressions for the components of the linear stiffness tensor are given by:

$$k^s_{(r-2)r}=k^s_{r(r-2)}=\frac{C_r^a}{l^2}\ ,\ r=3,\dots,N \tag{14}$$

$$k^s_{(r-1)r}=k^s_{r(r-1)}=-\frac{2}{l^2}\left(C_r^a+C_{r+1}^a\right)\ ,\ r=2,\dots,N \tag{15}$$

$$k^s_{rr}=\frac{1}{l^2}\left(C_r^a+4C_{r+1}^a+C_{r+2}^a\right)\ ,\ r=1,\dots,N \tag{16}$$

Take note that the other values of $k_{ij}^{sa}(x)$ are equal to zero.

In order to compute the $k_{ij}^{sa}(x)$ terms in equations (14), (15), and (16), the expression of $C_r^a(x)$ for $r=1,\dots,N+2$ must first be determined. The work presented in [10] gives the elementary potential energy dV_{br} in a tapered continuous beam, which corresponds to an elementary bar with length dx:

$$dV_r^l=\frac{1}{2}\frac{EI(x_r)}{l^2}\left(y_r-2y_{r-1}+y_{r-2}\right)^2$$
$$y_{N+2}=y_{N+1}=y_0=y_{-1}=0 \tag{17}$$

or

$$dV_r^l=\frac{1}{2}\frac{EI_0I^*(x_r)}{l^2}\left(y_r-2y_{r-1}+y_{r-2}\right)^2 \tag{18}$$

where, $I(x_r)$ and $I^*(x_r)$ are respectively the known average quadratic moment and a dimensional quadratic moment of a cross section located at position x_r. The local coordinates of the mass r in the x-axis can be presented by: $x_r=(r-1)L/(N+1)$ with $r=1,\dots,N+2$ where l is the distance between two successive masses.

The expression for the spiral spring stiffness $C_r^a(x)$ is found by identifying the two expressions (13) and (17) [10].

$$C_r^a=\frac{EI(x_r)}{l}=\frac{EI_0}{l}I^*(x_r)\ ,\ r=2,\dots,N+1 \tag{19}$$

We replace (19) in (14), (15) and (16), respectively and we find:

$$k^s_{(r\text{-}2)r}=k^s_{r(r\text{-}2)}=\frac{EI_0}{l^2}I^*(x_r)=\frac{EI_0}{l^2}k^{*s}_{r(r\text{-}2)}$$
$$r=3,\dots,N \tag{20}$$

$$k^s_{(r-1)r}=k^s_{r(r-1)}=-2\frac{EI_0}{l^3}\left(I^*(x_r)+I^*(x_{r+1})\right)=\frac{EI_0}{l^3}k^{*s}_{r(r-1)} \tag{21}$$
$$r=2,\dots,N$$

$$k^s_{rr}=\frac{EI_0}{l^3}\left(I^*(x_r)+4I^*(x_{r+1})+I^*(x_{r+2})\right)=\frac{EI_0}{l^3}k^{*s}_{rr} \tag{22}$$
$$r=1,\dots,N$$

Then the stiffness matrix representing the N-DOF system can be written as follows:

$$\left[K^s_{Ndof}\right]=\frac{EI_0}{l^3}\left[K^{*s}_{Ndof}\right] \tag{23}$$

The stiffness tensor corresponding to the Winkler foundation is given by the following formula:

$$k^f_{ij}=k^f_i\delta_{ij}=\frac{k^f(x_i)L}{N+1}\delta_{ij}\ \text{ for }\ i,j=1,\dots,N \tag{24}$$

or

$$k^f_{ij}=k^f_i\delta_{ij}=\frac{EI_0}{l^3}\frac{k^{*f}(x_i)}{(N+1)^4}\delta_{ij}\ \text{ for }\ i,j=1,\dots,N \tag{25}$$

where, $k^{*f}(x_i)$ is a dimensionless parameter that may be constant or variable, indicating the distribution of the foundation stiffness along the x-axis. The stiffness matrix corresponding the Winkler foundation can be described as Equation (26).

$$\left[K_{Ndof}^{f}\right]=\frac{EI_0}{l^3}\begin{bmatrix}\frac{k^{*f}(x_1)}{(N+1)^4} & 0 & 0 & \cdots & 0\\ 0 & \frac{k^{*f}(x_2)}{(N+1)^4} & \ddots & \ddots & 0\\ 0 & \ddots & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & 0 & \frac{k^{*f}(x_N)}{(N+1)^4}\end{bmatrix}= \quad (26)$$

$$=\frac{EI_0}{l^3}\left[K_{Ndof}^{*}\right]$$

Let us now consider the cracked beam shown in Figure 1. We keep the same linear stiffness coefficients established in the previous paragraph and we model the cracked part as follows:

The crack of depth ai present in the continuous beam of Figure 1(a) by a bar of depth ai is modelled by a spiral spring Figure 1(b) whose stiffness C_i^{ca} depends on its depth and location. It should be noted that the position of spring *i*, for *i*=1 to *N*, which models the crack *i* is taken as the position of the cross-section located at the center of the cracked bar. For this reason, the coefficient C_i^{ca} is known in average value.

Then the expression for the height corresponding to the cracked portion becomes $h_i^c=\left(h\left(X_i\right)-a_i\right)$. The spiral spring stiffness and the moment of inertia of the crack *i* are given by:

$$C_i^{ca}=\frac{E\,I^c\left(X_i\right)}{l}=\left(N+1\right)\frac{EI^c\left(X_i\right)}{L}\quad,\quad i=1,\dots,n \qquad (27)$$

$$I^c\left(X_i\right)=\frac{b^c\left(X_i\right)\left(h\left(X_i\right)-a_i\right)^3}{12}=\frac{b_i^c\left(h_i^c\right)^3}{12}\ ,\ i=1,\dots,n \qquad (28)$$

where, b_i^c and h_i^c are the width and thickness of the crack cross-section, respectively: $b_i^c=b\left(X_i\right)$ and $h_i^c=h\left(X_i\right)$ in Figure 1. Factoring by $\left(h\left(X_i\right)\right)^3$, equation (22) can be written in the form:

$$I^c\left(X_i\right)=\frac{1}{12}b\left(X_i\right)\left(h\left(X_i\right)\right)^3\left(1-\frac{a_i}{h\left(X_i\right)}\right)^3= \qquad (29)$$
$$=\frac{1}{12}b_i^c\left(h_i^c\right)^3Q_i\quad,\ i=1,\dots,n$$

with

$$Q_i=\left(1-\frac{a_i}{h\left(X_i\right)}\right)^3=\left(1-\eta_i^c\right)^3 \qquad (30)$$

where, $\eta_i^c=a_i/h\left(X_i\right)$ presents the reduced depth of the crack *i*. Replacing (30) in (29) leads to the final expression for stiffness of spiral spring simulating the cracked part:

$$C_i^{ca}=\frac{EI_0}{l}I^{c*}\left(X_i\right)Q_i \qquad (31)$$

To take into account the cracks in the structures studied by replacing expression (31) in (14), (15) and (16), respectively stiffness matrix in this case can be written as:

$$\left[K_{Ndof}^{cs}\right]=\frac{EI_0}{l^3}\left[K_{Ndof}^{*cs}\right] \qquad (32)$$

on the other hand, the total stiffness matrix of the N-DOF system is:

$$\left[K_{Ndof}\right]=\left[K_{Ndof}^{cs}\right]+\left[K_{Ndof}^{f}\right] \qquad (33)$$

2.3. Computation of the Dimensionless Frequencies Corresponding to the N-DOF System

The matrices corresponding to the rigidity (taking into account cracks and foundations) and the inertia the discrete system shown in Figure 1 are replaced in the equation of motion given by expression (8). A new equation that contains all of the information on the nature of the structure studied is obtained as expression (34).

$$-\frac{\rho S_0 L}{N+1}\left(\omega_{Ndof}^{l}\right)^2\left[M_{Ndof}^{*}\left(x\right)\right]\left\{A\right\}+$$
$$+\frac{EI_0}{l^3}\left[K_{Ndof}^{*}\left(x\right)\right]\left\{A\right\}=0 \qquad (34)$$

The non-dimensional frequencies are defined as the ratios of the natural frequencies of the N-dof system to the coefficient $\sqrt{\frac{EI_0}{\rho S_0 L^4}}$. This includes information about the right-hand side at *x*=0. Which allows us to write the following:

$$\omega_{r\,Ndof}^{*}=\frac{\omega_{r\,Ndof}}{\sqrt{\frac{EI_0}{\rho S_0 L^4}}}=(N+1)^2\sqrt{\beta_r} \qquad (35)$$

where, β_r are the eigenvalues, for *i* = 1 to *N*, corresponding to the *N* vibration modes. Note that the values of β_r vary with the boundary conditions.

3. NUMERICAL RESULTS

To validate the new model, we present some numerical applications in this part by solving the algebraic system (34). The findings for various forms of cracked tapered beams resting on elastic foundations are addressed in this study for various values of the reduced depth η_i^c and various values of $k^{*f}\left(x_i\right)$. The results are obtained using the MATLAB.

3.1. Numerical Result 1: Exponentially Cracked Tapered Beam

The beam considered in this example is the one presented in [8] for the end condition SS. It is an exponentially tapered beam whose width and height are given respectively by $b(x_r)=b_0\exp(\delta x_r)$ and $h(x_r)=h_0$. The cross-section area and its squared moment in x_r are given respectively by $S(x_r)=S_0\exp(\delta x_r)=S_0S^{*}(x_r)$ and $I(x_r)=I_0\exp(\delta x_r)=I_0I^{*}(x_r)$. δ is used to describe the variation of geometric properties along the *x*-axis. The beam is made of the structural material of low carbon alloy steel AISI 1050, its mechanical properties are $E=210$ GPA, $\rho=7860$ kg/m^3.

Table 1 lists the first four frequency values in Hz according to the example presented in (section 2.1) for N=49. The values of f are compared to the results published in [8] for the particular limit condition SS and for $\delta = -0.5$. The results for other end conditions not treated in [8] are calculated by intuition in this work as shown in Table 1.

The frequencies are calculated in 3 situations:

- Situation 1 (S_1): There is a single transverse crack at $X_1 = 0.1\times L$ such that: $a_1 = 0.3\times h(X_1)$.
- Situation 2 (S_2): There are two transverse cracks whose positions and depths are $X_1 = 0.1\times L$, $a_1 = 0.3\times h(X_1)$ for the first and $X_2 = 0.2\times L$, $a_2 = 0.3\times h(X_2)$ for the second.
- Situation 3 (S_3): There are three transverse cracks whose positions and depths are $X_1 = 0.1\times L$, $a_1 = 0.3\times h(X_1)$ for the first, $X_2 = 0.2\times L$, $a_2 = 0.3\times h(X_2)$ for the second and $X_3 = 0.3\times L$, $a_3 = 0.3\times h(X_3)$ for the third.

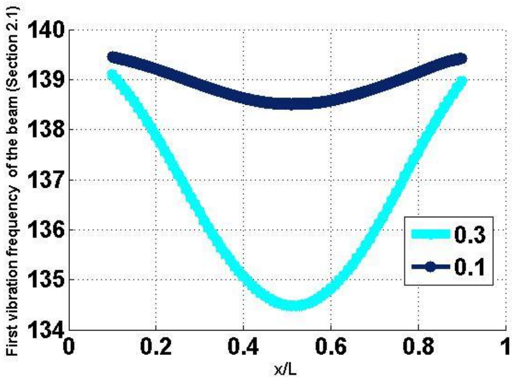

Figure 3. First vibration frequency (Hz) of a beam (simply supported exponentially tapered with parameter δ=–0.5 Situation 1) as a function of the relative position of the crack and for different values of the reduced depth of the crack $\eta^c = 0.1;\ 0.3$

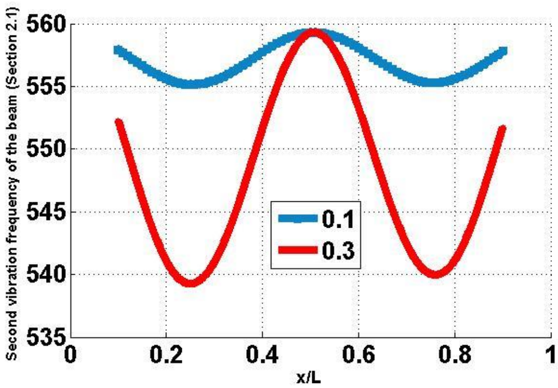

Figure 4. Second vibration frequency (Hz) of a beam (simply supported exponentially tapered with parameter δ=-0.5 Situation 1) as a function of the relative position of the crack and for different values of the reduced depth of the crack $\eta^c = 0.1;\ 0.3$

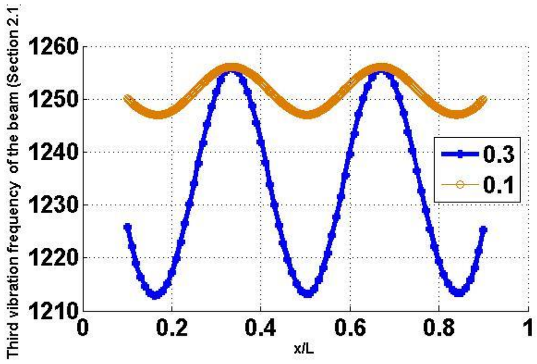

Figure 5. Third vibration frequency (Hz) of a beam (simply supported exponentially tapered with parameter δ=–0.5 Situation 1) as a function of the relative position of the crack and for different values of the reduced depth of the crack $\eta^c = 0.1;\ 0.3$

Table 1. Display of the first frequency of the beam described in section 2.1 for various end Conditions

		f_{1Ndof}			f_{2Ndof}			f_{3Ndof}			f_{4Ndof}		
		S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
SS	Theoretical [8]	139.93	137.82	133.99	553.16	533.29	517.60	1229.6	1193.2	1192.96			
	Present study N=49	139.09	137.43	134.35	552.21	534.59	519.87	1225.87	1192.80	1192.03	2156.63	2143.26	2103.48
CC	Present study N=49	322.48	322.14	320.59	904.54	895.50	868.63	1782.81	1732.19	1716.63	2932.01	2859.19	2843.97
CF	Present study N=49	56.98	54.94	53.66	332.69	332.56	330.10	918.92	909.51	882.82	1795.1	1745	1730

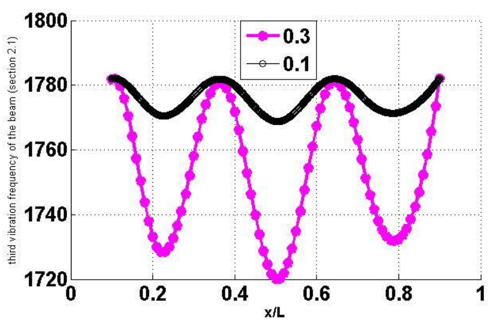

Figure 6. The third vibration frequency (Hz) of a beam (CC exponentially tapered with parameter δ=-0.5 Situation 1) as a function of the relative position of the reduced crack and for different values of the depth of the crack $\eta^c = 0.1$; 0.3

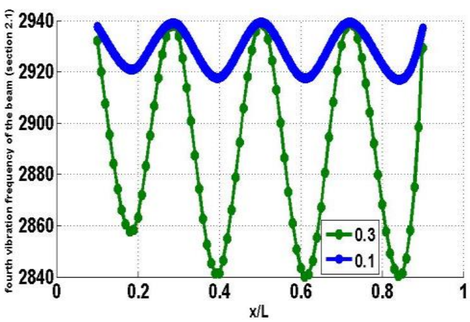

Figure 7. The fourth vibration frequency (Hz) of a beam (CC exponentially tapered with parameter δ=-0.5 Situation 1) as a function of the relative position of the crack and for different value of the reduced depth of the crack $\eta^c = 0.1$; 0.3

Figures 6 and 7 present the third and fourth vibration frequency of the beam, respectively which presented in section 2.1 for the condition CC and for different values of the crack position.

3.2. Numerical Result 2: Parabolic Tapered Beam under on a Winkler Foundation

In this part the foundation will be included in the calculation of the dimensionless frequencies, the example considered is a parabolic tapered beam of section $S(x)=S_0\left(1-0.8x^2\right)=S_0S^*(x)$, partially supported on a Winkler foundation of stiffness $k^{*f}(x)=500(1-0.5x)$ with $L/4\le x\le 2L/3$ Figure 8, which is written in the index form as follows $k^{*f}(x_r)=500(1-0.5x_r)$ the results of this study are given in Table 2 and are compared with the results presented in reference [6] for various values of N, Table 2.

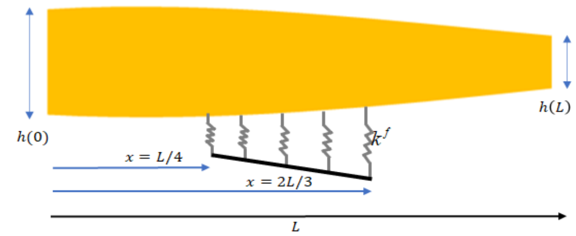

Figure 8. Parabolically tapered beam supported on a partial Winkler foundation

Table 2. Calculation of the first Five dimensionless frequencies for a partially supported beam (Section 3.2)

		$\sqrt{\omega^*_{1Ndof}}$	$\sqrt{\omega^*_{2Ndof}}$	$\sqrt{\omega^*_{3Ndof}}$	$\sqrt{\omega^*_{4Ndof}}$	$\sqrt{\omega^*_{5Ndof}}$
SS	[6]	3.382	5.266	7.737	10.253	
	N=39	3.021	5.141	7.618	10.082	12.533
	N=199	2.976	5.184	7.704	10.220	12.740
CC	[6]	4.025	6.318	8.856	11.422	
	N=71	3.886	6.307	8.893	11.471	14.014
	N=107	3.859	6.275	8.853	11.424	13.990
CF	[6]	2.624	4.677	6.895	9.306	
	N=79	2.484	4.565	6.899	9.350	11.839

3.3. Numerical Results 3: Cracked Linear Tapered Beams Resting on a Winkler Foundation

Consider now a linearly tapered beam such that $S(x)=S_0(1-0.8x)=S_0S^*(x)$, containing a single crack at the location X=0.4L of reduced depth $\eta^c=0.3$ and supported on a Winkler foundation assumed uniform of parameter $k^{*f}(x)=600$ as shown in Figure 9.

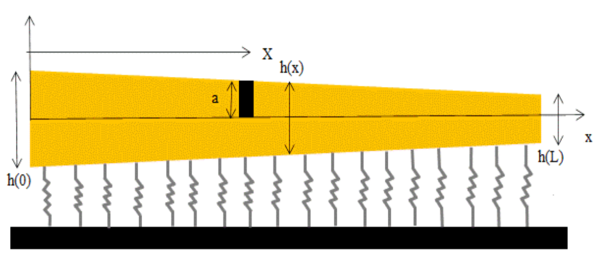

Figure 9. A linear tapered cracked beam supported on a Winkler foundation

The results of the dimensionless frequency are given in Table 3 for various end conditions as well as for different values of N.

Table 3. Calculation of the Four first dimensionless frequencies for a cracked tapered beam supported on a Winkler foundation (Section 3.3)

		$\sqrt{\omega^*_{1Ndof}}$	$\sqrt{\omega^*_{2Ndof}}$	$\sqrt{\omega^*_{3Ndof}}$	$\sqrt{\omega^*_{4Ndof}}$
SS	N=49	5.580	6.445	7.685	9.394
	N=99	5.600	6.485	7.727	9.484
	N=199	5.615	6.506	7.746	9.528
CC	N=49	5.925	6.907	8.5912	10.624
	N=99	5.941	6.931	8.5703	10.580
	N=199	5.948	6.945	8.560	10.557
CF	N=49	5.917	6.653	7.332	8.866
	N=99	5.933	6.702	7.3812	8.853
	N=199	5.941	6.727	7.407	8.846

The results for the frequency differ from one value of N to another, generally the convergence is reached when we approach N=100, then the most reliable results are those corresponding to N=99.

5. CONCLUSION

The new model used in this work may be very useful in industrial applications since it allows calculating easily the vibration frequencies of a tapered beam type structure resting on a foundation and containing n transverse cracks. It is applicable for all types of tapered beams as well as for a variety of laws of variation of the stiffness of the Winkler elastic foundation along the length. It also allows calculating the frequencies of any tapered beam containing n cracks positioned at n different places by making a small change in the stiffness matrix. The results provided by this work can be used to detect the location and depth of a crack in a tapered beam under an elastic Winkler foundation.

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