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USING ICT AND INTER-DIDACTIC CONTINUITY PHYSICS AND MATHEMATICS TO INTRODUCE INTEGRAL CONCEPT IN MATHEMATICS

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Abstract- This research proposes a redesign of the activity introducing the notion integral into Moroccan official guidelines and textbooks. This choice is based on the results of the didactic and epistemological analysis of the activity introducing the notion of integral in mathematics curriculum in the high school. The analysis concerned first the institutional relationship, then the pertinence of the approach activities¸ the semiotic registers and finally the interdisciplinary aspect. The activities proposed in the three textbooks studied are in conformity with the official pedagogical guidelines. However, although they are interesting¸ the definition of integrals from primitive functions is meaningless and the recommended approach seems to be based more on calculation techniques and parachuting as the relation

$$F(b)-F(a)=\int_a^b f(x)dx \tag{1}$$

without possibilities to interpret the meaning of Equation (1) and the meaning of Equation (2):

$$\int_a^b f(x)dx \tag{2}$$

We also note a rather tool status of the notion integral due to the ambiguous character of the relationship of the integral with the areas as it is presented in the textbooks. In order to give more meaning to the notion of integral and to ensure interdisciplinary continuity between physics and mathematics, we propose an introductory activity based on mechanics and using dynamic geometry, in this case Geogebra. We choice the notion of areas as approach to define integral concept. The model has been experimented with future teachers and the results are in the process of development.

Keywords: Integral, Primitive Functions, Mathematics¸ Interdisciplinary, ICT.

1. INTRODUCTION

The basic premises of the notion of the integral is attributed to Archimedes. But in fact, there are many other Greek mathematicians who contributed in this field before Archimedes as well as Arab mathematicians like Ibn Al-Haytam. The history of this rich and complex notion was marked by the work of Newton and Leibniz even though, it seems that Bhaskara was a pioneer in some of the principles of differential calculus [1]. Both, Newton and Leibniz created the infinitesimal calculus and consequently the foundations of the integral calculus. Newton, as a physicist, was interested in the notion of the integral in a more general context, namely the differential and integral calculus, in order to solve problems in physics, whereas Leibniz, who lived at the same period as Newton, focused on the notion of the integral defined in a purely mathematical context. Moreover, he who introduced the infinitesimal element "*d*" and the sign ∫ which is inspired by the Latin Summa.

Based on the work of Leibniz, Cauchy defines the integral as reported by Villeneuve [2] as the limit of the sums (called Cauchy sums), which correspond to the rectangles under the curve that approach the curve in limit.

Regarding the definite integral of a function between two values a and b of the interval $[a, b]$, Riemann divides the interval $[a, b]$ into n sub-intervals, multiplies the value of the function at a given point in each sub-interval by the length of the sub-interval and verifies whether the sum has a limit when the size of the sub-intervals tends to zero. For Riemann if the sum has a limit, it's the integral [3]. Moreover, the notion of the Riemann integral which was implicit in area and volume calculations became more explicit and acquired its status as real mathematical knowledge thanks to the work of Cauchy and Riemann [4].

As for the teaching integral, it's generally introduced at the end of high school. This is probably due that its formalization requires function and the limit. This applies also to Morocco and the notion of integral is presented, in accordance with the official pedagogical guidelines specific to the teaching, via the primitives where Equation (2) is defined as the difference in Equation (3).

$$F(b)-F(a) \tag{3}$$

where, f is a continuous function on the interval $[a,b]$ and F is the primitive of f Thus, we admit the Equation (1).

Does this approach by primitives allow the establishment of the link between integral and primitive? Is the meaning of the ostensive of the Equation (2) and the unit dimension well explained?

The work of this research has attempted to answer these questions especially since the difficulties related to the teaching of the integral concept still arouse the interest of several researchers and trainers [5-9]. Considering the close relationship between mathematics and physics, the interest of ensuring an interdidactic continuity in the teaching of these two disciplines [10-13] and in a perspective to propose appropriate alternatives that may make the notion of integral more meaningful, a situation from mechanics was elaborated. Generally, the interdidactic continuity is considered in the mathematical direction to physics. But in our study, we propose to explore a physics situation to introduce a mathematical notion.

Due to the importance of ICT in education, we have integrated dynamic geometry in our proposed activity model in this case Geogebra. Indeed, several studies have shown the benefits of integrating ICT into mathematics and science education as supporting and transforming teaching and learning tool [14-18].

2. RESEARCH METHODOLOGY

In this descriptive and analytical research¸ we were interested in the introduction of the notion of integral in the Moroccan mathematics programs. The research work was articulated in three phases. The first phase was dedicated to the choice of the textbooks to be studied¸ to the definition of the objects of content analysis as well as the indicators and to the coding of the latter.

The second phase concerns firstly the analysis of the institutional relationship that could possibly explain the didactic choices adopted in the three textbooks studied, and then, as a second step, the analysis of the contents of the three textbooks selected, in this case the approach activities for the introduction of the notion of integral, as well as the properties and functional aspects of this notion.

This in-depth analysis was preceded by a quick reading of the textbooks, which enabled the indicators defined in the first phase to be improved. Thus, the analysis grid¸ presented in detail in the results and discussion section¸ was elaborated from the orientations and key recommendations consigned in the official document relating to the pedagogical orientations and the specific programs for the teaching of mathematics at the high school. Then we added other indicators covering the meaning of the ostensive of the Equation (2) as well as the dimension unit of area.

The last phase was devoted to the choice of the approaching activity that may give more meaning to the notion of integral and to the ostensive of Equation (2).

2.1. Analytical Objects and Indicators

The objects of analysis of the institutional report were conceived from each recommendation or requirement relating to the teaching integral notion. These data¸ which we have translated from Arabic¸ are recorded in pages 92 and 99 of the official documents. Evaluation's indicators of the pertinence of introduction activities were defined from the official pedagogical orientations then improved following a fast reading of the textbooks. For example, we note the addition of indicators concerning the ostensive constituting the Equation (2) and the justification of the notion of unit of area. Then¸ we proceeded to their codification. The objects of the analysis as well as the indicators are shown in Table 1.

Table 1. Objects of analysis and indicators adopted

Subject	Indicators	Coding of indicators
Activity to introduce the concept of integral	The approach taken is pertinent	I_1
	The approach taken is coherent	I_2
	The origin of the symbol ∫ is specified	I_3
	The meaning of dx explained	I_4
	The unit of area is justified	I_5
	Various semiotic registers	I_6
	Integrated ICT	I_7

3. MAIN RESULTS AND DISCUSSION

3.1. Pre-Analysis Phase

In order to understand and be able to interpret the choice of approach activities for the introduction of the notion of integral and the didactic approaches adopted in the textbooks¸ we proceeded to the analysis of the official document relating to the pedagogical orientations and the specific programs to the teaching of mathematics in the cycle of the scientific secondary qualifying. This document was published in November 2007 by the Department of Programs and Curricula. and since then, no other revised edition has been published to date. Based on this observation and the fact that the education system has undergone several reforms since then, this pre-analysis phase allowed us to further refine the objectives of our study as well as the objects of analysis and the indicators

The textbooks selected for the study are, to our knowledge, the most widely used in the teaching of mathematics in the end year of the high school. As illustrated in the description of the textbooks below¸ we have chosen three textbooks for all science except the mathematical sciences option.

Table 2. Textbooks studied

Manual	Code	Option	Language	Publisher	Edition
The Math Oasis BIOF*	M1	Physical Sciences – Life and Earth Sciences – Electrical and Mechanical Sciences – Technologies Agricultural Sciences	French	Al Madariss Publishing and Distribution Company	2016
Al Wadih Fi Riadiat	M2		Arabic	Dar Arachad Alhaditha	2007
FI Rihab Riadiat	M3		Arabic	Dar Alalamia lilkitab	2016

* International Baccalaureate, French Option

3.2. Analysis Phase

The results of the analysis of the textbooks show that they conform to the guidelines specified in the official document. Thus¸ in all the textbooks studied¸ the integral is defined from the primitive function and it is admitted the

Equation (1). f is a continuous function on the interval $[a, b]$ and F a primitive of the function f. The properties of the integral: Chasles relation; linearity; integrals and order; mean value is admitted according to the pedagogical guidelines. It is indicated that it is possible to interpret these properties geometrically via the areas. In this effect, we note that in the M2 textbook, the geometric interpretation of the integral of a continuous positive function has been integrated before the properties mentioned above, which could facilitate its exploitation for their interpretation. However, in textbooks M1 and M3, the integral was used to calculate area and volume.

As for the integral calculation techniques, all textbooks are limited to the use of the primitive function and integrating by parts in conformity with the official teaching guidelines. The integration of situations from physics, in particular mechanics and electricity (the work of a force¸ the power ...) is proposed in all the textbooks as suggested in page 92 of the official guidelines. In fact, in the M1 student's book, the mechanics situations are exploited as deep learning exercises (Exercises 34 and 35; page 168) and as synthesis problems (Exercise 47; page 170).

In the textbook M2¸ two electricity situations are exploited, one in the section entitled "Uses" situated towards the end of the chapter (page 207) and the other in the section "Synthesis problems" (exercise 44; page 214) where we note the presence of another problem related to the work of a force (exercise 46; page 214). In the textbooks M3, we note the presence of only one solved exercise in the section "I integrate my learning" and which relates to mechanics (page 214) and in the section the deepening and the synthesis exercises, two problems of electricity and a problem of mechanics are integrated.

Concerning the integration of ICT¸ no requirement or recommendation is recorded in the pedagogical guidelines this could be explained by the fact that the official document has not been updated since 2007. However, in the M1 manual¸ towards the end¸ we note the use of the Excel spreadsheet as an application of the computer tool in the framing of an integral but not in the M2 and M3 manuals.

3.3. Pertinence of Activities Adopted in the Textbooks

In all the textbooks studied, the approach activity introducing the notion of integral is based on the primitive function in conformity with the official guidelines. The expression (2) is defined as equal to the expression (3) where F is the primitive of function f. The general approach adopted is illustrated below. Note that in M2, the function is $f(x)=3x^2$ and in M3, the function $f(x)=3x^2-2x+1$

Algorithm 1. Approach activity defining calculus, page 153 of the textbook M1

Let f be the numerical function of a real variable defined by $f(x)=3x^2-4x$.
Give three primitives functions F, G and H of the function f defined on $\mathbb{R}$.
Calculate $F(2)-F(1)$, $G(2)-G(1)$, $H(2)-H(1)$.
What do we find?
Let a and b be two real numbers, Calculate $F(b)-F(a)$, $G(b)-G(a)$ and $H(b)-H(a)$.
What do we find?
The number $F(b)-F(a)$ the number doesn't depend on the choice of a primitive of the function f.
The number $F(b)-F(a)$ is called Integral of f from a to b. It's noted: $\int_a^b f(x)dx$.
We read Integral of $f(x)$ from a to b or sum from a to b of $f(x)$

The notion of primitive permits the definition of the integral of a continuous function and constitutes a tool for calculating the integral. However, the Equation (1) is not explained and the ostensive dx seems to have no status. Further on, it is mentioned that in the Equation (2) ¸ the letter x is a dummy variable that can be replaced by any other letter:

$$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(u)du \tag{4}$$

In all the textbooks, the following is included in addition to the Equation (2) the Equation (5), as equal to the Equation (3).

$$\left[F(x)\right]_a^b \tag{5}$$

This variation Equation (5) seems more explicit than the Equation (1). In the approach activity adopted for the introduction of the notion of integral in the M2 and M3, the notion of sum from a to b of Equation (6) is not mentioned.

$$f(x)dx \tag{6}$$

This makes more sense with harmony to the choice of the primitive function as an approach. However, this notion of summation is parachuted into the M1 textbook given the absence of any mention of the origin of the symbol ∫ which could possibly explain it.

These results of our analysis of the three textbooks have challenged us as trainers of future teachers and researchers: The transition from the Equation (3) to the Equation (2) is parachuted without demonstration and without any possibility to interpret the meaning of this equality and the meaning of the Equation (2) Indeed¸ what is the interest of introducing the notion of integral by the notion of primitive if one cannot justify this choice and specify the roles and the sense of the ostensive constituting this expression such as the notation dx and the sum from a to b of $f(x)dx$? Why mention the sum of a to b of $f(x)dx$? What is the meaning of the unit of area mentioned later when calculating areas?

These observations don't seem to be in favor of an efficient teaching-learning and a making meaning of the mathematical notions studied. It's seeming rather favorize a fragmented teaching-learning and based on calculative techniques. Such approaches support the regurgitation of isolated facts and hinder the development of students' reasoning skills and mobilization of what they have learned to understand concepts.

Moreover, the relationship between integral, primitive and area are not well explained. We notice also the exploitation of only one register, in this case algebraic register, in the new notion' definition whereas the comprehension of a conceptual content as specified by Duval [19] relies on the coordination of at least two registers of representation.

For all these considerations¸ we opted for another activity to introduce the integral allowing to make more sense while integrating ICT and at least two semiotic registers, namely the geometric and algebraic register.

3.4. Suggested Introductory Activity

The activity we propose could, and this is really our main objective¸ give more meaning to the symbols associated with the integral object and the Equation (2) and to further explain the relationship between the integral and area concepts and between the integral and primitives. This activity has been experimented with future teachers of mathematics and the results are under analysis and development.

This activity is based on a problem situation from mechanics whose resolution mobilizes the Riemann sums of a function f on an interval [a, b] as well as the ICT via the use of dynamic geometry as Geogebra tool. These interdisciplinary aspects including the didactic continuity physics-mathematics are among our research priorities [10, 12].

3.4.1. Triggering Situation

The choice of the triggering situation is motivated by the principle of inter-didactic continuity between physics and mathematics since the students have already studied it in physics in the first year of high school where they had to determine the expression of the work of a force $W_{AB}\left(\vec{F}\right)$ on an interval [A, B].

The approach consists in first defining the work of a constant force $\vec{F}$ whose point of application moves from A to B, via the relation:

$$W_{AB}\left(\vec{F}\right)=\vec{F}\cdot\overrightarrow{AB}=\left\|\vec{F}\right\|\times\left\|\overrightarrow{AB}\right\|\times\cos\left(\vec{F},\overrightarrow{AB}\right) \tag{7}$$

However, this relation is no longer valid in the case where the force is not constant during the displacement. To determine the work of the force $\vec{F}$ we proceed to the dividing of the trajectory AB into n elementary displacements $\vec{dl}$ which can be considered to be rectilinear and for any of these displacements, the force can be considered as constant. Thus, the elementary work can be expressed as:

$$\delta W\left(\vec{F}\right)=\vec{F}\cdot\vec{dl} \tag{8}$$

The work W_{AB} of the force $\vec{F}$ is defined as being equal to the sum of the elementary works and we obtain the expression:

$$W_{AB}\left(\vec{F}\right)=\sum_{A}^{B}\delta W\left(\vec{F}\right)=\sum_{A}^{B}\vec{F}\cdot\vec{dl} \tag{9}$$

It should be noted that the notion of integral is not mentioned because at this school level it is not yet covered in mathematics.

3.4.2. Mathematical Modelling of the Problem

If we consider that the force $\vec{F}$ is described by the continuous function f defined on [a,b]. We chose one of the usual functions studied in mathematics, namely the

$$f(x)=\sqrt{x} \tag{10}$$

Determining the work done would be equivalent to finding the area of the part of the plane bounded by the vertical line x=a, x the vertical line x=b, the x-axis, and the graph of the function f reported to an orthonormal reference frame $\left(o,\vec{i},\vec{j}\right)$.

To do this, the interval [a,b] is divided into n sub-intervals of the same amplitude $\frac{b-a}{n}$ then, using the representative curve of f as a basis, we draw n rectangles "below" the curve and n rectangles "above" the curve of the same width $(b-a)/n$ thus the sums of the areas of these rectangles are defined by:

$$G_n=\sum_{i=0}^{n-1}f(x_i)\Delta x \tag{11}$$

$$D_n=\sum_{i=1}^{n}f(x_i)\Delta x \tag{12}$$

where, $\Delta x=(b-a)/n$ and $x_i=a+i\Delta x$, respectively.

Contrary to mathematical science students, learners don't have to demonstrate that:

$$\lim_{n\to+\infty}\sum_{i=1}^{n}f(x_i)\Delta x=\lim_{n\to+\infty}\sum_{i=0}^{n-1}f(x_i)\Delta x \tag{13}$$

but can be visualized by using Geogebra. Thus, learners realize that when n tends to $+\infty$ $\lim_{n\to+\infty}\sum_{i=1}^{n}f(x_i)\Delta x$ is equal to $\lim_{n\to+\infty}\sum_{i=0}^{n-1}f(x_i)\Delta x$ and the lower and upper areas correspond to the total area A sought. Then The teacher leads the students to notice that the width of the subintervals becomes very small and tends to 0, which is noted dx. To estimate the area under the curve, we use the sum of the areas of the approximating rectangles:

$$A=\sum_{a}^{b}f(x)dx \tag{14}$$

Approximate the total area is based on the principle of the summing the areas of the approximating rectangle. This summation is symbolized by the symbol $\int$ whose origin comes from the Latin word summa. This mathematical symbol represents integration and called the sign of the integral. At this level, the teacher could introduce the notion of integral by replacing the sign $\sum$ with the sign $\int$ in units of area:

$$A=\sum_{a}^{b}f(x)dx=\int_{a}^{b}f(x)dx \tag{15}$$

The structured integration of ICT in teaching-learning, in this case Geogebra, can favorite socioconstructivist approaches and give meaning to the modeling process and strengthen students' understanding of mathematics.

A study on the effectiveness of Geogebra using in students 'understanding about the circle notion shows a significant increase in experimental group conceptual understanding compared with the control group [20]. Concerning the unit, it can be further clarified by referring to the initial mechanics situation:

$$W_{AB}(\vec{F}) = \int_a^b Fdl \cos(\vec{F}.\overrightarrow{dl}) \tag{16}$$

In this case, the unit of area is newton-meter (N.m) or joule. It's defined as the work done by the force of one newton causing a displacement of one meter. Note that in Equation (15) the variable of integration is x and in Equation (16) the variable is l.

3.5. Integral and Primitive

Once the notion of integral is defined, it is possible to establish the relationship between integral and primitive by using Geogebra software which allows the learners to realize that the Equation (1) is valuable whatever x. We can also establish this relationship by showing that for a function f continuous on an interval $[a, b]$ the function F defined on $[a, b]$ by Equation (17) is the primitive of f which cancels at a. Then, we show the Equation (1).

$$F(x) = \int_a^x f(t)dt \tag{17}$$

Thus, the use of Geogebra could bring mathematic to life and support and facilitate teaching and learning as shown by many researches [14, 15, 16, 17, 18, 20]. In fact, Arbain, et al. [15], based on the results of their study, highly recommend the use of GeoGebra software because of its positive effects on the performance of the experimental group ($p < 0.05$) compared to the control group. The same is observed for enthusiasm, confidence and motivation of the learners.

However, the use of Geogebra requires a structured environment to avoid regression in student performance. In this sense, Nouhou and Jaillet [18] conducted a study with three homogeneous groups having initially a low level of knowledge related to the numerical function with real variables. It concerns a control group GC that benefited from a traditional course and two experimental groups GE1 and GE2 that benefited from the use of Geogebra but in different environments The results reveal a relevant progression in the learning of the group GE2 with which Geogebra has been used in a structured way compared to the second group GE1. In addition, the results of the latter group decreased in comparison with those of the control group GC.

4. CONCLUSION AND PERSPECTIVIES

In this research, we analyzed the approach and process adopted in three Moroccan mathematics textbooks to introduce the notion of the integral. Our main objective was to analyze, among others, the pertinence of introducing this notion from the primitive notion.

The main findings show that the approach and process adopted don't seem to favor coherent and meaningful teaching and learning. Indeed, the fact of defining the Equation (2) from the Equation (3), f is a continuous function on the interval $[a, b]$ and F is the primitive of the function f without explicating the meaning of the ostensive of the Equation (2). The Equation (1) is to be admitted and this according to the official pedagogical guidelines. Concerning the proposed application exercises, one notes a predominance of the calculation type.

We also deplore the introduction of a new notion via a single semiotic register and without integration of ICT. Moreover, in this context, the area approach seems to us relevant since it allows to exploit more than one semiotic register for a better understanding of the notion of integral and to explain the meaning of the symbol $\int$, of dx as well as the unit of area or those of the physical quantities mobilizing the notion of integral.

Based on these observations, the approach activity we have developed in this article was designed by considering the interdisciplinary continuity between physics and mathematics, the mobilization of the algebraic and graphical registers and the integration of ICT. Ensuring didactic continuity between mathematics and physics would promote decompartmentalization and coordination between Moroccan teachers of both disciplines

Experimentation with learners would require authorization as teachers are obliged to conform to official guidelines which recommend the adoption of the primitive approach to introduce the notion of the integral. However, we have experimented it with a sample of trainee mathematics teachers and the results are being developed.

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