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PROGRAMMABLE MOVEMENT OF THE WHEEL ROBOT

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Abstract- We have utilized a kinematic nonholonomic model of a wheeled robot in order to delve into two possible approaches to controlling motion along programmed curves: an active controlled central wheel and a passive support central wheel in various mathematical forms - both explicit and parametric. In case of an active central wheel, we have solved a control law design problem related to a steering angle of a controlled wheel module. In case of a passive central wheel, we have solved a control law design problem related to an angular velocity of rotating drive wheels. Use cases of our Maple-based software for closed trajectories are available.

Keywords: Wheeled Robot, Kinematic Nonholonomic Model, Motion Control, Programmed Trajectory.

1. INTRODUCTION

Wheeled robots are finding more and more uses in everyday life [1]. Robots can deliver pharmaceuticals and other products to infectious patients when it is advised to avoid direct contact. Such robots can emit UV light to disinfect premises without human assistance. They can act as sappers and participate in special operations endangering human life. Most of these wheeled robots belong to nonholonomic mechanical rolling systems (in addition to geometric constraints, kinematic constraints, which cannot be reduced to geometric ones, are imposed on the system). Such kinematic constraints are exemplified by no longitudinal and lateral slip when wheels roll on a supporting surface, which, in turn, makes a specific impact on motion control in such systems to be implemented [1-8]. There is a wide variety of kinematic diagrams with various modes of motion that can be classified as follows:

1) Classical (regular wheels)
2) Combined (regular wheels and rollers)

A kinematic model of a wheeled robot enables us to consider two possible approaches to implementing motion control of a model along programmed curves in case of an active controlled central wheel and a passive support wheel [3-6]. In this case, the object of research is the control processes of the autonomous wheeled robot - three-wheeled Robot Tima by Infocom Ltd - when implementing classical nonholonomic constraints between wheels and supporting surfaces.

The goal of research is to develop the necessary mathematical tool and software in order to implement a programmed motion of a nonholonomic model of a wheeled robot designed for repeated maneuvers along closed trajectories. At the initial stage, we have had a task to determine the required range of kinematic characteristics specific to a controlled wheel module of a robot when simulating a complex periodic motion along programmed curves in various mathematical forms.

2. CONTROLLING MOTION OF A WHEELED ROBOT ALONG PROGRAMMED CURVES

2.1. Setting a Motion of a Nonholonomic Model of a Wheeled Robot Along a Programmed Trajectory in Case When a Heading Angle or a Curvature of a Programmed Curve is Assigned as a Known Function of Time

The system of differential Equations setting a programmed trajectory is as follows:

$$\begin{aligned}&\dot{\psi}=\dot{\psi}(t)\\&\dot{x}=v\cos\psi\\&\dot{y}=v\sin\psi\end{aligned}\tag{1}$$

In these Equations, v is a velocity of motion along a programmed curve (assumed to be uniform) and ψ is a heading angle, a change rate of which is related to a curvature of a trajectory by the following formula: $\dot{\psi}=vK_r$ where, K_r is curvature.

In case of wheel modules, the law of variation of a steering angle can be determined by the condition that a heading angle is equal to an angle of arrival that is tangent to a programmed curve and a heading angle of a trajectory for a rear uncontrolled wheel:

$$\mathrm{tg}(\theta)=1/\rho=K_r$$

In this case, feature point trajectories of a wheeled robot are assigned by a system of differential Equations:

$$\begin{aligned}&\dot{x}=v\cos(\psi(t)),\quad \dot{y}=v\sin(\psi(t)),\quad \dot{\psi}=v/\rho(t)\\&\dot{x}_B=v\cos(\psi_1(t))/\cos(\theta(t)),\quad \dot{y}_B=v\sin(\psi_1(t))/\cos(\theta(t))\\&\dot{\psi}_1=v/\rho(t)+\dot{\theta}(t)\end{aligned}\tag{2}$$

where, $\psi_1=\psi+\theta$ is a heading angle of a front controlled wheel, and $\dot{\theta}(t)$ is a rate at which a steering angle of a controlled wheel module changes (known function of time). As an example, consider a case when a heading angle changes: "snaking motion" $\dot{\psi}(t)=v\times1.5\sin(5t)$

Figure 1. Wheeled Robot Prototype, (Three-Wheeled Robot Tima by Infocom Ltd)

Numerical integration of a system of differential Equations (1) and (2) in a Maple system enables us to visualize a motion of a nonholonomic model of a wheeled robot (Figure 2).

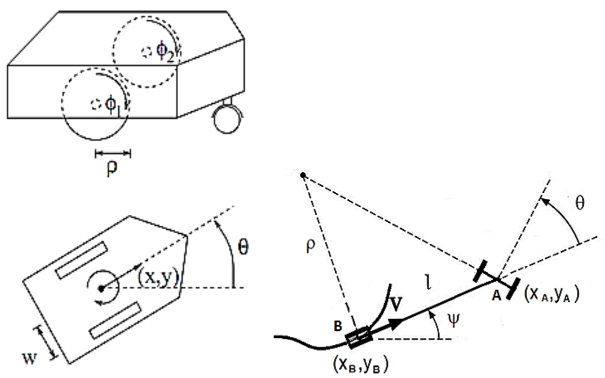

Figure 2. Kinematic Diagram of a Three-Wheeled Robot

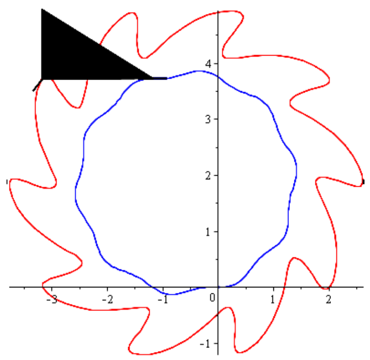

Figure 3. Visualized programmed motion of a robot

In this case, blue represents a trajectory of a rear wheel, while red represents a trajectory of a midpoint between front wheels.

2.2. Defining a Motion of a Nonholonomic Model of a Wheeled Robot When a Programmed Curve is Set in a Polar Coordinate System

In this case, we can set a trajectory of Point B parametrically (contact points of a rear uncontrolled wheel):

$$r = r(\varphi),\quad x_B = r(\varphi)\cos(\varphi),\quad y_B = r(\varphi)\sin(\varphi)$$

$$\dot{x}_B = \frac{\partial x_B}{\partial \varphi}\dot{\varphi}(t),\quad \dot{y}_B = \frac{\partial y_B}{\partial \varphi}\dot{\varphi}(t) \tag{3}$$

$$\dot{\psi} = v\frac{\frac{\partial x}{\partial \varphi}\frac{\partial^2 y}{\partial \varphi^2} - \frac{\partial y}{\partial \varphi}\frac{\partial^2 x}{\partial \varphi^2}}{\left(\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2\right)^{\frac{3}{2}}}$$

Consider a case of a "three-leafed rose" programmed curve:

$$x = \sin(3\varphi)\cos(\varphi)$$

$$x = \sin(3\varphi)\sin(\varphi)$$

$$\frac{\partial x}{\partial \varphi} = 3\cos(3\varphi)\cos(\varphi) - \sin(3\varphi)\sin(\varphi)$$

$$\frac{\partial y}{\partial \varphi} = 3\cos(3\varphi)\sin(\varphi) + \sin(3\varphi)\cos(\varphi)$$

$$\frac{\partial^2 x}{\partial \varphi^2} = -10\sin(3\varphi)\cos(\varphi) - 6\cos(3\varphi)\sin(\varphi)$$

$$\frac{\partial^2 y}{\partial \varphi^2} = -10\sin(3\varphi)\sin(\varphi) + 6\cos(3\varphi)\cos(\varphi)$$

$$\rho = \frac{\left(\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2\right)^{\frac{3}{2}}}{\frac{\partial x}{\partial \varphi}\frac{\partial^2 y}{\partial \varphi^2} - \frac{\partial y}{\partial \varphi}\frac{\partial^2 x}{\partial \varphi^2}}$$

$$\theta = \arctan\left(\frac{1}{\rho}\right)$$

where, $\dot{\theta} = \frac{\partial \theta}{\partial t}$ is a rate at which a control angle θ changes.

The results of Maple-based numerical integration of a relevant system of differential Equations (Figure 3) are as

$$\begin{aligned}
&\dot{x}_B = v\cos(\psi(t))\\
&\dot{y}_B = v\sin(\psi(t))\\
&\dot{\psi} = v / \rho(t)\\
&\dot{x}_A = v\cos(\psi(t)) + \theta(t)) / \cos(\theta(t))\\
&\dot{y}_A = v\sin(\psi(t) + \theta(t)) / \cos(\theta(t))\\
&\dot{\varphi}(t) = \frac{v}{\left(\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2\right)^{\frac{1}{2}}}
\end{aligned} \tag{4}$$

Initial conditions:

$$x_A(0) = 1,\ y_A(0) = 0,\ x_B = 0,\ y_B = 0,\ \theta(0) = \theta_0,\ \varphi(0) = 0$$

where, $l = 2\,\text{m},\ v = 1\,\text{m/s},\ \theta_0 = \theta|_{t=0} = 0.197395598\,\text{rad}$

A Maple-based program listing is as the following:

Listing 1

```
> x: =sin(3*phi) *cos(phi);
> y: =sin(3*phi) *sin(phi);
> Xphi: =diff(x,phi);
> Yphi: =diff(y,phi);
```

```
> XXphi: =diff (Xphi,phi);
> YYphi: =diff (Yphi,phi);
> Xphi: =subs(phi=phi(t), Xphi);
> Yphi: =subs(phi=phi(t), Yphi);
> XXphi: =subs(phi=phi(t), XXphi);
> YYphi: =subs(phi=phi(t), YYphi);
> ro: =(Xphi^2+Yphi^2) ^(3/2)/(YYphi*Xphi-
XXphi*Yphi):
> l: =2; v: =1;
> thet: =arctan(l/ro):
> Dtheta is a change rate of a theta control angle, but thet
appears in the program as well (it is identical in meaning,
though it was necessary to introduce two functionally
different dependencies):
> Dtheta: =diff(thet,t):
> Dtheta: =subs(diff(phi(t), t) =v/(Xphi^2+Yphi^2) ^
(1/2), Dtheta):
> theta0: =evalf(subs(phi(t)=0, thet));
> with(plots):
> F: =dsolve({diff(xB(t), t) =v*cos(psi(t)),
diff(yB(t), t) =v*sin(psi(t)),
diff(psi(t), t) =v/ro,
diff(xA(t), t) =v*cos(psi(t)+theta(t))/cos(theta(t)),
diff(yA(t), t) =v*sin(psi(t)+theta(t))/cos(theta(t)),
diff(thta(t), t) =Dtheta,
diff(phi(t), t) =v/(Xphi^2+Yphi^2) ^ (1/2, xB (0) =0, yB
(0) =0, psi (0) =0,
xA (0) =l,yA(0)=0,theta(0)=theta0,phi(0)=0},
[xB(t), yB(t), psi(t), xA(t), yA(t), theta(t), phi(t)],
numeric,output=listprocedure;
```

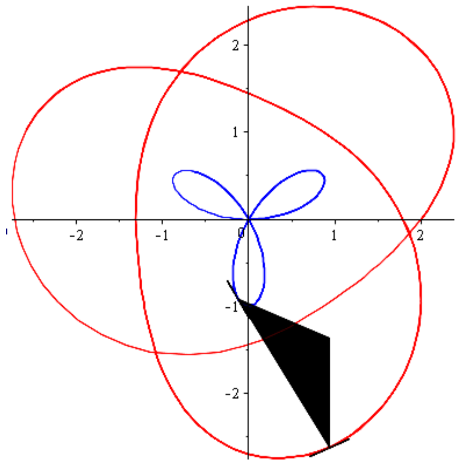

Figure 4. Visualized Programmed Motion in a Polar Coordinate System

In this case, blue represents a trajectory of a rear wheel, while red represents a trajectory of a midpoint between front wheels. A control function ($\theta(t)$ is a steering angle of controlled wheels) is represented by a phase trajectory in Figure 5.

The above examples assume that a steering angle of front controlled wheels (a wheel), $\theta(t)$ is set by a known function of time (arctan(l/ρ) at every instant, and a velocity of a rear wheel (wheels) (Point B) is a constant that corresponds to a bicycle scheme of a robot with an active controlled (guided) wheel.

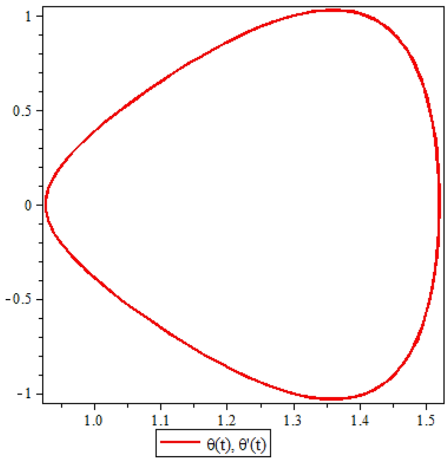

Figure 5. Phase trajectory: Steering angle and angular velocity

2.3. Three-Wheeled Robot with a Passive Central Support

A three-wheeled robot model with a passive central support (castor) can move along a programmed trajectory by controlling an angular velocity of rotating drive wheels [4].

$$\omega = \frac{v_B}{\rho} = \frac{v_2 - v_1}{Kol};\ v_B = \frac{v_2 + v_1}{2}$$

Velocities of wheel centers are related to angular velocities of their own rotation around B_1B_2 axis and radius of drive wheels R:

$$\omega_1 = \frac{v_1}{R};\ \omega_2 = \frac{v_2}{R}$$

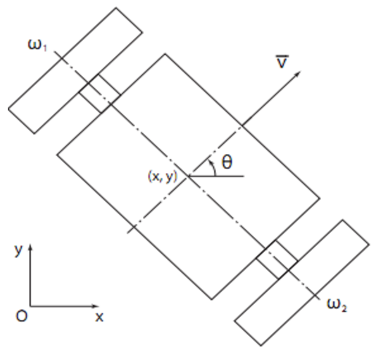

Figure 6. Wheeled robot with a passive support

An angular velocity of a robot in its rotating motion around a vertical axis (a change rate of a heading angle) is determined by the following equation:

$$\omega = \dot{\psi}$$

where, ψ is a heading angle formed by a longitudinal axis of robot symmetry.

Taking into consideration a curvature of a trajectory, we have the following equation for Point B:

$$\dot{\psi} = v_B / \rho(t)$$

On the other hand, if there is no longitudinal slip of drive wheels, we have the following ratios for velocities of drive wheel centers [1-4]:

$$v_B / \rho(t) = \frac{(\omega_2 - \omega_1)R}{Kol};\ v_B / R = \frac{\omega_2 + \omega_1}{2}$$

where,

$$\omega_1 = \frac{v_B}{R} - \frac{v_B Kol}{2R\rho(t)};\ \omega_2 = \frac{v_B}{R} + \frac{v_B Kol}{2R\rho(t)}$$

or for linear velocities of drive wheels centers:

$$v_1 = v_B - \frac{\omega Kol}{2};\ v_B = v_B + \frac{\omega Kol}{2}$$

Consider the implementation of robot motion along a complex closed trajectory:

$$\rho = \frac{1}{0.5 + 1.5\sin(5t)}$$

Data:

$m = 13.3$ kg, $J = 0.35$ kgm^2, $l = 0.36$ m, $a = 0.13333$ m, $b = 0.226666$ m, $Kol = 0.25$ m

In this case, velocities of drive wheels are as follows:

$$v_1 = v - \frac{vKol}{2\rho};\ v_2 = v + \frac{vKol}{2\rho}$$

System of differential equations:

$$\dot{x}_{B1} = v_1 \cos(\psi(t))$$
$$\dot{y}_{B1} = v_1 \sin(\psi(t))$$
$$\dot{x}_{B2} = v_2 \cos(\psi(t))$$
$$\dot{y}_{B2} = v_2 \sin(\psi(t))$$
$$\dot{\psi} = v / \rho(t)$$

Initial conditions:

$x_{B1}(0) = 0,\ y_{B1}(0) = 0,\ x_{B2}(0) = 0,\ y_{B2}(0) = 0.125,$ $\psi(0) = 0$

The relevant program is shown in Listing 2.

Listing 2

```
> v: =1;
> l: =0.36; Kol: =0.25;
> ro: =2;
> ro: =1/(0.5+1.5*sin(5*t));
> v1: =v-v/ro*Kol/2; v2: =v+v/ro*Kol/2;
>F: =dsolve({diff(xB1(t), t) =v1*cos(psi(t)),
diff(yB1(t), t) =v1*sin(psi(t)),
diff(psi(t), t) =v/ro,
diff(xB2(t), t) =v2*cos(psi(t)),
diff(yB2(t), t) =v2*sin(psi(t)),
xB1(0) =0, yB1(0) =0.25/2, psi (0) =0, xB2(0) =0, yB2(0)
=-0.25/2},
[xB1(t), yB1(t), psi(t), xB2(t), yB2(t)],
numeric,abserr=0.1e-8,output=listprocedure);
> XB1: =subs (F, xB1(t)):
> YB1: =subs (F, yB1(t));
> XB2: =subs (F, xB2(t)):
> YB2: =subs (F, yB2(t));
> List: = [];
> for i from 0 by 1 to N-1 do List: = [op (List), [[rhs (F
[2] (1.25*i)), rhs (F [3] (1.25*i))], [rhs (F [5] (1.25*i)),
rhs (F [6] (1.25*i))]]] end do:
> TB1:=plot([XB1, YB1, t0...tN],
style=LINE,linestyle=SOLID,
color=RED,scaling=constrained):
>B2:=plot([XB2, YB2, t0...tN],
style=LINE,linestyle=SOLID,
color=RED,scaling=constrained):
>L: =plot
(List,style=LINE,color=BLUE,scaling=constrained):
> display (TB1, TB2, L);
```

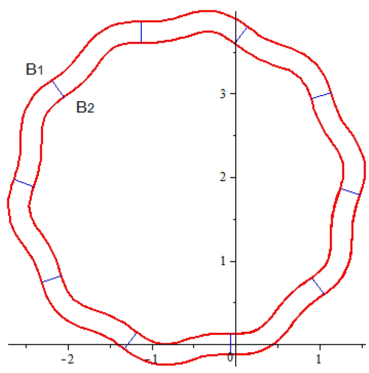

Figure 7. Drive wheel trajectories

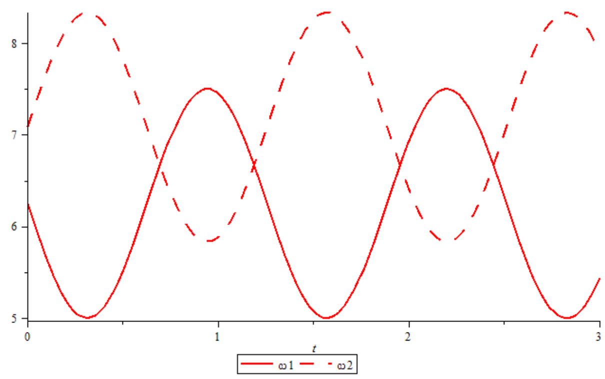

Figure 8. Control synthesis of drive wheel angular velocities

Within the framework of a kinematic model of a wheeled robot, we have considered two possible approaches to implementing a motion control of a model along programmed curves (for an active controlled central wheel and a passive support wheel). The practical implementation of robot motion along programmed trajectories requires using dynamic equations of nonholonomic systems [1, 9-14], which is supposed to be the subject for further consideration.

3. CONCLUSIONS

We have developed the mathematical tool and software that solves control problems of nonholonomic kinematic models of three-wheeled robots with various diagrams (active and passive symmetric supports) and various options for setting programmed trajectories both in explicit and parametric form, including the setting of a curve in a polar coordinate system).

NOMENCLATURES

1. Symbols / Parameters

$\psi(t)$: heading angle of the module frame
$\theta(t)$: steering angle of the front wheel unit

$\rho(t)$: radius of curvature of the rear wheel path
t: temporary variable
v: rear wheel speed
x_A, y_A: coordinates of the center of the front axle
x_B, y_B: coordinates of the center of the rear axle
ω_1: angular velocity of the left front wheel
ω_2: angular velocity of the right front wheel
R: radius of the front wheels
r, φ: polar coordinates
K_r: curvature of the trajectory
l: distance between the axles of the front and rear wheels

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