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INVESTIGATION OF OSCILLATORY MOTION OF INHOMOGENEOUS LONGITUDINALLY REINFORCED CYLINDRICAL SHELLS WITH A VISCO-ELASTIC MEDIUM

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Abstract- The theory of elasticity of inhomogeneous bodies deals with boundary value problems for systems of linear partial differential equations with variable coefficients - this causes difficulties and specifics of methods for solving problems for inhomogeneous bodies. The development of modern technology and technologies imposes new theoretical and applied requirements to the mechanics of deformable solids, as well as to all branches of science. These requirements are imposed on the properties (physical, mechanical, anisotropic, inhomogeneous, viscoelastic, etc.) of materials selected to accurately determine the boundaries of working resources, structural elements and devices in general. To consider its properties and clarify the equations interpreting the tense-deformation state. On the other hand, in addition to the external influences to which objects and structural elements are exposed, taking into account the effects of the environment in which they are located is also an integral part of the requirements for certain models. The presented article is devoted to a very complex field of the theory of reinforced cylindrical shells, which is widely used in practice. In the article, a cylindrical shell is chosen as the design. The shell is reinforced with rectilinear rods along the axis. It is assumed that the coating material is not homogeneous, that is, the axis passing through the middle surface of the shell and the perpendicular to this axis change the elastic modulus linearly in both (one of which is circular) directions. Since there is a viscoelastic medium inside the structure under study, its influence, in addition to the hydrostatic action of one medium, also takes into account the friction between the medium and the coating (Pasternak model). When solving the problem, the condition of stationery of the Hamilton-Ostrogradsky variation principle is used, which refers to the integral principle of mechanics. With this condition, all parameters of the oscillation of the structure are determined.

Keywords: Shell, Medium, Pasternak Model, Variation Principle, Relaxation Core, Heterogeneity, The Frequency Equation, Visco-Elastic Medium, Filler, Lagrange Function.

1. INTRODUCTION

The articles [1, 9, 10] is devoted to the topic "Oscillation of a reinforced inhomogeneous cylindrical shell with various media", proposed by the author's supervisor, which is the initial part when it is not complicated. In this article, it was assumed that the shell material consists of a composite (fiberglass) material, and in this article, it is also accepted. that the inhomogeneity depends on three coordinates, that is, the displacement of the shell points linearly depends on all three coordinates. The problem posed in the article was solved to the end, the energy method (the principle of variation) was chosen as the solution method, the resulting equation for determining the natural frequency of vibrations of the structure was solved numerically and characteristic curves were constructed. The results were analyzed.

In the article [2], the tense-deformation state of a cylindrical shell made of orthotropic material was investigated if the structure is reinforced externally with rectilinear rods parallel to its axis. The density of the structural material, taking into account the axis directed in the direction of the normal of the shell surface and the modulus of elasticity, is chosen as a linear function of coordinates (in the special case, parabolic), varying in this direction. The tense-deformation state of the structure under consideration was studied in the case of free oscillatory motion during the movement of a liquid medium inside it. In accordance with the requirement of the oscillation theory, an intrinsic frequency oscillation equation was constructed, the resulting equation was solved numerically, and the results are presented in the form of graphs and widely analyzed.

In the article [3], the stress-strain state of the structural element was investigated with a relatively complex formulation. The complexity of the problem lies in the fact that the expression between deformation and displacement is geometrically nonlinear. On the other hand, the structure in question is reinforced with ribs forming an orthogonal network, while the structure is exposed to a viscoelastic medium from the outside and pressure from the inside. The influence of the external environment was taken into account in the case of simultaneous application of hydrostatic pressure and friction force. Parametric vibrations of the structure are considered here.

In the article [4], the problem of nonlinear formulation was considered, which is a more complex area of deformable solid mechanics. In the question, a cylindrical shell made of viscoelastic material was chosen and the refined Timoshenko theory was applied. This theory mainly takes into account the sliding deformation, the force of inertia of rotation and geometric nonlinearity.

The solution of the problem under consideration is reduced to a system of integro-differential nonlinear relaxation equations with a singular kernel. The Bubnov-Galerkin method was chosen as a solution in combination with the numerical method using quadrature formulas, and the convergence of the result was verified. The influence of viscoelasticity on nonlinear oscillations of the cylindrical shell and dynamic stability is analyzed.

In the paper [5] based on V.Z. Vlasov moment less theory, a problem of dynamical stability of an isotropic cylindrical shell variable along the generatrix of thickness and density under the action of symmetric external pressure variable along the generatrix under different boundary conditions was considered. The exact solution was obtained at one ratio of change in thickness, pressure and density. Structural elements of long and medium shells with variable thickness of materials are used in different fields of machine-building and aero cosmic engineering for mass optimization.

In the case of five boundary value problems minimum values of the excitation coefficients were obtained in relation to the possible arise of nongaming vibrations for the first and second instability domains that have great importance for engineering practice. The assessment of the accuracy of the WKB method was performed for the considered boundary value problems and laws of changes in thickness and density.

The results of experimental studies of frequencies and the forms of natural vibrations of heterogeneous cylindrical shells with holes were given in [6]. The study was performed by the method of holographic interferometry. Influence of holes and other constructional features on the main dynamical characteristics of shells was established. The procedure for conducting the experiment was described. Experimental data were compared with numerical results obtained by the finite elements method.

The review of the works given in the paper shows that the vibrations of a strengthened, viscous-elastic medium contacting cylindrical shell whose material is heterogenous along the thickness in the direction of generatrix and annular directions were not studied. In the article [1] a similar problem was solved; reinforcement was done with rings. The shell material is heterogeneous (depends on all three coordinates), inside the liquid medium and the ring materials are also homogeneous.

2. PROBLEM STATEMENT

The design under consideration is a closed cylindrical shell, inside of which there is a viscoelastic fluid, the body of which is mutually orthogonal to all three coordinate axes with a density varying in the direction and Young's modulus, reinforced from the outside by rectilinear rods parallel to the central axis (Figure 1).

Consider a three-dimensional functional that takes into account the heterogeneity of the coating material depending on stress, relative deformation and density:

$$V=\frac{1}{2}\iint\int_{-h/2}^{h/2}\Big(\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}+\sigma_{12}\varepsilon_{12}+ +\rho\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\Big)dx\,dy\,dz \quad (1)$$

Constants included in the equations expressing the stress-strain states of structures whose material is inhomogeneous (Young's modulus, density, etc.). They should be replaced with variables [8]. In order not to complicate the solution of the problem under consideration too much, the Poisson's ratio is assumed to be constant.

The relationship between the stresses and relative deformations of the coating points, the question of which is considered in the oscillation, can be chosen as follows:

$$\sigma_{11}=\frac{E(x,\theta,z)}{1-\nu^2}(\varepsilon_{11}+\nu\,\varepsilon_{22})$$
$$\sigma_{22}=\frac{E(x,\theta,z)}{1-\nu^2}(\varepsilon_{22}+\nu\,\varepsilon_{11}) \quad (2)$$
$$\sigma_{12}=G(x,\theta,z)\,\varepsilon_{12}$$

$$\varepsilon_{11}=\frac{\partial u}{\partial x};\ \varepsilon_{22}=\frac{\partial \vartheta}{\partial y}+\frac{w}{R};\ \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x} \quad (3)$$

For the shell material, we take the modulus of elasticity and density as follows:

$$E(x,\theta,z)=E_0 f_1(z)\,f_2(x)\,f_3(\theta)$$
$$\rho(z,x)=\rho_0 f_1(z)\,f_2(x)\,f_3(\theta) \quad (4)$$

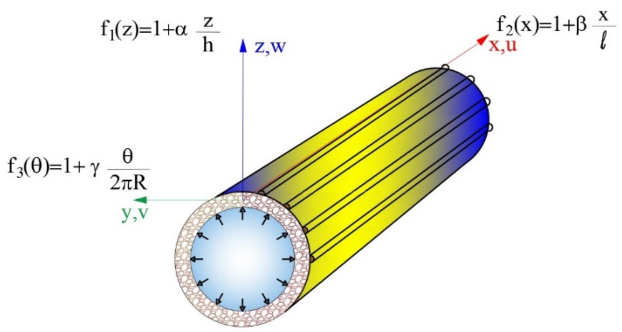

Figure 1. Longitudinally stiffened inhomogeneous cylindrical shell (general view)

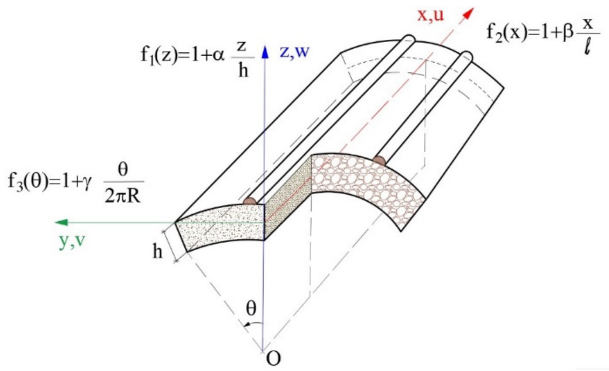

Figure 2. Working scheme of the construction

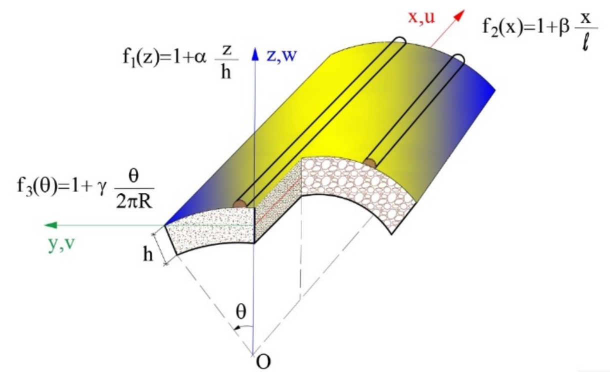

Figure 3. Body of the construction

Taking into account (4) in (2), we obtain:

$$\sigma_{11} = \frac{E_0}{1-\nu^2}(\varepsilon_{11} + \nu\,\varepsilon_{22}) f_1(z)\, f_2(x)\, f_3(\theta)$$
$$\sigma_{22} = \frac{E_0}{1-\nu^2}(\varepsilon_{22} + \nu\,\varepsilon_{11}) f_1(z)\, f_2(x)\, f_3(\theta) \qquad (5)$$
$$\sigma_{12} = G\,\varepsilon_{12} = \frac{E_0}{2(1+\nu)}\,\varepsilon_{12}\, f_1(z)\, f_2(x)\, f_3(\theta)$$

where, E_0 is an elasticity modulus of the heterogeneous material of the shell, ρ_0 is the density of the homogeneous shell material (Figure 3). Allowing for Equation (5) the functional of the total energy of the cylindrical shell is of the form [8]:

$$V = \frac{RE_0}{2(1-\nu^2)} \int_{-h/2}^{h/2} f_1(z)\,dz \times \iint \{\varepsilon_{11}^2 + 2(1-\nu)\varepsilon_{11}\varepsilon_{22} +$$
$$+\varepsilon_{22}^2 + \varepsilon_{12}^2\} \times f_2(x) f_3(\theta)\,dx\,d\theta + \int_{-h/2}^{h/2} f_1(z)\,dz \times \qquad (6)$$
$$\times \iint \rho_0 \left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right) f_2(x) f_3(\theta)\, R\,dx\,d\theta$$

The expression for the potential energy of elastic deformation of the ith longitudinal rib is:

$$\Pi_i = \frac{1}{2}\int_0^l \left[\tilde{E}_i F_i \left(\frac{\partial u_i}{\partial x}\right)^2 + \tilde{E}_i J_{yi} \left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + \right.$$
$$\left. + \tilde{E}_i J_{zi} \left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + \tilde{G}_i J_{kpi} \left(\frac{\partial \varphi_{kpi}}{\partial x^2}\right)^2 \right] dx \qquad (7)$$

Kinetic energy of ribs is written as follows:

$$K_i = \rho_i F_i \int_0^l \left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \right.$$
$$\left. + \left(\frac{\partial w_i}{\partial t}\right)^2 + \frac{J_{kpi}}{F_i}\left(\frac{\partial \varphi_{kpi}}{\partial t}\right)^2 \right] dx \qquad (8)$$

Equations (7) and (8) are written depending on the kinetic and potential energy of the ith rod on the geometric and physical parameters of the rod. They are called as follows: u_i, ϑ_i, w_i corresponding displacements in the direction of the coordinate axes of the points of contact of the ith rod with the coating (so that there are no repetitions, the designations refer to the ith rod), F_i the cross-sectional area of the rod in the direction generating the coating, $\tilde{E}_i$ the elastic modulus of the rod in tension, J_{yi}, J_{zi} are the moment of inertia of the y axis, z in the plane of the shaft cross-section, J_{kpi} is the moment of inertia during torsion, ρ_i is the density, t is the time, respectively, the angles of rotation and torsion. Let's write down the expressions of the torsion and rotation angles:

$$\varphi_{kpi}(x) = \varphi_2(x, y_i) = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right)\Bigg|_{y=y_i} \quad \varphi_i(x) =$$
$$= \varphi_1(x, y_i) = -\frac{\partial w}{\partial x}\Bigg|_{y=y_i}$$

Since potential energy is work performed with the opposite sign, the work performed under the action of an elastic medium on the coating as an external force can be written as follows:

$$A_0 = -R\int_0^l \int_0^{2\pi} q_r\, w\,dx\,d\theta \qquad (9)$$

Let's write down the total energy of the structure in question: when recording the total energy, it is necessary to consider the structure in question as a mechanical system and take into account the energy of each element included in it (shell, k_i ribs, the influence of the medium):

$$J = V + \sum_{i=1}^{k} (\Pi_i + k_i) + A_0 \qquad (10)$$

Accounting for the influence of external q_r forces by various models is given in the scientific literature. Since the vibrations of the reinforced cylindrical shell are studied in this issue, it is necessary to take into account the friction force. In this case, the Pasternak model is considered more suitable:

$$q_r = \mu w - \int_{-\infty}^{t} \Gamma(t-\tau)\, w(\tau)\, d\tau \qquad (11)$$

where, these quantities μ, Γ, $\Gamma(t-\tau) = Ae^{-\lambda(t-\tau)}$, A, λ were interpreted in [1].

Rigid contact conditions between the shell and the rods must be rigid, which means that deformations cannot occur in a jump:

$$u_i(x) = u(x, y_i) + h_i \varphi_1(x, y_i)$$
$$\vartheta_i(x) = \vartheta(x, y_i) + h_i \varphi_2(x, y_i)$$
$$w_i(x) = w(x, y_i),\ \varphi_i(x) = \varphi_1(x, y_i) \qquad (12)$$
$$\varphi_{kpi}(x) = \varphi_2(x, y_i), h_i = 0.5h + H_i^1$$

where, H_i^1 and h_i are the linear dimensions of the rod.

The boundary (Navier) conditions of the considered construction can be written as follows if $x = 0$ and $x = l$:

$$\vartheta = 0,\ w = 0,\ N_{11} = 0,\ M_{11} = 0 \qquad (13)$$

where, the values T_{11}, M_{11} are the internal forces arising in the shell of length l (Figures 2 and 3).

The solution of the issue considered in the article implies finding the frequency of its oscillation. To do this, you first need to get the frequency equation.

The simplest and most reliable method of obtaining the frequency equation is the principle of variation [1]:

$$\delta W = 0 \tag{14}$$

the integral expression w is shown in [1].

A mathematical apparatus has already been obtained for studying and completing the proper vibrations of the structure under consideration. In the design under consideration, it is assumed that the shell body is inhomogeneous in three directions, the rods are homogeneous, a viscoelastic medium moves inside the shell. In the design under consideration, it is assumed that the shell body is inhomogeneous in three directions, the rods are homogeneous, a viscoelastic medium moves inside the shell. To determine the stress-strain state of the described structure (in particular, its own oscillatory movements), it is possible to integrate the total energy (10) of the system in question into (12), (13) boundary and contact conditions and obtain the desired results.

3. PROBLEM SOLUTION

The values u, ϑ, w, included in the full energy expression of the system (10), contains inside the desired frequency ω, as well as unknown constants $u_i, \vartheta_i, w_i\ (i=0,1,2,3)$ and wave numbers k, n, it can be expressed by trigonometric functions as follows.

$$\begin{aligned} u &= (u_0 + u_1 x + u_2 R\theta + u_3 z)\cos kx \cos n\theta \sin \omega t \\ \vartheta &= (\vartheta_0 + \vartheta_1 x + \vartheta_2 R\theta + \vartheta_3 z)\sin kx \sin n\theta \sin \omega t \\ w &= (w_0 + w_1 x + w_2 R\theta + w_3 z)\sin kx \cos n\theta \sin \omega t \end{aligned} \tag{15}$$

Applying (15), we can calculate the work (9):

$$\begin{aligned} A_0 = {} & \frac{\pi^2 h l}{4\omega}\left(\mu - \frac{A\beta}{\omega^2+\beta^2}\right)\times\Big[w_0^2 + l w_0 w_1 + \\ & +\left(\frac{l^2}{3} - \frac{1}{2k^2}\right) w_1^2 + 2R w_0 w_2 + \pi l R w_1 w_2 + \\ & + \frac{R^2 l}{2}\left(\frac{4\pi^2}{3} + \frac{1}{n}\right) w_2^2 + \frac{h^3}{12} w_3^2 \Big] \end{aligned} \tag{16}$$

Simplifying (10), the following dependences were accepted (Figure 2) [7]:

$$f_i(x \cdot y \cdot \theta) = \delta_i^j \begin{cases} 1 + \alpha \dfrac{z}{h} & z,\ i=1 \\ 1 + \beta \dfrac{x}{l}, & x,\ i=2 \\ 1 + \gamma \dfrac{\theta}{2\pi R} & \theta,\ i=3 \end{cases} \tag{17}$$

$(i = 0,1,2,3;\ j = 1,\ 2,\ 3)$.

where, is the $\delta_i^j = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$ Kronecker symbol, α, β, γ are constant coefficients and are indicators of the change in the inhomogeneity of the shell in the direction of the corresponding coordinates;

$x \in [0,1]$, $y \in [0, 2\pi]$ and $t \in [0, \pi/\omega]$

Finally, if we integrate the variables x, y, t in the appropriate bounds, we can write the Lagrange function J depending on the desired quantities $u_i, \vartheta_i, w_i\ (i=0,1,2,3)$:

$$\begin{aligned} J = {} & t_0 a_1 + \sum_{i=2}^{39} t_i a_i + \rho_0 \omega^2 [(u_0^2 + \vartheta_0^2 + w_0^2) C_1 + \\ & + 2(u_0 u_1 + \vartheta_0 \vartheta_1 + w_0 w_1) C_2 + u_1^2 C_3 + (\vartheta_1^2 + w_1^2) C_4 + \\ & + 2(u_0 u_2 R + w_0 w_2) C_5 + 2\vartheta_0 \vartheta_2 R C_6 + \\ & + 2(u_0 u_3 + \vartheta_0 \vartheta_3 + w_0 w_3) C_7 + u_1 u_2 R C_8 + R(\vartheta_1 \vartheta_2 + w_1 w_2) C_9 + \\ & + u_1 u_3 C_{10} + (\vartheta_1 \vartheta_3 + w_1 w_3) C_{11} + (u_2^2 + w_2^2) C_{12} + \vartheta_2^2 C_{13} + u_2 u_3 C_{14} + \\ & + (\vartheta_2 \vartheta_3 + w_2 w_3) C_{15} + (u_3^2 + \vartheta_3^2 + w_3^2) C_{16} + \\ & + \frac{1}{2}\sum_{i=1}^{k_1} (E_i F_i + \rho_i F_i \omega^2) \frac{k^2 l}{2} (H_i - \frac{h}{2}) \cdot [u_0^2 + \\ & + u_0 u_1 l + u_1^2 \left(\frac{l^2}{3} - \frac{1}{2k^2}\right) + 2R u_0 u_2 \theta_i + u_0 u_3 \left(H_i + \frac{h}{2}\right) + \\ & + 2R u_1 u_2 l \theta_i + u_1 u_3 l \left(H_i + \frac{h}{2}\right) + R^2 u_2^2 \theta_i^2 + R u_2 u_3 \theta_i \left(H_i + \frac{h}{2}\right) + \\ & + \frac{1}{3} u_3^2 \left(H_i + \frac{H_i h}{2} + \frac{h^2}{4}\right) \Big] \cos^2 n\theta_i + \\ & + \frac{1}{2}\sum_{i=1}^{k_1} \left[E_i J_{yi} \frac{k^2 l}{2}\left(H_i - \frac{h}{2}\right) + \omega^2 \rho_i F_i \right] \cdot [w_0^2 + w_0 u_1 l + \\ & + w_1^2 \left(\frac{l^2}{3} - \frac{1}{2k^2}\right) + 2R w_0 w_2 \theta_i + w_0 w_3 \left(H_i + \frac{h}{2}\right) + \\ & + R w_1 w_2 l \theta_i + w_1 w_3 l \left(H_i + \frac{h}{2}\right) + R^2 w_2^2 \theta_i^2 + R w_2 w_3 \theta_i \left(H_i + \frac{h}{2}\right) + \\ & + \frac{1}{3} w_3^2 \left(H_i^2 + \frac{H_i h}{2} + \frac{h^2}{4}\right) \Big] \cos^2 n\theta_i + \\ & + \frac{1}{2}\sum_{i=1}^{k_1} \left[E_i J_{zi} \frac{k^2 l}{2}\left(H_i - \frac{h}{2}\right) + \omega^2 \rho_i F_i \right] \cdot [\vartheta_0^2 + \vartheta_0 \vartheta_1 l + \\ & + \vartheta_1^2 \left(\frac{l^2}{3} - \frac{1}{2k^2}\right) + 2R \vartheta_0 \vartheta_2 \theta_i + \vartheta_0 \vartheta_3 \left(H_i + \frac{h}{2}\right) + R \vartheta_1 \vartheta_2 l \theta_i + \\ & + \vartheta_1 \vartheta_2 l \left(H_i + \frac{h}{2}\right) + R^2 \vartheta_2^2 \theta_i^2 + R \vartheta_2 \vartheta_3 \theta_i \left(H_i + \frac{h}{2}\right) + \\ & + \frac{1}{3} \vartheta_3^2 \left(H_i^2 + \frac{H_i h}{2} + \frac{h^2}{4}\right) \Big] \sin^2 n\theta_i + \\ & + \frac{1}{2}\sum_{i=1}^{k_1} \left[\frac{l}{2R^2}\left(H_i - \frac{h}{2}\right) G_i J_{kpi} + \omega^2 \rho_i F_i \right] \cdot \{ [(\vartheta_0 - n w_0)^2 + \\ & + l(\vartheta_0 - n w_0)(\vartheta_1 - n w_1) + (\vartheta_1 - n w_1)^2 + \\ & + 2R(\vartheta_0 - n w_0)(\vartheta_2 - n w_2)\theta_i + \\ & + (\vartheta_0 - n w_0)(\vartheta_3 - n w_3)\left(H_i + \frac{h}{2}\right) + \\ & + Rl(\vartheta_1 - n w_1)(\vartheta_2 - n w_2)\theta_i + \\ & + (\vartheta_1 - n w_1)(\vartheta_3 - n w_3)\frac{l}{2}\left(H_i + \frac{h}{2}\right) + R^2 (\vartheta_2 - n w_2)\theta_i^2 + \\ & + \frac{R}{2}(\vartheta_2 - n w_2)(\vartheta_3 - n w_3)\left(H_i + \frac{h}{2}\right)\theta_i + \frac{1}{3}(\vartheta_3 - n w_3)^2 \cdot \end{aligned}$$

$$\cdot\left(H_i^2+\frac{H_ih}{2}+\frac{h^2}{4}\right)\Bigg]\sin^2 n\theta_i-\left[\frac{n}{R}\omega_2(\vartheta_0-nw_0)+\frac{l}{2}(\vartheta_1-nw_1)+\right.$$
$$+R(\vartheta_2-nw_2)\theta_i+\frac{l}{2}(\vartheta_3-nw_3)\left(H_i+\frac{h}{2}\right)\Bigg]\sin 2n\theta_i+$$
$$+n^2w_2^2\cos^2 n\theta_i\}+\frac{\pi^2hl}{4\omega}\left(\mu-\frac{AB}{\omega^2+\beta^2}\right)[w_0^2+lw_0w_1+$$
$$+\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)w_1^2+2Rw_0w_2+\pi lRw_1w_2+$$
$$+\frac{R^2l}{2}\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)w_2^2+\frac{h^3}{12}w_3^2\Bigg] \tag{18}$$

$P_0=ku_1u_0;\ P_1=ku_1^2;\ P_2=ku_1u_2;$

$P_3=ku_1u_3;\ P_4=k^4u_0^2;\ P_5=2k^2u_0u_1;\ P_6=P_1;$

$P_7=2k^2u_0u_2;\ P_8=2k^2u_0u_3;\ P_9=2k^2u_1u_2;$

$P_{10}=2k^2u_1u_3;\ P_{11}=k^2u_2^2;\ P_{12}=k^2u_2u_3;\ P_{13}=u_3^2k^2$

$t_0=u_1^2+u_2^2;\ t_{1+i}=-P_i+\frac{1-\nu}{R}l_i\quad(i=0,1,2,3);$

$t_{i+5}=P_{i+4}+\frac{1}{R^2}T_i+l_{4+i}(i=0,1,2);$

$t_8=P_7+\frac{1}{R^2}T_4+l_8;\ t_9=P_8+\frac{1}{R^2}T_5+l_7;$

$t_{10}=P_9+\frac{1}{R^2}T_6+l_{11};\ t_{11}=P_{10}+\frac{1}{R^2}T_7+l_{12};$

$t_{12}=P_{11}+\frac{1}{R^2}T_3+l_9;\ t_{13}=P_{12}+\frac{1}{R^2}T_9+l_{13};$

$t_{14}=P_{13}+\frac{1}{R^2}T_7+l_{10};\ t_{15}=S_0R-\frac{1}{2}kRu_0\vartheta_2;$

$t_{16}=S_1R-\frac{1}{2}kRu_1u_2;\quad t_{17}=S_2R-\frac{1}{2}kRu_2\vartheta_2;$

$t_{18}=S_3R-\frac{1}{2}kRu_3\vartheta_2;\quad t_{19}=\frac{1}{R^2}\vartheta_2^2;\ t_{20}=u_0u_2;$

$t_{21}=u_1u_2;\quad t_{22}=u_2^2;\quad t_{23}=u_2u_3;$

$t_{24}=\frac{1}{2R}\vartheta_1u_2+\frac{1-\nu}{2}u_1\vartheta_2;\ t_{25}=\vartheta_2^2R;$

$t_{26}=n^2u_0^2+k^2\vartheta_0^2-2nku_0\vartheta_0;$

$t_{27}=2n^2u_0u_1+2k^2\vartheta_0\vartheta_1-2nku_0\vartheta_2-2nku_2\vartheta_0;$

$t_{28}=2n^2u_0u_2+2k^2\vartheta_0\vartheta_2-2nku_0\vartheta_2-2nku_2\vartheta_0;$

$t_{29}=2n^2u_0u_3+2k^2\vartheta_0\vartheta_3-2nku_0\vartheta_3-2nk\vartheta_0u_3;$

$t_{29+i}=n^2u_i^2+k^2\vartheta_i^2-2nku_i\vartheta_i\quad(i=1,2,3);$

$t_{33}=2n^2u_1u_2+2k^2\vartheta_1\vartheta_2-2nku_1\vartheta_2-2nku_2\vartheta_1;$

$t_{34}=2n^2u_1u_3+2k^2\vartheta_1\vartheta_3-2nku_1\vartheta_3-2nku_3\vartheta_1;$

$t_{35}=n^2u_2u_3+k^2\vartheta_2\vartheta_3-2nku_2\vartheta_3-2nku_3\vartheta_2;$

$t_{36+i}=-nu_i\vartheta_1+k\vartheta_1\vartheta_i\quad(i=0,1,2,3);$

$S_i=\vartheta_2(n\vartheta_i+w_i)\quad(i=0,1,2,3);$

$l_i=u_1(n\vartheta_i+w_i)\quad(i=0,1,2,3);$

$l_{4+i}=-nku_0\vartheta_i-nk\vartheta_0\vartheta_i-ku_0w_i-kw_0u_i\quad(i=1,2,3);$

$l_{7+i}=-knu_i\vartheta_i-ku_iw_i\quad(i=1,2,3);$

$l_{11}=-knu_1\vartheta_2-nku_2\vartheta_1-ku_1w_2-ku_2w_1;$

$l_{12}=-knu_1\vartheta_3-nku_3\vartheta_1-ku_1w_3-ku_3w_1;$

$l_{13}=-knu_2\vartheta_3-nku_3\vartheta_2-ku_2w_3-ku_3w_2;$

$T_0=n^2\vartheta_0^2+2nw_0\vartheta_0+w_0^2;$

$T_i=2n^2\vartheta_0\vartheta_i+2n\vartheta_0w_i+2nw_0\vartheta_i+2w_0w_i\quad(i=1,2,3);$

$T_{3+i}=n^2\vartheta_i^2+2n\omega_i\vartheta_i+w_i^2\quad(i=1,2,3);$

$T_7=2n^2\vartheta_1\vartheta_2+2nw_2\vartheta_1+2nw_1\vartheta_2+2w_1w_2;$

$T_8=2n^2\vartheta_1\vartheta_3+2nw_3\vartheta_1+2nw_1\vartheta_3+2w_1w_3;$

$T_9=2n^2\vartheta_2\vartheta_3+2nw_3\vartheta_2+2nw_2\vartheta_3+w_2w_3;$

$a_1=\frac{h\pi l}{2}\left(1+\frac{\gamma}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right);$

$a_2=\frac{h\pi l}{2}\left(1+\frac{\gamma}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right);$

$a_3=\frac{R\beta}{2k}\left[\pi^2+\frac{\gamma l}{2\pi^2}\left(\frac{8\pi^3}{3}-\frac{\pi}{2n^2}\right)\right];$

$a_4=\frac{\pi\alpha\beta h^2}{24k}\left(l+\frac{\gamma}{2}\right);$

$a_5=\frac{\pi lh}{2n}\left(1+\frac{\gamma}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{8l}\right);$

$a_6=\frac{\pi h}{2}\left[\frac{l^2}{2}+\frac{\gamma l^3}{4}+\pi h\beta\left(\frac{l^3}{3}+\frac{1}{2k^2}\right)+\frac{\beta\gamma}{2}\left(\frac{l^2}{3}+\frac{1}{2k^2}\right)\right];$

$a_7=\frac{\pi l}{2}\left[\frac{l^2}{3}+\frac{1}{2k^2}+\frac{\gamma h}{2}\left(\frac{l^2}{3}+\frac{1}{2k^2}\right)+\frac{\beta\gamma h}{8}\left(l^2-\frac{3}{k^2}\right)\right];$

$a_8=\frac{hlR}{2}\left[\pi^2+\frac{\gamma}{2}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)+\frac{\beta\pi^2}{2}+\frac{\beta\gamma}{4}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)\right];$

$a_9=\frac{\pi h^2\alpha}{3}\left(\frac{1}{l}+\frac{\gamma l}{2}+\frac{\beta l}{2}+\frac{\beta\gamma l}{4}\right);$

$$a_{10}=hR\left[\frac{\pi^2l^2}{4}+\frac{\gamma hl^2}{8}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)+\beta h\pi^2\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)\right]+$$
$$+\frac{\beta\gamma h}{2}\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)\Bigg];$$

$a_{11}=\alpha h^2\left(\frac{\pi l^2}{6}+\frac{\pi\gamma l^2}{12}-\frac{\beta}{k}-\frac{\pi\beta\gamma}{6k}\right);$

$$a_{12}=\frac{\pi hlR^2}{2}\left[\frac{8\pi^2}{3}-\frac{1}{2n^2}+\frac{\beta}{2}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)+\right.$$
$$+\gamma\left(\pi^2+\frac{2}{n^2}\right)+\frac{\beta\gamma}{2}\left(\pi^2+\frac{1}{n^2}\right)\Bigg];$$

$$a_{13}=\frac{Rh^2l\alpha}{24}\left[\pi^2+\frac{\gamma}{2}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)+\right.$$

$$+\frac{\pi^2\beta}{2}+\frac{\beta\gamma}{4}\left(\frac{8\pi^2}{3}-\frac{1}{2n^2}\right)\Bigg];$$

$$a_{14}=\frac{\pi l h^3}{24}\left(1+\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right);\ a_{15}=-\frac{\gamma h l}{2}\left(1+\frac{\beta}{2}\right);$$

$$a_{16}=\frac{\gamma h}{4}\left[l^2+\beta\left(\frac{l^2}{9}+\frac{1}{k^2}\right)\right];$$

$$a_{17}=-\pi h l R\left(1+\frac{\gamma}{2n}+\frac{\beta}{2}+\frac{\beta\gamma}{4n}\right);\ a_{18}=-\frac{\alpha\gamma h^2 l}{24}\left(1+\frac{\beta}{2}\right);$$

$$a_{19}=-\frac{\pi l}{2}\left(1+\frac{\gamma h}{2}+\frac{\beta h}{2}+\frac{\beta\gamma h}{4}\right);\ a_{20}=-\frac{\gamma h l}{2}\left(1+\frac{\beta}{2}\right)=a_{15};$$

$$a_{21}=-\gamma h\left[\frac{l^2}{4}-\frac{\beta}{2}\left(\frac{l^3}{3}-\frac{1}{2k^2}\right)\right];$$

$$a_{22}=-\pi h R\left(\frac{1}{l}+\frac{\gamma l}{2n}+\frac{\beta l}{2}+\frac{\beta\gamma l}{4n}\right);$$

$$a_{23}=-\frac{\alpha\gamma h^2 l}{24}\left(1+\frac{\beta}{2}\right)=a_{18};\quad a_{24}=-\frac{\beta\gamma h}{2k};$$

$$a_{25}=-\frac{h\pi l}{2}\left(1+\frac{\pi\gamma}{24}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right)=b_7;$$

$$a_{26}=-\frac{\pi h l}{2R^2}\left(1+\frac{\pi\gamma}{24}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right);$$

$$a_{28}=\frac{\pi^2 h l}{2R}\left[1+\frac{\pi\gamma}{18}+\frac{\beta}{2}+\frac{\beta\gamma}{3}\right];$$

$$a_{27}=\frac{\pi h}{R^2}\left[\frac{l^2}{4}+\frac{\pi\gamma l^2}{96}+\beta\left(\frac{l^2}{9}+\frac{1}{4k^2}\right)+\frac{\pi\beta\gamma}{2}\left(\frac{l^2}{9}+\frac{1}{4k^2}\right)\right];$$

$$a_{29}=-\frac{\pi h^2 l\alpha}{24R^2}\left(1+\frac{\pi\gamma}{18}+\frac{\beta}{2}+\frac{\beta\gamma}{3}\right);$$

$$a_{30}=\frac{1}{R^2}\left(\pi h+\frac{\gamma h}{2}\right)\left[\frac{l^8}{6}+\frac{l}{4k^2}+\beta\left(\frac{l^2}{8}+\frac{3l}{8k^2}\right)\right];$$

$$a_{31}=\pi\left(\frac{hl}{2}+\frac{\beta h}{2}\right)\left[\frac{4\pi^2}{3}+\left(\gamma+\frac{\beta\gamma}{l}\right)\left(\pi^2-\frac{2}{n^2}\right)\right];$$

$$a_{32}=\frac{\pi h^3 l}{24R^2}\left[1+\frac{\gamma}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right];$$

$$a_{33}=\frac{h\pi^2}{R}\left[\frac{l^2}{4}+\beta\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)+\frac{\gamma l^2}{6}+\frac{2\beta\gamma}{3}\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)\right];$$

$$a_{34}=\frac{\pi h^2}{12R^2}\left[\frac{\alpha l^2}{4}+\alpha\beta\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)+\frac{\alpha\gamma l^2}{8}+\frac{\alpha\beta\gamma}{2}\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)\right];$$

$$a_{35}=\frac{\pi^2 a l h^2}{24R}\left[1+\frac{\beta}{2}+\frac{2\gamma}{3}+\frac{\beta\gamma}{3}\right];\quad a_{36}=\frac{\pi\beta h}{2kR^2}\left(1+\frac{\gamma}{2}\right);$$

$$a_{37}=\frac{\pi^2 a l h^2}{24R}\left[l-\beta+\frac{\gamma l}{2}-\frac{\beta\gamma}{2}\right];$$

$$a_{38}=\frac{\pi^2\beta h}{Rk}\left(\frac{1}{2}+\frac{\gamma}{3}\right);\quad a_{39}=\frac{\pi\alpha\beta h^2}{24kR^2}\left(1+\frac{\gamma}{2}\right);$$

$$c_1=\frac{\pi h l}{2}\left[1+\frac{\gamma}{2}+\frac{\beta}{2}+\frac{\beta\gamma}{4}\right];$$

$$c_2=\frac{\pi h}{2}\left[\frac{l^2}{2}+\frac{\gamma l^2}{4}+\beta\left(\frac{l^2}{3}+\frac{1}{2k^2}\right)+\frac{\gamma l}{2}\left(\frac{l^2}{3}+\frac{1}{2k^2}\right)\right];$$

$$c_3=\pi h\left[\left(\frac{l^2}{6}+\frac{1}{4k^2}\right)l\left(1+\frac{\gamma}{2}\right)+\left(\frac{l^2}{8}+\frac{3}{8k^2}\right)l\beta\left(1+\frac{\gamma}{2}\right)\right];$$

$$c_4=\pi h\left[\left(\frac{l^2}{6}-\frac{1}{4k^2}\right)l\left(1+\frac{\gamma}{2}\right)+\left(\frac{l^2}{8}-\frac{3}{8k^2}\right)l\beta\left(1+\frac{\gamma}{2}\right)\right];$$

$$c_5=h\left[\frac{\pi^2 l}{2}+\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)\left(\frac{\gamma l}{2}+\frac{\beta\gamma}{4}\right)+\frac{\pi^2\beta}{2}\right];$$

$$c_6=h\left[\frac{\pi^2 R}{2}+\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)\left(\frac{\gamma l}{4}+\frac{\beta\gamma}{4}\right)+\frac{\pi^2\beta}{2}\right];$$

$$c_7=\frac{\pi\alpha h^2}{24}\left(1+\frac{\gamma l}{2}+\frac{\beta l}{2}+3\beta\gamma l\right);$$

$$c_8=\pi R\left[\frac{l^2}{2}+\frac{\gamma l^2}{4}+\left(\frac{l^2}{3}+\frac{1}{2k^2}\right)\left(\beta+\frac{\beta\gamma}{2}\right)\right];$$

$$c_9=\pi R\left[\frac{l^2}{2}+\frac{\gamma l^2}{4}+\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)\left(\beta+\frac{\beta\gamma}{2}\right)\right];$$

$$c_{10}=\frac{\alpha h^2}{12R}c_8;\quad c_{11}=\frac{\alpha h^2}{12R}c_9;$$

$$c_{12}=\frac{\pi^2 R}{2}\left[(l+\beta)\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)+(\gamma l+\beta\gamma)\left(\pi^2+\frac{1}{n^2}\right)\right];$$

$$c_{13}=\frac{\pi^2 R}{2}\left[(l+\beta)\left(\frac{4\pi^2}{3}-\frac{1}{n}\right)+(\gamma l+\beta\gamma)\left(\pi^2-\frac{1}{n^2}\right)\right];$$

$$c_{14}=\frac{R\alpha h^2 l}{12}\left[\pi^2\left(1+\frac{\beta}{2}\right)+\left(\frac{\gamma}{2}+\frac{\beta\gamma}{4}\right)\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)\right];$$

$$c_{15}=\frac{\alpha h^2 lR}{12}\left[\pi^2\left(1-\frac{\beta}{2}\right)+\left(\frac{\gamma}{2}+\frac{\beta\gamma}{4}\right)\left(\frac{4\pi^2}{3}-\frac{1}{n}\right)\right];\ c_{16}=\frac{h^2}{12}c_1$$

4. CONCLUSIONS

For the construction under consideration (18), we vary the expression of the total energy and obtain a system of algebraic Equations based on the coefficients of independent variations u_i, ϑ_i, w_i $(i=0,1,2,3)$. By making the main determinant of the resulting system of equations equal to zero, we obtain its solution. This equation will be the frequency equation of the system, and this equation can be solved by taking the following geometric, physical and mechanical quantities.

$$\rho_0=\rho_j=1850\ \frac{\text{kg}}{\text{m}^3};\ \tilde{E}_i=6.67\times10^9\ \frac{\text{N}}{\text{m}^2};\ m=1;\ n=8;$$

$$h_i=0{,}45;\ R=160\,\text{cm};\ n=8;\ \mu=10^6\ \frac{\text{N}}{\text{m}^2};\ h_i=0.45\,\text{mm};$$

$$\nu=0.35;\ \frac{l}{R}=3,\ \frac{h}{R}=\frac{1}{6},\ \alpha=0.4;\ R=160\,\text{cm};$$

$F_i = 5.2\,\text{mm}^2$; $I_{kp,i} = 0.23\,\text{mm}^4$; $I_i = 5.1\,\text{mm}^4$;
$I_{Zi} = 1.3\,\text{mm}^4$; $\lambda = 0.005$; $A = 0.1615$.

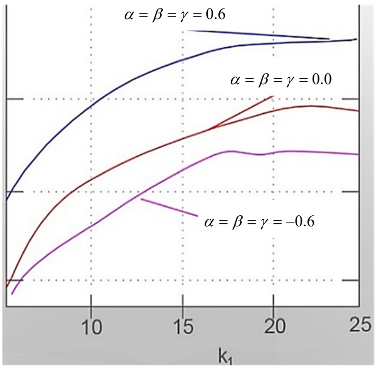

Figure 4. Dependence of the frequency parameter on k_1

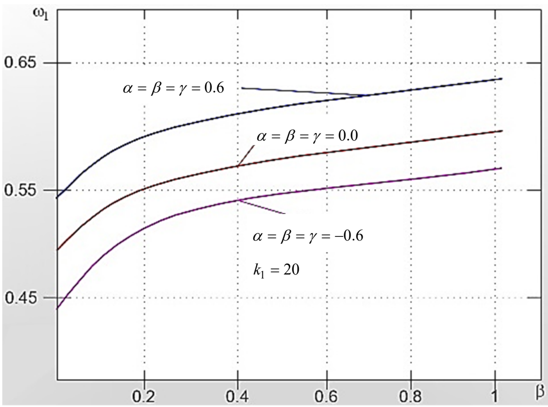

Figure 5. Dependence of the frequency parameter on β

Numerical calculations have been completed and the results are described in two graphs. The graphs are constructed of the frequency parameter of the coating $\omega_1 = (\rho_0 R^2 \omega^2 / b_{11})^{1/2}$ depending on the number of longitudinally reinforcing rods k_1 and on the parameter β of inhomogeneity in the direction of the coordinate axis x. When the number of rods directed in the longitudinal direction changes, the frequency of the structure also changes, which after reaching the highest value, it begins to decrease. Increasing the number of rods creates the effect of inertia in the oscillatory motion. In the problem under consideration, the change in the value of β, which is a parameter of inhomogeneity only in the direction of the forming shell, was taken into account in the calculation. Here it was observed that with increasing positive values of the inhomogeneity parameter of the system (Figure 5) the frequency of the system increases.

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BIOGRAPHY

Javad Mahdavi Tabatabaei was born in Tabriz, Iran, on July 06, 1981. He received the B.Sc. degree from Mahabad Branch, Islamic Azad University, Iran and the M.Sc. degree from Azerbaijan Architecture and Construction University, Baku, Azerbaijan, both in Civil Engineering in 2003 and 2009, respectively. He is studying Ph.D. in Civil Engineering at Azerbaijan Architecture and Construction University. Currently, he is an Academic Member in Department of Civil Engineering, Faculty of Engineering and Architecture, Istanbul Rumeli University, Istanbul, Turkey. He is also Director of International Relations Office at the same university. He has published 15 papers in international conferences and journals. His scientific interests are construction with suitable energy managing, optimum architecture, energy efficiency and saving in buildings, and sustainable energies-based buildings.